

EVIDENCE OF ISOSPIN-SYMMETRY VIOLATION IN HIGH-ENERGY COLLISIONS OF ATOMIC NUCLEI: THEORETICAL AND PHENOMENOLOGICAL CONSIDERATIONS

WOJCIECH BRYLINSKI 

Warsaw University of Technology, Warsaw, Poland

MAREK GAZDZICKI , FRANCESCO GIACOSA [†]


Jan Kochanowski University, Kielce, Poland

MARK GORENSTEIN 

Bogolyubov Institute for Theoretical Physics, Kyiv, Ukraine

ROMAN POBEREZHZNIUK 

University of Houston, Houston, Texas, USA

SUBHASIS SAMANTA 

Department of Physics, School of Applied Sciences
Kalinga Institute of Industrial Technology, Bhubaneswar, Odisha, India

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Recently, the NA61/SHINE Collaboration at the CERN SPS reported evidence of isospin-symmetry violation in high-energy nuclear collisions. The effect was observed in the relative yields of charged and neutral kaons and cannot be explained by known sources of isospin-symmetry breaking. In this work, we extend the theoretical and phenomenological aspects of that study. We discuss the historical background and introduce the concepts of isospin transformations and symmetry. Importantly, we relate isospin symmetry to the QCD flavour symmetry, and we present both conceptual and analytical proofs demonstrating the equality of the mean multiplicities of charged and neutral kaons for an initial ensemble of colliding systems that is invariant under charge-symmetry transformation.

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[†] Corresponding author: francesco.giacosa@ujk.edu.pl

1. Introduction

In May 2025, the NA61/SHINE Collaboration at the CERN SPS published a significant excess of charged over neutral kaon production in high-energy nucleus–nucleus collisions [1]. For completeness, we recall the key plot of the paper (see Fig. 1), which shows the ratio of charged-to-neutral kaons as a function of collision energy. The NA61/SHINE results are presented alongside a compilation of worldwide data.

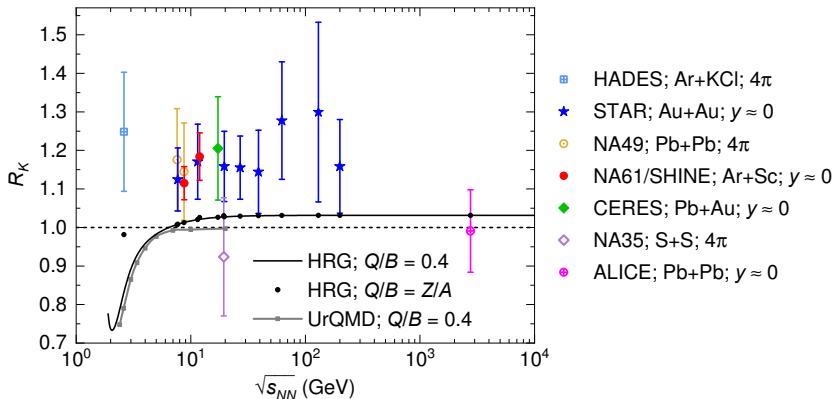


Fig. 1. Ratio R_K of charged-to-neutral K meson yields in nucleus–nucleus collisions as a function of collision energy. The lines show model predictions; see below for explanations. Figure adapted from Ref. [1] with a new preliminary Ar+Sc 8.8 GeV point added [2].

The largely experimental paper included a theoretical interpretation of the results in terms of evidence for the violation of isospin symmetry beyond known effects. The latter part of the paper was formulated in collaboration with a group of theoretical colleagues. This paper extends the theoretical and phenomenological aspects of the experimental discovery reported in *Nature Communications*.

The paper is organised as follows. Section 2 briefly overviews the history of the isospin concept, whereas an introduction to the isospin transformation and symmetry is presented in Section 3. In Section 4, we recall that the isospin symmetry follows from the flavour symmetry of up and down quarks. The method used by NA61/SHINE for testing the isospin symmetry by checking the charge-symmetry invariance in the ensemble of charge-uniform collisions is elaborated in Sections 5 and 6. In particular, we present the conceptual and analytical proofs of the method. In Section 7, we estimate the charge-symmetry violation (CSV) within the Hadron Resonance Gas (HRG) model, and we discuss the formally analogous case of D meson production. Finally, in Section 8, we present closing remarks and outlook.

2. Historical background

In 1932, Heisenberg suggested that the neutron and the proton could be seen as different manifestations of the same particle, the nucleon [3], explaining their similar masses and interaction properties. Wigner introduced the concept of ‘isotopic spin’, or ‘isospin’ of nucleons, following the formal similarity with the more familiar spin [4] and postulated invariance (symmetry) of strong interactions with respect to rotations in the isospin space. Today, isospin is an important element of nuclear physics and nuclei classification [5]. The concept of isospin was promptly extended to the other strongly interacting particles, the hadrons. In particular, Kemmer [6] applied it to the pion triplet postulated by Yukawa in 1935 [7]. In general, when considering the hadron mass distribution, hadrons form groups (isospin multiplets) with members having similar masses but different electric charges. This suggests that strong interactions predominantly determine the masses and strong interaction properties of hadrons, whereas electromagnetic — and, to a much smaller degree, weak — interactions give rise to only small corrections. For a historical outline, see Refs. [8, 9].

With the continuous discovery of new hadron species, it has become clear that they are not elementary particles of strong interactions. The quark model of hadron classification proposed by Gell-Mann [10] and Zweig [11] in 1964 started a 15-year-long term in which sub-hadronic particles, quarks and gluons were discovered, and a theory of their interactions, Quantum Chromodynamics (QCD) [12] was established. There are six types (flavours) of quarks: up (u), down (d), strange (s), charm (c), bottom (b), and top (t) with their QCD masses increasing by five orders of magnitude. Each quark carries a colour charge (red, green, or blue), while the gluons are the gauge bosons of QCD and mediate the colour interactions.

