ASYMPTOTIC NORMALIZATION COEFFICIENT FOR $^{13}\mathrm{C}+n\to^{14}\mathrm{C}$ FROM THE $^{13}\mathrm{C}(t,d)^{14}\mathrm{C}$ REACTION

K.I. Tursunmakhatov

Gulistan State University, Gulistan 120100, Uzbekistan
tursunmahatovkahramon@gmail.com

E.Sh. Ikromkhonov

Institute of Nuclear Physics, Tashkent 100214, Uzbekistan
and
Tashkent State Medical University Chirchik branch, 111700 Chirchik, Uzbekistan

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The analysis of the differential cross section of the $^{13}\mathrm{C}(t,d)^{14}\mathrm{C}$ reaction measured at an incident energy of 38 MeV was performed within the framework of the Modified Distorted Wave Born Approximation. The peripheral character of the reaction was tested. It was shown that the considered reaction is peripheral at the main maximum of the angular distribution of the reaction under consideration. The squared ANC value for the $^{13}\mathrm{C}+n\to{}^{14}\mathrm{C}$ configuration was found to be 15.50 ± 0.55 fm^{-1}.

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1. Introduction

The first thorough review of the nuclear vertex constant (NVC) and its relationship with the asymptotic normalization coefficient (ANC) was presented in Ref. [1]. That work also introduced the theoretical foundation of the ANC and discussed its role in the nuclear reaction theory. In Ref. [2], the role of the ANC in the theory of nuclear reactions with charged particles was given. The residue in the poles of the elastic scattering S-matrix corresponding to bound states [3] and in resonances [4] can be expressed in terms of the ANC in the scattering theory.

The asymptotic normalization coefficient is a fundamental nuclear characteristic of bound states [1] and the partial resonance width is expressed in terms of the ANC [5]. The ANC plays an important role in low-energy elastic scattering, nuclear transfer, and radiative capture reactions [6]. The experimentally determined ANCs can be used to calculate the astrophysical S factor of the peripheral radiative capture reactions [7].

The radiative capture $^{13}\mathrm{C}(n,\gamma)^{14}\mathrm{C}$ reaction plays an important role in nucleosynthesis both in stars and the early universe [8]. In the nucleosynthesis in stellar helium burning, it is supposed that the $^{13}\mathrm{C}(n,\gamma)^{14}\mathrm{C}$ reaction may act as a neutron poison for the slow neutron capture process of the heavy element synthesis [9]. ANC for the $^{13}\mathrm{C}+n\rightarrow{}^{14}\mathrm{C}$ configuration extracted from the $^{13}\mathrm{C}(t,d)^{14}\mathrm{C}$ process could be used for the prediction of characteristics of the radiative capture $^{13}\mathrm{C}(n,\gamma)^{14}\mathrm{C}$ reaction. In this regard, it is of great interest to extract the value of ANC for the $^{13}\mathrm{C}+n\rightarrow{}^{14}\mathrm{C}$ configuration.

In this study, we extracted the ANC value for the $^{13}\mathrm{C}+n\rightarrow{}^{14}\mathrm{C}$ configuration from the analysis of the peripheral neutron transfer reaction $^{13}\mathrm{C}(t,d)^{14}\mathrm{C}$, which was measured in [10], within the framework of the Modified Distorted Wave Born Approximation (MDWBA)[11].

2. Analysis of the $^{13}\mathrm{C}(t,d)^{14}\mathrm{C}$ reaction

2.1. Basic formulas of the modified DWBA

In the Modified Distorted Wave Born Approximation (MDWBA), the differential cross section in the region of the main peak of the angular distribution for the peripheral transfer reaction $A(x,y)B$ ($x\rightarrow y+a$ and $B\rightarrow A+a$) is parameterized in terms of the product of the squares of ANCs, and the differential cross section (DCS) has the form (see Refs. [12, 13] and references therein)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=C^2_{aA;\,l_B,\,j_B}R\left(E,\theta;b_{ya;\,l_x,\,j_x},b_{aA;\,l_B,\,j_B}\right)\,. \tag{1}$$