Within QCD, hadrons are viewed as confined bags of quarks and gluons. They are further classified into mesons with integer spin, typically quark–antiquark states such as pions and kaons, and baryons with semi-integer spin, typically three-quark states such as protons and neutrons. However, to date, attempts to derive quantitative QCD predictions for hadron properties and multi-hadron production in high-energy collisions have not been successful. Predictions of QCD-based and QCD-inspired models suffer from large uncertainties. For recent review, see Ref. [13]. In Section 4, we argue that the exception here is the isospin symmetry and, in particular, its part called charge-symmetry invariance.

Within QCD, the hadron isospin symmetry is considered as a result of the flavour symmetry postulated by QCD. It means that gluons interact ‘democratically’ with all quark flavours. But the results of the interactions depend heavily on quark masses. However, the two of them,

up and down quarks, have masses close to zero ($m_u = 2.16 \pm 0.07$ MeV and $m_d = 4.70 \pm 0.07$ MeV [14]) in comparison to the typical strong interaction scale ($\Lambda_{\text{QCD}} \approx 250$ MeV [15]). Thus, one traces the hadron isospin symmetry to the ud -flavour symmetry¹ of QCD [16, 17]. In the limit where the u and d quarks can be considered massless (the so-called chiral limit), the interactions do not depend on their chirality, leading to a larger symmetry called chiral symmetry. The latter is spontaneously broken in the QCD vacuum, generating (quasi-)Goldstone bosons, pions, kaons, and the η -meson [17, 18].

Experiments show that the isospin symmetry is well fulfilled in pion–pion and pion–nucleon elastic or quasi-elastic scatterings [19, 20]. Moreover, the smallness of isospin-breaking decays (for example, $\eta' \rightarrow \pi\pi\pi$ [21, 22]) is observed.

The above suggests that the isospin symmetry should also be well obeyed in hadron production processes. The results of NA61/SHINE [1] seem to contradict this expectation [23].

3. Isospin transformation and symmetry

In order to formulate the isospin symmetry of strong interactions, one introduces the isospin vector and the isospin transformation. This is done at the level of the system’s quantum state, representing either an individual hadron or many hadrons. The isospin transformation is a rotation of the isospin vector in the isospin space. It is analogous to a rotation of the spin vector in 3-dimensional space, but acting on states related by the (approximate) $SU(2)$ isospin symmetry of the strong interaction — the state multiplets.

More specifically, the isospin operator triplet $\hat{\mathbf{I}} = (\hat{I}_x, \hat{I}_y, \hat{I}_z)$ refers to an $SU(2)$ internal rotation within a given multiplet. A generic isospin rotation by the angle θ_j takes place via the operator

$$\hat{O} = e^{i\theta_j \hat{I}_j}. \quad (1)$$

Importantly, while isospin I refers to a multiplet of states, I_z is a property of a single state within this multiplet. Moreover, I_z can be inferred from the measured hadron properties using the Nishijima–Nambu–Gell-Mann (NNG) formula [24, 25], which for light hadrons reads

$$I_z = Q - (B + S)/2, \quad (2)$$

where Q , B , and S are the electric charge, baryon number, and strangeness of the assembly of measured hadrons. In general, a given value of I_z obtained

¹ The textbooks refer to this symmetry as the quark isospin symmetry. Here, to avoid a possible confusion whether the symmetry is meant at the quark or hadron level, we call it the ud -flavour symmetry.

from the experimental data corresponds to a distribution of I values within the range

$$I_{\min} \leq I \leq I_{\max}, \quad (3)$$

where $I_{\min} = |I_z|$ and I_{\max} is the maximum total isospin when all isospin vectors are aligned. This makes the experimental test of full isospin symmetry significantly more difficult than the tests of its I_z -related sub-symmetry, the so-called charge symmetry.

The charge-symmetry transformation is obtained by setting $\theta_1 = \theta_3 = 0$ and $\theta_2 = \pi$ in Eq. (1). The corresponding operator reads

$$\hat{C} = e^{i\pi\hat{I}_y}. \quad (4)$$

When applied to a hadronic state, this operator inverts the I_z component of isospin.

Let us start by considering examples for a single hadron. Here, the measurement of hadron mass identifies the multiplet, allowing us to assign both I and I_z to the underlying state of each measured hadron.

The simplest isospin multiplet is realised for isospin singlets, also denoted as isoscalar. For example, there is only one isospin state for the ω meson and for the Λ hyperon ($I = 0$).

The pair proton (p) and neutron (n) forms an ‘isodoublet’ ($I = 1/2$), where p has eigenvalue $I_z = 1/2$ and n has $I_z = -1/2$. This also holds for the pair K^+ and K^0 . Thus, an isospin transformation on the doublet $(p, n)^T$ mixes proton and neutron and is obtained by setting $\hat{I}_j = \tau_j/2$, with τ_j being the Pauli matrices. The antinucleon isodoublet $(-\bar{p}, \bar{n})^T$ transforms just as the nucleon one (note the minus sign [26]).

An isospin triplet ($I = 1$), for example, the pion triplet, is represented by a column vector

$$\vec{\pi} = \begin{pmatrix} -\pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}.$$

Then an isospin transformation is described by identifying the isospin operators \hat{I}_j with the 3×3 matrices \hat{J}_j . These matrices realise the spin-1 irreducible representation of $SU(2)$, also called the adjoint representation.

The charge-symmetry transformation, Eq. (4), replaces a hadronic state $|I, I_z\rangle$ by its multiplet partner $|I, -I_z\rangle$. This results in swapping between measured hadrons as $p \leftrightarrow n$, $\pi^+ \leftrightarrow \pi^-$, $K^+ \leftrightarrow K^0$, $\bar{K}^0 \leftrightarrow K^-$, etc.

We now turn to assemblies of two or more hadrons and atomic nuclei. The selected examples will be used later in the paper.

- A. Firstly, we consider an example of two nucleons. For two protons, using Eqs. (2) and (3), one gets $I_z = +1$ and $I = 1$ ($I_{\min} = I_{\max} = 1$). Similarly, for the two-neutron state, one gets $I_z = -1$ and $I = 1$. Thus, in both cases, I is uniquely defined. However, for a proton–neutron state, the situation is more complicated: here $I_z = 0$, but $I_{\min} = 0$ and $I_{\max} = 1$, allowing for two possible values of the total isospin, $I = 0$ and $I = 1$. Consequently, the $|p, n\rangle$ and $|n, p\rangle$ states are not eigenstates of total isospin, as they are linear combinations of the singlet ($I = 0$) and triplet ($I = 1$) components. The charge-symmetry transformation changes the $|p, p\rangle$ state into the $|n, n\rangle$ one and *vice versa*, and similarly maps $|p, n\rangle$ into $|n, p\rangle$. This means that an ensemble of $|p, p\rangle$ and $|n, n\rangle$ states with an equal number of each is invariant under charge symmetry.
- B. Secondly, we discuss the case of an atomic nucleus with equal numbers of protons and neutrons², $Z = N$, $A = Z + N$. For a single nucleus, one gets $I_z = 0$ and $0 \leq I \leq (Z + N)/2$. Thus, the state of two nuclei $|A, A\rangle$ has $I_z = 0$. It is transformed into itself by the charge-symmetry transformation independently of its I value. These states are charge-symmetry invariant. This property will later be used to test charge-symmetry invariance in nucleus–nucleus collisions. From the phenomenology of nuclear physics follows that, for almost all nuclei of the $Z = N$ type, the total isospin is zero [27]. Then the $|A, A\rangle$ state has $I_z = I = 0$. This would allow for the extension of tests of isospin symmetry beyond charge symmetry by studying nucleus–nucleus collisions.
- C. Thirdly, we consider $|\pi, A\rangle$ states for $Z = N$ nuclei. The $|\pi^+, A\rangle$ state has $I_z = 1$, whereas the $|\pi^-, A\rangle$ state has $I_z = -1$. Similar to the $|p, p\rangle$ and $|n, n\rangle$ states, the charge-symmetry transformation swaps $|\pi^+, A\rangle$ and $|\pi^-, A\rangle$ states. The ensemble of equal numbers of $|\pi^+, A\rangle$ and $|\pi^-, A\rangle$ states is invariant under charge-symmetry transformation, whereas each state is not.

Isospin symmetry refers to the invariance of the strong interaction between hadrons under global SU(2) rotation in isospin space. The transition probability from a given initial state to a given final state equals the transition probability between the states obtained by applying the isospin transformation to these states. This symmetry has many empirical consequences, for example:

² In terms of conserved quantities, the condition $Z = N$ is given by $Q/B = 1/2$, where $Q = Z$ and $B = A$ are the nuclei's electric charge and baryon number, respectively.

- (i) The elastic cross sections of $p + p$ and $n + n$ scattering should be the same.
- (ii) The mean multiplicity of K^+ mesons produced in $p + p$ interactions is equal to the mean multiplicity of K^0 mesons produced in $n + n$ interactions

$$\langle K^+ \rangle_{pp} = \langle K^0 \rangle_{nn} . \quad (5)$$

Many additional relations of this kind follow from the same symmetry principle.

4. Isospin symmetry from ud -flavour symmetry

In QCD, the fundamental degrees of freedom are quarks and gluons. Restricting ourselves to the two lightest flavours (u and d), the QCD Lagrangian reads

$$\mathcal{L}_{\text{QCD}} = \bar{u} (i\cancel{D} - m_u) u + \bar{d} (i\cancel{D} - m_d) d + \dots , \quad (6)$$

where $\cancel{D} = \cancel{D} - ig_{\text{QCD}}\gamma^\mu A_\mu$, with A_μ being the gluon field. If the up and down quarks were mass-degenerate ($m_u = m_d$), this part of the Lagrangian would be invariant under global SU(2) flavour rotations mixing the u and d states

$$\begin{pmatrix} u \\ d \end{pmatrix} \longrightarrow \hat{O} \begin{pmatrix} u \\ d \end{pmatrix}, \quad \hat{O} \in \text{SU}(2). \quad (7)$$

The pair u and d forms an ‘isodoublet’, where the quark u has eigenvalue $I_z = 1/2$ and d has $I_z = -1/2$ (see *e.g.* Ref. [16] and the quark summary in the PDG [14]). Thus, the ud -flavour transformation on the doublet $(u, d)^T$ mixes both quarks and is obtained by setting $\hat{I}_j = \tau_j/2$, with τ_j being the Pauli matrices. The antiquark isodoublet $(-\bar{d}, \bar{u})^T$ transforms just as the quark one (note the minus sign [26]). Since hadrons are colour-singlet bound states of quarks and gluons, they inherit this symmetry: the spectrum of hadrons is arranged into isospin multiplets such as the nucleons (p, n) or the pions (π^+, π^0, π^-).

The fact that the ud -flavour transformation translates into the one for isodoublets can be seen by using, for example, microscopic currents used in lattice calculations [28], Bethe–Salpeter approaches [29], and effective models of QCD [30, 31].