Here,

$$R\left(E,\theta;b_{ya;\,l_x,\,j_x},b_{aA;\,l_B,\,j_B}\right)=C^2_{ya;\,l_x,\,j_x}\frac{\sigma^{\mathrm{DWBA}}_{l_x,\,l_B,\,j_B}\left(E,\theta;b_{ya;\,l_x,\,j_x},b_{aA;\,l_B,\,j_B}\right)}{b^2_{ya;\,l_x,\,j_x}\;b^2_{aA;\,l_B,\,j_B}}\,. \tag{2}$$

The peripheral character of the considered reaction in the region of the main peak of the angular distribution can be formulated by the condition [11]

$$R\left(E,\theta;b_{ya;\,l_x,\,j_x},b_{aA;\,l_B,\,j_B}\right)=\mathrm{const.} \tag{3}$$

as a function of free parameters $b_{ya;\,l_x,\,j_x}=b_{ya;\,l_x,\,j_x}(r_0,a)$ and $b_{aA;\,l_B,\,j_B}=b_{aA;\,l_B,\,j_B}((r_0,a)$ which, in turn, are functions of the geometric parameters of the radius and diffusion of the Woods–Saxon potential, for all scattering angles and a fixed value of the energy E. If condition (3) is satisfied, then the value of ANC for the $A+a\rightarrow B$ vertex can be determined from the condition

$$C^2_{aA;\,l_B,\,j_B} = \frac{\left(\frac{\mathrm{d}\sigma^{\exp}}{\mathrm{d}\Omega}\right)_{\theta=\theta_{\mathrm{peak}}}}{R\left(E,\theta;b_{ya;\,l_x,\,j_x},b_{aA;\,l_B,\,j_B}\right)} = \mathrm{const.}, \tag{4}$$

which must be fulfilled for fixed energy E, scattering angle θ, and the corresponding function $R(E,\theta;b_{ya;\,l_x,\,j_x},b_{aA;\,l_B,\,j_B})$ from Eq. (3).

We note that the spectroscopic factor $Z_{aA;\,l_B,\,j_B}$, which is the norm of the radial overlap function of the bound state B wave function in the (A+a) channel, is related to the ANC $C_{aA;\,l_B,\,j_B}$ by the equation [1]

$$C_{aA;\,l_B,\,j_B} = Z^{1/2}_{aA;\,l_B,\,j_B}\,b_{aA;\,l_B,\,j_B}\,. \tag{5}$$

2.2. Analysis of the ^{13}C$(t,d)^{14}$C reaction within the modified DWBA

The DCS of the neutron transfer reaction $^{13}\mathrm{C}(t,d)^{14}\mathrm{C}$ populating the ground state of ^{14}C was measured at a tritium bombarding energy of 38 MeV [10], and the spectroscopic factor was extracted from the experimental data using the DWBA calculation. In this section, we performed the analysis of the DCS of the considered reaction. In the $^{14}\mathrm{C} = {}^{13}\mathrm{C} + n$ system, the transferred neutron has orbital momentum $l_{n^{13}\mathrm{C}} = 1$ for the ground state of ^{14}C, and the corresponding total angular momentum is equal to $j_{n^{13}\mathrm{C}} = 1/2$. The orbital and total angular momenta of the transferred neutron in the $t = d + n$ system are equal to $l_{dn} = 0$ and $j_{dn} = 1/2$, respectively.

The calculations were performed using the DWUCK5 code within the Nuclear Reaction Video (NRV) program [15]. The ANC value for the $d + n \to t$ configuration was taken to be 4.29 ± 0.1 fm^{-1} from Ref. [16]. The sets of optical potential (OP) for the entrance and exit channels are given in Table 1. The parameters of OP for the entrance $^{13}\mathrm{C} + t$ channel, labelled A1–A3, were taken from Ref. [10]. The parameters of OP for the exit $^{14}\mathrm{C} + d$ channel were taken from Refs. [10, 18, 19] and they are labelled B1–B3.

The peripheral character of the considered reaction was checked in two ways. First, we change the cutoff radius which is the lower limit in the radial integration over the distance between the colliding particles. We made calculations at different values of the cutoff radius for all sets of OPs from Table 1. The dependence of the differential cross section of the considered reaction on the cutoff radius only for the OP set (A1+B1) at an angle of $\theta = 11.38^\circ$ is presented in Fig. 1. The results of calculations show that the DCS at $r_{\mathrm{cut}} \leq 4$ fm is not sensitive to the variation of the cutoff radius. The second, we systematically varied the parameters r_0 and a of the adopted Woods–Saxon (W–S) potential within the ranges $1.10 \leq r_0 \leq 1.40$ fm and $0.50 \leq a \leq 0.80$ fm, relative to the standard values $r_0 = 1.25$ fm and $a = 0.65$ fm. For each (r_0, a) pair, the depth of the W–S potential was adjusted to reproduce the neutron binding energy.