The ud -flavour symmetry of the QCD Lagrangian means that certain Noether currents and charges are conserved. Namely, for the global SU(2) transformation of the light quark doublet, the Noether currents are given by

$$J_i^\mu = \bar{q} \gamma^\mu \frac{\tau_i}{2} q ,$$

where $q^T = (u, d)$. Their divergences follow from the QCD equations of motion,

$$\partial_\mu J_i^\mu = \bar{q} i \overleftrightarrow{D} \frac{\tau_i}{2} q = (m_u - m_d) \bar{q} \frac{\tau_i}{2} q.$$

Hence, in the exact ud -flavour symmetry limit $m_u = m_d$, the divergences vanish and the currents are conserved. Turning back to hadrons, isospin symmetry implies that only transitions preserving the total isospin I and its third component I_z are allowed. This property is the already mentioned isospin conservation.

In particular, when the initial and final states are eigenstates of both I and I_z , the consequences of the symmetry are straightforward. However, in heavy-ion collisions, the initial state is typically an eigenstate of I_z (since Q , B , and S are conserved event-by-event) but not necessarily an eigenstate of I . In such cases, isospin conservation must be interpreted with some care.

If the initial state is a *classical mixture* of components with different total isospin values I , the symmetry applies separately to each component, and the composition of the mixture remains unchanged. If, instead, the initial state is a *quantum superposition* of several states with distinct values of I , the interaction cannot mix them. Consequently, the relative weights of the different I components of the superposition are independent of time, and observables depending on I are conserved at the level of expectation values.

The QCD u and d quark masses extracted from the experimental data³ are different but small. They are much smaller than Λ_{QCD} . Thus, one argues that ud -flavour symmetry is well obeyed. Yet, a closer inspection of the link between the approximate ud -flavour and isospin symmetries shows that other features of QCD are also needed to obey isospin symmetry approximately. We discuss them below.

- a. Neglecting at first the Goldstone bosons of QCD (pions, kaons, ...), the masses of the light hadrons can be successfully described by the constituent quark model approach with $m_u^* \simeq m_d^* \sim \Lambda_{\text{QCD}} \sim 250$ MeV, see Refs. [32, 33]. For instance, the masses of conventional $\bar{q}q$ mesons, such as the $\rho(770)$ meson, can be described as bound states of a constituent quark and a constituent antiquark: their mass is proportional to the sum of the constituent quark masses that build it. As shown by microscopic models, such as the Nambu–Jona-Lasinio one [30] and

³ The QCD ‘current’ quark masses $m_u = 2.16 \pm 0.07$ MeV and $m_d = 4.70 \pm 0.07$ are not measured directly but are obtained by fitting experimental data on hadron properties [14]. Light-quark masses are fitted within lattice QCD and chiral perturbation theory, while heavy-quark masses come from perturbative QCD analyses of quarkonium and collider data. They are always quoted in a particular renormalisation scheme.

Bethe–Salpeter approaches [34], the difference of the constituent quark masses is proportional to the difference of QCD masses, $m_d^* - m_u^* \simeq m_d - m_u$, thus suppressed. For example, the physical $\rho(770)$ -meson masses lead to the mass difference $(m_{\rho^+} - m_{\rho^0}) = -0.7 \pm 0.8$ MeV. Within this context, one can easily understand that $m_{u,d} \ll \Lambda_{\text{QCD}}$ implies that isospin symmetry is well fulfilled. The symmetry is also well respected in decays and scattering properties; see Refs. [31, 35].

- b. The pseudoscalar Goldstone bosons (pions, kaons, and $\eta(547)$) are special, since their mass squared is proportional to the QCD quark masses that form them. For charged pions, one has $m_{\pi^\pm}^2 \propto (m_u + m_d)$. For the four kaons, central to this work, one has

$$m_{K^-}^2 = m_{K^+}^2 = c_K(m_u + m_s), \quad m_{\bar{K}^0}^2 = m_{K^0}^2 = c_K(m_d + m_s), \quad (8)$$

where c_K is an appropriate constant that depends on the quark condensates, *e.g.* [31]. Thus, the similarity of K^+ and K^0 masses (and similarly K^- and \bar{K}^0) is reflected in the QCD masses of up, down, and strange quarks. In particular, the strange quark mass has to be large enough, $m_s = 93.5 \pm 0.8$ MeV [14]. For a cross-check, one may evaluate the ratio

$$\frac{m_{K^+}}{m_{K^0}} = \sqrt{\frac{m_u + m_s}{m_d + m_s}} = 0.9870 \pm 0.0005, \quad (9)$$

whose central value is close to the corresponding experimental value 0.99209 ± 0.00004 . Yet, due to the small errors, these values are not compatible. The reason is the small but decisive role of the electromagnetic interaction [36], which slightly increases the mass of K^+ (both constituents u and \bar{s} have positive charge), but slightly decreases the mass of K^0 (the constituents d and \bar{s} have opposite charge).

Using the statistical approach, one can estimate the ratio of mean multiplicities of charged and neutral kaons caused by the mass difference as

$$\frac{\langle K^+ \rangle}{\langle K^0 \rangle} \approx e^{-\frac{m_{K^+} - m_{K^0}}{T}} \approx 1.03, \quad (10)$$

where $T = 150$ MeV is the temperature parameter of the kaon production process. This estimate assumes direct kaon production. The detailed estimate of the effect, including resonance decays, is presented in Section 7. It is about several % at $\sqrt{s} \simeq 10\text{--}20$ GeV. Note that this estimate includes both strong and electromagnetic effects that modify vacuum masses of hadrons and their branching ratios (assuming they have been measured).

- c. A special remark about the neutral members of the pseudoscalar nonet is needed. The neutral pion has a mass $m_{\pi^0}^2 \propto (m_u + m_d)$ just as the charged ones. The isoscalar members $\eta_N = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_S = s\bar{s}$ have masses $m_{\eta_N}^2 \propto (m_u + m_d) + 2c_A$ and $m_{\eta_S}^2 \propto m_s + c_A$, as well as a mixing proportional to $\sqrt{2}c_A$. Here, c_A is a constant proportional to the $U(1)_A$ chiral (or axial) anomaly. The latter refers to a specific $U(1)_A$ -chiral-phase symmetry of quarks that is a symmetry of the classical QCD, which is broken by gluonic quantum fluctuations [18, 37]. Since the coefficient c_A is large (in fact, dominant compared to the contributions of the QCD quark masses), the masses of the isoscalar mesons η_N and η_S become much larger than that of the neutral pion π^0 . The states η_N and η_S effectively decouple from π^0 . Instead, η_N and η_S mix to form the physical mesons $\eta(547)$ and $\eta'(958)$. The latter is particularly massive and cannot be regarded as a Goldstone boson. The neutral pion π^0 is almost degenerate with the charged states π^\pm , forming an isospin triplet. As noted in Refs. [21, 38], the situation would have been different if the chiral anomaly had been small. In this case, the π^0 and the η_N states would be almost mass degenerate. A residual π^0 - η_N small mixing term, proportional to $(m_d - m_u)$, would generate a large mass splitting. The lighter state would have a mass of about 77 MeV. Thus, pions would not form an isospin triplet of almost degenerate masses. Colloquially, we may say that the large chiral anomaly protects isospin symmetry at the level of pions⁴. Importantly, the charge symmetry is not affected by the chiral anomaly.

The relation between the phenomenological isospin and the QCD ud -flavour symmetries can be summarised as follows:

- A. The exact flavour symmetry of up and down quarks ($m_u = m_d$) results in the exact isospin symmetry of hadrons.
- B. Taking the QCD quark masses leads to a small breaking of the ud -flavour symmetry, which in turn causes a small breaking of isospin symmetry. In particular, the charge symmetry is weakly affected.
- C. The magnitude of isospin-symmetry violation cannot be calculated exactly in QCD. For mean kaon multiplicities, the estimate is approximately several percent; see Section 7 for details.

⁴ This is not the case for other nonets. Even if the chiral anomaly is non-zero, it is never the dominating component as for the η' -meson [39, 40].

5. Charge-uniform ensembles and kaons

The simplest conceptual test of charge symmetry in kaon production is given by Eq. (5), which for completeness we repeat here: $\langle K^+ \rangle_{pp} = \langle K^0 \rangle_{nn}$. One needs to measure the mean multiplicity of K^+ mesons in proton–proton interactions and compare it with the corresponding measurement for the K^0 mesons in neutron–neutron interactions. The K^+ ($u\bar{s}$) and K^0 ($d\bar{s}$) mesons are charge-symmetry partners. At the quark level, they transform by swapping up and down quarks. Importantly, kaons are copiously produced in high-energy collisions.

However, this test is almost impossible to perform experimentally. Neither neutron beams nor neutron targets are easy to produce. Measurements of K^0 production are possible only through its strong interactions with the detector material. It is also a challenging task. In view of these difficulties and based on previous consideration [23], NA61/SHINE performed the test of the charge-symmetry invariance in the kaonic sector, measuring the ratio of charged ($K^+ + K^-$) to neutral ($K^0 + \bar{K}^0 = 2K_S^0$) kaons in nucleus–nucleus ($Z \approx N$) collisions. In this and the following sections, we elaborate on this test method.

5.1. Conceptual proof

The conceptual proof of the correctness of the NA61/SHINE test reads:

- (i) Assume the ensemble of initial states is invariant under charge-symmetry transformation. It means the ensemble has an equal population of charge-symmetry partners. It is the charge-uniform ensemble. Here, the important example is the ensemble of two colliding nuclei, each with equal numbers of protons Z and neutrons N , $Z = N = A/2$. For instance, this can be an ensemble of $^{16}\text{O} + ^{16}\text{O}$ states. As discussed in Section 3, point B, each state of the ensemble has $I_z = 0$, and thus it is invariant under charge transformation. Obviously, the ensemble of these states is charge-uniform. The second example is the ensemble of an equal number of $\pi^+ + C$ and $\pi^- + C$ systems. As discussed in Section 3, point C, the states have $I_z = \pm 1$. Thus, the overall ensemble is charge-uniform, despite each initial state, taken separately, not being charge-symmetry invariant. The experimental study of $^{16}\text{O} + ^{16}\text{O}$ as well as $\pi^+ + C$ and $\pi^- + C$ collisions was recently initiated by NA61/SHINE [41].
- (ii) Assume that strong interactions are charge-symmetry invariant; see Section 3 for details.
- (iii) From the assumptions (i) and (ii), it trivially follows that the ensemble of final states is charge-symmetry invariant (charge uniform).

A charge-symmetric initial ensemble yields a charge-symmetric final ensemble. This means that the mean multiplicities of charge-symmetry partners have to be equal

$$\langle K^+ \rangle = \langle K^0 \rangle, \quad (11)$$

and

$$\langle K^- \rangle = \langle \bar{K}^0 \rangle. \quad (12)$$

Thus,

$$\langle K^+ \rangle + \langle K^- \rangle = \langle K^0 \rangle + \langle \bar{K}^0 \rangle, \quad (13)$$

where, as shown in Section 6, the sum of neutral kaon multiplicities can be replaced by the $2\langle K_S^0 \rangle$.

5.2. Analytical proof

Here, we present an alternative proof of Eqs. (11) and (12) based on the quantum-mechanical formalism. For simplicity, we restrict to the $A + A$ collisions with $N = Z = A/2$.

In terms of the statistical operator $\hat{\rho} = \sum_n p_n |\Psi_n\rangle \langle \Psi_n|$ describing the initial state of $A + A$ collisions of two nuclei with $N = Z = A/2$, and thus $I_z = 0$, invariance under charge-transformation \hat{C} implies the equality $\hat{C}\hat{\rho}\hat{C}^\dagger = \hat{\rho}$. Here, p_n represents the probability for a specific initial state $|\Psi_n\rangle$. As usual, $\text{Tr}[\hat{\rho}] = \sum_n p_n = 1$.