Table 1. The parameters of OPs for the initial and exit channels for the $^{13}C(t,d)^{14}C$ reaction. The depth of the potentials is given in MeV and the geometry parameters are in fm.

Parameters	A1	A2	A3	B1	B2	B3
V_0	305.0	199.6	276.8	68.9	69.62	73.05
r_0	1.10	1.10	1.13	1.25	1.25	1.174
a_0	0.603	0.664	0.591	0.700	0.752	0.809
W			21.28			2.226
r_W			1.434			1.563
a_W			0.867			0.808
W_D	19.89	17.12		10.2	13.0	11.79
r_D	0.966	1.25		1.25	1.25	1.328
a_D	0.847	0.723		0.700	0.689	0.573
V_{SO}					6.0	3.703
r_{SO}					1.25	1.234
a_{SO}					0.743	0.813
r_C	1.11	1.11	1.30	1.11	1.30	1.698
Refs.	[10]	[10]	[10]	[10]	[18]	[19]

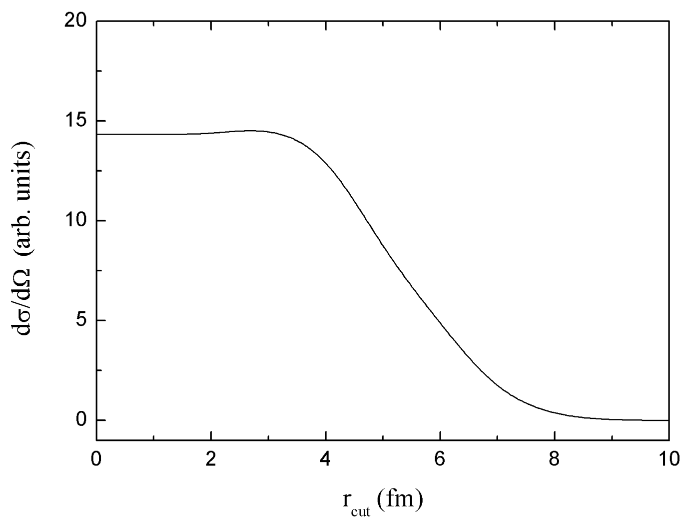

Fig. 1. The dependence of the DCS of the $^{13}C(t,d)^{14}C$ reaction on the cutoff radius. The calculations have been performed with the geometric parameters of the W–S potential $r_0 = 1.25$ fm and $a = 0.65$ fm for OP set (A1+B1) at angle $\theta_{\rm peak} = 11.38°$.

Variations of the parameters r_0 and a result in changes of the single-particle ANC within the interval $2.328 \leq b_{n^{13}\mathrm{C};1\,1/2} \leq 4.204$ fm$^{-1/2}$ for the ground state of the ^{14}C nucleus. Figure 2 illustrates the dependence of the $R(\theta, b_{n^{13}\mathrm{C};1\,1/2})$ function on the single-particle ANC $b_{n^{13}\mathrm{C};1\,1/2}$ for the $^{13}\mathrm{C}(t,d)^{14}\mathrm{C}$ reaction. The width of the band reflects the minor residual dependence of the $R(\theta, b_{n^{13}\mathrm{C};1\,1/2})$ function on the parameters r_0 and a. For instance, the arithmetic mean value of the $R(\theta, b_{n^{13}\mathrm{C};1\,1/2})$ function is equal to 0.92 mb/sr fm at an angle $\theta = 11.38°$. Variations of the parameters r_0 and a cause R function to change in the range of $0.87 \leq R(\theta, b_{n^{13}\mathrm{C};1\,1/2}) \leq 0.946$ mb/sr fm.