The time-evolution of the statistical operator reads $\hat{\rho}(t) = \hat{U}(t)\hat{\rho}\hat{U}(t)^\dagger$, with $\hat{U}(t) = e^{-i\hat{H}_{\text{QCD}}t}$. We introduce the kaonic number operators \hat{N}_{K^+} and \hat{N}_{K^0} , which are related by the charge-symmetry operator as $\hat{N}_{K^+} = \hat{C}^\dagger\hat{N}_{K^0}\hat{C}$. Finally, using the charge symmetry of strong interactions, $[\hat{U}(t), \hat{C}] = 0$, the kaon mean multiplicities at a given time $t \geq 0$ read

$$\begin{aligned} \langle K^+ \rangle &= \text{Tr}[\hat{\rho}(t)\hat{N}_{K^+}] = \text{Tr}[\hat{U}(t)\hat{\rho}\hat{U}^\dagger(t)\hat{N}_{K^+}] = \text{Tr}[\hat{U}(t)\hat{\rho}\hat{U}^\dagger(t)\hat{C}^\dagger\hat{N}_{K^0}\hat{C}] \\ &= \text{Tr}[\hat{U}(t)\hat{C}\hat{\rho}\hat{C}^\dagger\hat{U}^\dagger(t)\hat{N}_{K^0}] = \text{Tr}[\hat{U}(t)\hat{\rho}\hat{U}^\dagger(t)\hat{N}_{K^0}] \\ &= \text{Tr}[\hat{\rho}(t)\hat{N}_{K^0}] = \langle K^0 \rangle, \end{aligned} \quad (14)$$

obeying Eq. (11) for the ensemble of $A + A$ collisions at any later time t . Similarly, one proves Eq. (12), $\langle K^- \rangle = \langle \bar{K}^0 \rangle$.

5.3. Comments on isospin-uniform ensembles

The conceptual proof (Section 5.1) of the relations (11) and (12) shows similarity to the setup considered by Shmushkevich [42, 43] in the mid-1950s.

The setup allowed him to formulate an easy method for relating cross sections for various processes, following from the invariance of strong interactions under isospin transformations; see also Refs. [16, 44, 45]. The main idea is that an initial ‘isospin-uniform’ ensemble of hadronic states (that is, one with an equal mean number of each member of any isospin multiplet) evolves into an isospin-uniform final-state ensemble.

Here, we illustrate the Shmushkevich method considering a simple example of $\rho(770)$ ($I = 1$) meson decays. If initially we have equal number of $\rho(770)^+$, $\rho(770)^0$, and $\rho(770)^-$ independent states, the corresponding statistical operator reads

$$\hat{\rho} = \frac{1}{3} (|\rho(770)^+\rangle \langle \rho(770)^+| + |\rho(770)^0\rangle \langle \rho(770)^0| + |\rho(770)^-\rangle \langle \rho(770)^-|) . \quad (15)$$

As such, it is invariant under any isospin transformation $\hat{O} = e^{i\theta_j \hat{I}_j}$ with $\hat{O} \hat{\rho} \hat{O}^\dagger = \hat{\rho}$ for an arbitrary choice of the angles θ_j . With this initial ensemble, the multiplicities of the members of any isospin multiplet must be equal at all times t . This point can be shown by considering the isospin transformation $\hat{C}^{1/2} = e^{iI_y \pi/2}$, leading to⁵

$$\langle \pi^+ \rangle = \frac{1}{4} \langle \pi^+ \rangle + \frac{1}{2} \langle \pi^0 \rangle + \frac{1}{4} \langle \pi^- \rangle . \quad (16)$$

Utilising charge symmetry, $\langle \pi^+ \rangle = \langle \pi^- \rangle$, one gets $\langle \pi^+ \rangle = \langle \pi^0 \rangle$.

The discussion above is general. An initial ensemble $\hat{\rho}$ being isospin-uniform at $t = 0$

$$\hat{\rho} \text{ uniform} \iff \hat{O} \hat{\rho} \hat{O}^\dagger = \hat{\rho} \quad \forall \hat{O} , \quad (17)$$

remains isospin-uniform at any later time:

$$\hat{\rho}(t) = \hat{U}(t) \hat{\rho} \hat{U}^\dagger(t) \quad \text{is such that} \quad \hat{O} \hat{\rho}(t) \hat{O}^\dagger = \hat{\rho}(t) \quad \forall t \geq 0 , \quad (18)$$

because $[\hat{U}(t) = e^{-i\hat{H}_{\text{QCD}} t}, \hat{O}] = 0$ is a consequence of isospin symmetry of the underlying Hamiltonian.

In a nucleus–nucleus collision, the initial states are, in general, not isospin-uniform, see Section 3. Only if both nuclei have vanishing isospin, the initial state has $I = 0$ and is trivially uniform⁶. For example, the

⁵ Under this transformation, $u \rightarrow (u + d)/\sqrt{2}$, $d \rightarrow (-u + d)/\sqrt{2}$. Note, a number operator for a certain isospin multiplet member r is given by $\hat{N}_r = \sum_{\mathbf{p}} a(\mathbf{p}, r)^\dagger a(\mathbf{p}, r)$, with \mathbf{p} being the momentum. Hence, a transformation with arbitrary θ_3 but with $\theta_{1,2} = 0$ assures that all expectations of mixed products $\langle a(\mathbf{p}, r)^\dagger a(\mathbf{p}, l) \rangle$ vanish for $r \neq l$. Then, under $\hat{C}^{1/2}$, the operator N_{π^+} leads to Eq. (16).

⁶ Within the vector meson analogy, the case with $I = 0$ corresponds to $\hat{\rho} = |\omega(782)\rangle \langle \omega(782)|$, where $|\omega(782)\rangle$ is an isosinglet state [14].

ensemble of $I = 1$ and $I_z = 0$ states is not isospin-uniform. Namely, the uniformity would require having, besides the $I_z = 0$ member, also the ones with $I_z = \pm 1$, which, however, are not present in the system since only $I_z = 0$ is allowed for $N = Z$ nuclei. Thus, we cannot predict that each member of an isospin multiplet is produced with equal probability (for example, one cannot show that $\langle \pi^+ \rangle = \langle \pi^0 \rangle$).

Using the $\rho(770)$ -meson example, the case encountered in nucleus–nucleus collisions with $Q/B = 1/2$ and $I = 1$ is analogous to the charge-invariant statistical operator of the initial ensemble, the $\hat{\rho} = |\rho^0(770)\rangle\langle\rho^0(770)|$, where only the member with $I_z = 0$ is retained. This implies that the final ensemble of states is also charge-symmetry invariant, hence $\langle \pi^+ \rangle = \langle \pi^- \rangle$, $\langle K^+ \rangle = \langle K^0 \rangle$, $\langle K^- \rangle = \langle \bar{K}^0 \rangle$, $\langle p \rangle = \langle n \rangle$, but it does not imply that $\langle \pi^+ \rangle = \langle \pi^0 \rangle$. (In fact, $\rho^0(770)$ decays solely into $\pi^+\pi^-$, implying no π^0 production.)

In summary, the initial ensemble of nucleus–nucleus collisions with an equal number of protons and neutrons is the charge-uniform ensemble of states. It thus leads to the charge-uniform ensemble of final states. In general, these ensembles do not fulfil the requirement of isospin-uniform ensembles that is necessary for the Shmushkevich method to be valid, not even for $Q/B = 1/2$. Nevertheless, charge-symmetry invariance of the initial charge-uniform ensemble is sufficient for having the same mean multiplicities of charged and neutral kaons.

6. Neutral kaons and R_K

The neutral kaons are eigenstates of the strong interaction and oscillate ($K^0 \leftrightarrow \bar{K}^0$) due to the weak interaction. The weak eigenstates are the long-lived ‘ K -long’ K_L^0 and short-lived ‘ K -short’ K_S^0 , which decay into three and two pions with mean lifetimes of 15 m/c and 2.8 cm/c, respectively. By neglecting at first the small CP violation (see below), the weak and strong kaonic eigenstates are related by

$$\begin{pmatrix} |K_S^0\rangle \\ |K_L^0\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}, \quad (19)$$

hence, for the mean number of K_S^0 and K_L^0 , one gets

$$\langle K_S^0 \rangle = \frac{1}{2} \langle K^0 \rangle + \frac{1}{2} \langle \bar{K}^0 \rangle = \langle K_L^0 \rangle. \quad (20)$$

Thus, we have an equal number of long-lived and short-lived kaons, even if $\langle K^0 \rangle$ and $\langle \bar{K}^0 \rangle$ are different due to the non-zero baryon number of the initial state.

By combining Eqs. (11) and (12) and using Eq. (20), one finds

$$\langle K^+ \rangle + \langle K^- \rangle = 2 \langle K_S^0 \rangle, \quad (21)$$

where all multiplicities are well accessible to experiments. Consequently, the exact charge-symmetry prediction for the charged-to-neutral kaon ratio in nucleus–nucleus collisions with $Q/B = 1/2$ reads

$$R_K \equiv \frac{\langle K^+ \rangle + \langle K^- \rangle}{\langle K^0 \rangle + \langle \bar{K}^0 \rangle} = \frac{\langle K^+ \rangle + \langle K^- \rangle}{2 \langle K_S^0 \rangle} = 1. \quad (22)$$

This prediction serves as the reference for testing charge symmetry using results from the NA61/SHINE experiment [1] and previous experiments; see the next section.

Including CP-symmetry breaking into the mixing modifies the mixing matrix as [46]

$$\begin{pmatrix} |K_S^0\rangle \\ |K_L^0\rangle \end{pmatrix} = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} \begin{pmatrix} 1+\varepsilon & -(1-\varepsilon) \\ 1+\varepsilon & 1-\varepsilon \end{pmatrix} \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}, \quad (23)$$

which leads to

$$\langle K^0 \rangle + \langle \bar{K}^0 \rangle \simeq 2 \langle K_S^0 \rangle \left(1 - 2 \operatorname{Re} \varepsilon \frac{\langle K^+ \rangle - \langle K^- \rangle}{\langle K^+ \rangle + \langle K^- \rangle} \right). \quad (24)$$

In turn: the value of R_K ,

$$R_K = \frac{\langle K^+ \rangle + \langle K^- \rangle}{\langle K^0 \rangle + \langle \bar{K}^0 \rangle} \simeq \frac{\langle K^+ \rangle + \langle K^- \rangle}{2 \langle K_S^0 \rangle} \left(1 + 2 \operatorname{Re} \varepsilon \frac{\langle K^+ \rangle - \langle K^- \rangle}{\langle K^+ \rangle + \langle K^- \rangle} \right), \quad (25)$$

with $\operatorname{Re} \varepsilon \simeq 1.62 \times 10^{-3} \ll 1$. For instance, in the case of NA61/SHINE, one gets

$$R_K \simeq \frac{\langle K^+ \rangle + \langle K^- \rangle}{2 \langle K_S^0 \rangle} (1 + 0.001). \quad (26)$$

The bias due to CP-symmetry violation is about 0.1%.