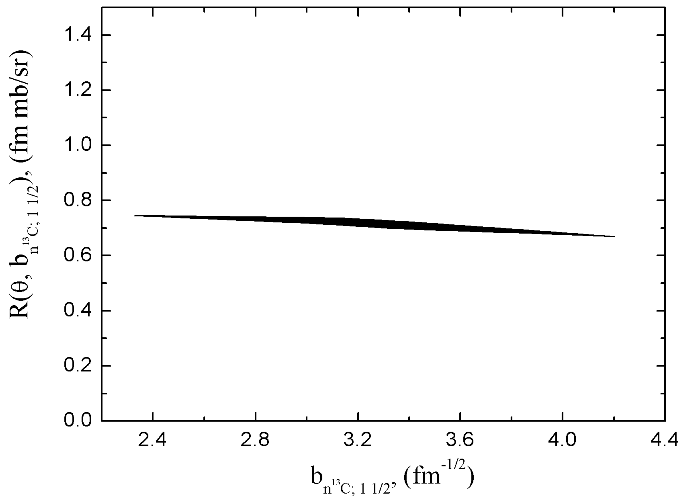

Fig. 2. The dependence of the $R(\theta, b_{n^{13}\mathrm{C};1\,1/2})$ function on the single-particle ANC $b_{n^{13}\mathrm{C};1\,1/2}$ at energy of 38 MeV using OP set (A1+B1) at the angle $\theta_{\mathrm{peak}} = 11.38°$. The bandwidth in the graphs corresponds to the variation of the geometric parameters r_0 and a within the ranges $1.10 \leq r_0 \leq 1.40$ fm and $0.50 \leq a \leq 0.80$ fm.

Figure 3 shows the dependence of the ANC $C^2_{n^{13}\mathrm{C};1\,1/2}$ and the spectroscopic factor $Z_{n^{13}\mathrm{C};1\,1/2}$ on the single-particle ANC $b_{n^{13}\mathrm{C};1\,1/2}$ at $\theta_{\mathrm{peak}} = 11.38°$. The ANC $C^2_{n^{13}\mathrm{C};1\,1/2}$ varies slightly within the narrow range of $15.15 \leq C^2_{n^{13}\mathrm{C};1\,1/2} \leq 16.47$ fm^{-1} when changing the Woods–Saxon potential parameters r_0 and a within the above-mentioned interval. This implies that ANC is relatively insensitive to the single-particle ANC variations. In contrast, the spectroscopic factor $Z_{n^{13}\mathrm{C};1\,1/2}$ changes significantly in the range of $0.932 \leq Z_{n^{13}\mathrm{C};1\,1/2} \leq 2.795$ under the same variations of the parameters r_0 and a. This indicates a strong dependence of $Z_{n^{13}\mathrm{C};1\,1/2}$ on the single-particle ANC $b_{n^{13}\mathrm{C};1\,1/2}$. Similar trends, namely weak ANC dependence and strong sensitivity of the spectroscopic factor, are observed at other experimental angles. In summary, the results demonstrate the reliability of the ANC determination for the $^{13}\mathrm{C} + n \to ^{14}\mathrm{C}$ configuration from the transfer reaction $^{13}\mathrm{C}(t, d)^{14}\mathrm{C}$, while highlighting the strong model dependence that affects the extraction of the spectroscopic factor. For all optical potential sets listed in Table 1, condition (2) for the $R(\theta, b_{n^{13}\mathrm{C};1\,1/2})$ function is fulfilled at angles within the main peak of the angular distribution when the Woods–Saxon potential parameters r_0 and a are varied across the above-mentioned ranges. The $R(\theta, b_{n^{13}\mathrm{C};1\,1/2})$ function value fluctuates between 1.91% and 2.03% considering all sets of OPs. The results indicate that the reaction under study is peripheral at the main peak of the angular distribution. Therefore, the experimental DCS for the $^{13}\mathrm{C}(t, d)^{14}\mathrm{C}$ reaction can be employed to extract the values of ANC $C^2_{n^{13}\mathrm{C};1\,1/2}$ in a manner that is independent of the geometric parameters (r_0 and a) of the W–S potential.

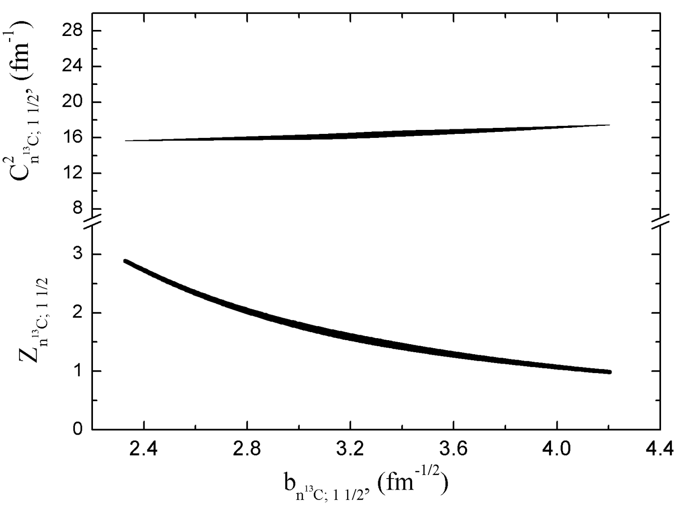

Fig. 3. The behavior of the $C^2_{n^{13}\mathrm{C};1\,1/2}$ [in upper panel] and $Z_{n^{13}\mathrm{C};1\,1/2}$ [in lower panel] functions on the single-particle ANC $b_{n^{13}\mathrm{C};1\,1/2}$ at an energy of 38 MeV using the optical potential set (A1+B1) at the angle $\theta_{\mathrm{peak}} = 11.38°$. The width of the bands corresponds to the variation of the geometric parameters r_0 and a within the ranges $1.10 \le r_0 \le 1.40$ fm and $0.50 \le a \le 0.80$ fm.

2.3. Determination of ANC for $^{13}C + n \to ^{14}C$ configuration

By normalizing the calculated DCS within the framework of MDWBA to the experimental one at the forward angles $\theta < 20°$, we extracted the values of the ANC for the $^{13}\mathrm{C}+n \to ^{14}\mathrm{C}$ configuration. For each of the four experimental points of angles θ, the value of ANCs for the $^{13}\mathrm{C}+n \to ^{14}\mathrm{C}$ configuration is determined by using relation (3) and the central value $R(\theta, b_{n^{13}\mathrm{C};1\,1/2})$ function, corresponding to the standard values of the parameters r_0 and a of the adopted W–S potential ($r_0 = 1.25$ fm and $a = 0.65$ fm).

The calculated values of the squared ANCs for the $^{13}\mathrm{C} + n \rightarrow {}^{14}\mathrm{C}$ configuration are presented in Fig. 4 for each experimental data point in the main peak of the angular distribution, considering all sets of OPs. Additionally, the weighted ANCs for the $^{13}\mathrm{C} + n \rightarrow {}^{14}\mathrm{C}$ configuration associated with each OP set are summarized in Table 2. The uncertainty in the ANC values represents the root mean square error derived from the combined uncertainties of the calculated R function, which depends on the free parameter $b_{n^{13}\mathrm{C};1\,1/2}$, alongside the experimental DCS error. Our recommended weighted average ANC value, obtained from OP sets listed in Table 1, is $C^2_{n^{13}\mathrm{C};1\,1/2} = 15.50 \pm 0.55\ \mathrm{fm}^{-1}$ for the $^{14}\mathrm{C}$ nucleus ground state.

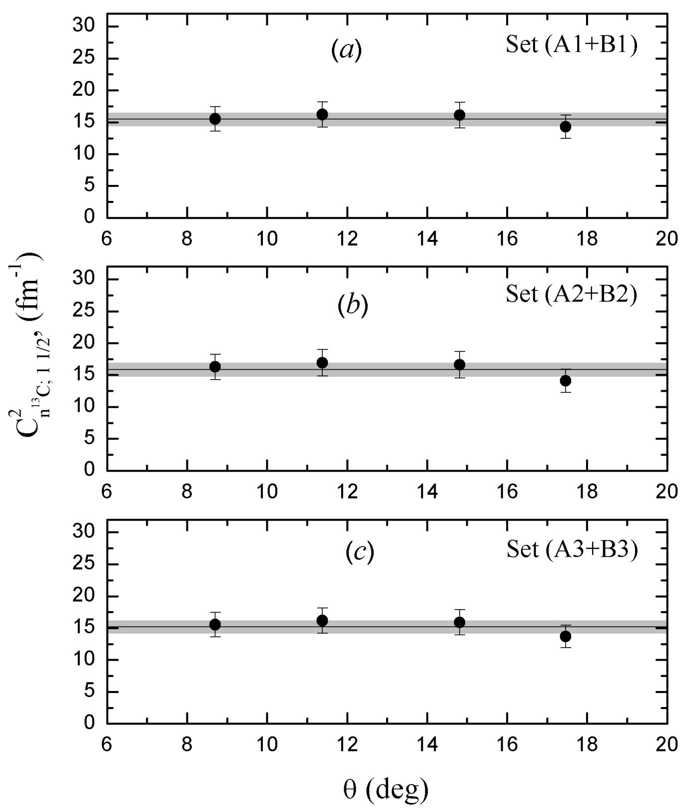

Fig. 4. The values of ANCs for the $^{13}\mathrm{C}+n \rightarrow {}^{14}\mathrm{C}$ configuration for each experimental point of angles in the main peak of the angular distribution. The band widths are the corresponding weighted uncertainties.

Table 2. The weighted values of ANCs for the ^{13}C+$n \to ^{14}$C configuration, obtained from the analysis of the peripheral reaction ^{13}C$(t,d)^{14}$C for different sets of OPs. ANCs are given in fm^{-1}.

Sets	$C^2_{n^{13}\mathrm{C};1\,1/2}$
Set (A1+B1)	15.47 ± 0.96
Set (A2+B2)	15.85 ± 0.96
Set (A3+B3)	15.85 ± 0.99
Weighted mean value	15.50 ± 0.55

It should be noted that the ANC values for the $^{13}\mathrm{C} + n \to ^{14}\mathrm{C}$ configuration have been extracted from the experimental data for the first time.

The weighted ANC values listed in Table 2 for each set of optical potentials (OPs) were employed to calculate the DCSs using equation (1). The calculated DCS results for the ^{13}C$(t,d)^{14}$C reaction, populating the ground state of the ^{14}C nucleus, are presented in Fig. 5 alongside the corresponding experimental data from Ref. [10]. The curves, which are shown in Fig. 5, represent calculations performed within the framework of the MDWBA, utilizing the weighted ANC values from Table 2, across different OP sets listed in Table 1. As demonstrated in the figure, the calculated DCSs within the framework of MDWBA exhibit good agreement with the experimental data, particularly within the main peak region of the angular distribution.

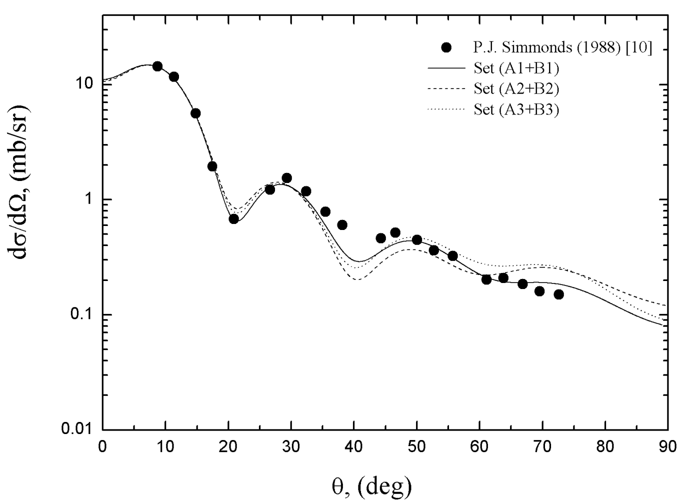

Fig. 5. The DCSs for the ^{13}C$(t,d)^{14}$C reaction populating the ground (0^+; $E^* = 0$ MeV) state of the ^{14}C nucleus at an energy of 38 MeV. The experimental points are taken from [10]. The solid, dashed, and dotted lines are the calculated DCSs corresponding to set (A1+B1), set (A2+B2), and set (A3+B3), respectively.

3. Conclusion

The analysis of the differential cross section of the $^{13}\mathrm{C}(t,d)^{14}\mathrm{C}$ reaction measured at an incident energy of 38 MeV was performed within the framework of the Modified Distorted Wave Born Approximation. The peripheral character of the reaction was tested in two ways. It was shown that the considered reaction is peripheral at the main maximum of the angular distribution of the reaction under consideration. It was established that the spectroscopic factor demonstrates a significant sensitivity to the parameters of the Woods–Saxon potential. The squared ANC for the $^{13}\mathrm{C}+n \to {}^{14}\mathrm{C}$ configuration was found to be 15.50 ± 0.55 fm^{-1}.

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