7. Violation of charge-symmetry invariance by known effects

As discussed in Sections 3 and 4, isospin symmetry is an approximate symmetry of strong interactions. This is because the light quark masses, $m_u \neq m_d$, are different. In addition, the symmetry is explicitly broken by weak and electromagnetic processes. Violation due to CP-symmetry

breaking is negligible, as shown in the previous section. Electromagnetic effects have been discussed in Ref. [1]. While simple estimates suggest they are small, further studies will be required.

As pointed out in Section 5, the NA61/SHINE method tests the charge-symmetry invariance, which is a part of isospin symmetry. The experimental quantity used in the test is the ratio of charge-to-neutral kaons, R_K . Here, we extend the previous discussion in [1] about the R_K sensitivity on effects violating the charge-symmetry invariance within strong interactions. The quantitative calculations are performed within the statistical Hadron Resonance Gas model [47]. In doing so, the fact that typically $Q/B < 1/2$ must also be taken into account. For Ar+Sc collisions measured in NA61/SHINE, $Q/B = 0.459$, while for lead-lead collisions, $Q/B = 0.394$. Thus, the choice $Q/B = 0.4$ shall also be investigated in HRG calculations.

The section concludes with a discussion of charge-symmetry-invariance violation in the charm sector.

7.1. Violation in the kaon sector

Different u and d quark masses violate the ud -flavour symmetry and consequently charge symmetry. We list below the considered effects and quantify their impact on R_K using the HRG model [47]. In Ref. [1], the HRG results were cross-checked within the microscopic, UrQMD transport model [48–50].

- (i) In Fig. 2 (left), we have verified that for $Q/B = 0.5$, the ratio $R_K = 1$ is reproduced in the isospin-symmetric limit by the HRG. For this test, the masses of hadrons, resonances, and their decays were set to values that exactly obey isospin symmetry. For $Q/B = 0.4$ (the right panel), the isospin-symmetric HRG predicts an increase in R_K with collision energy that reaches one above 10 GeV. More neutrons than protons (more d than u quarks) in the initial ensemble favours neutral kaon production. The effect of the initial ensemble asymmetry diminishes at high energies due to the dominance of created up and down quarks over the constituent quarks. In this limit, the model predictions are similar to the $Q/B = 1/2$ case.
- (ii) Smaller masses of charged kaons than neutral ones, $m_{K^+} = m_{K^-} = 493.7$ MeV and $m_{K^0} = m_{\bar{K}^0} = 497.6$ MeV, lead to an increase of R_K resulting from direct kaon production by about 2%, see Fig. 2. This was estimated by removing resonances from the HRG particle list.

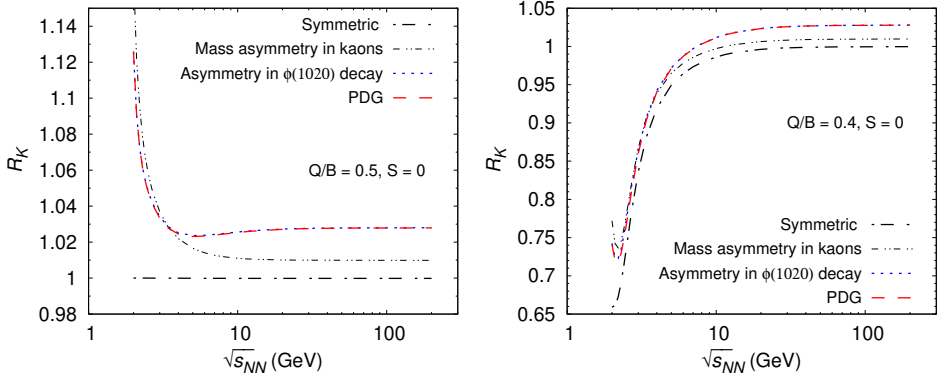


Fig. 2. Left: R_K as a function of $\sqrt{s_{NN}}$ calculated within HRG [47] for $Q/B = 0.5$ for different cases (from down upwards): assuming exact isospin symmetry: $R_K = 1$ as expected; including PDG kaon masses; including PDG $\phi(1020)$ branching ratios; including PDG data for all particles. Right: as in the left panel, but for $Q/B = 0.4$.

(iii) Different kaon masses impact the branching ratios of hadronic resonances decaying into kaons. The most relevant example is the $\phi(1020)$ meson, which decays twice as frequently to charged kaons as to neutral ones. This large difference is a consequence of the mass of $\phi(1020)$ being just above the threshold for decays into two kaons. Including the kaon production from resonance decays increases R_K by about 4%; see Fig. 2. This figure also shows that the ϕ meson decays have the largest effect on R_K among all known resonance decays.

(iv) In connection with the previous point, other potentially relevant $\bar{K}K$ decays refer to the resonances $a_0(980)$ and $f_0(980)$, whose masses are close to the kaon–kaon threshold. In the PDG table for both resonances, only $\bar{K}K$ is quoted. The standard HRG model has the same branching ratios to K^+K^- and $K^0\bar{K}^0$ channels. In particular, in the HRG used in this work, $a_0^0(980)$ has two decay channels: $\pi^0\eta$ (85%) and $K\bar{K}$ (15%), with 7.5% for K^+K^- and 7.5% for $K^0\bar{K}^0$. The resonance $f_0(980)$ has in HRG the decay channels $\pi^+\pi^-$ (52.7%), $\pi^0\pi^0$ (26.3%), and $K\bar{K}$ (21%), with 10.5% for K^+K^- and 10.5% for $K^0\bar{K}^0$. Yet charge-symmetry violation may be relevant for these resonances. Using the Flatté-like spectral functions as in Refs. [51–53] and taking into account the difference of the charged and neutral kaon masses, we find for both resonances a charged-to-neutral ratio of decay widths of about $K^+K^-/K^0\bar{K}^0 \simeq 1.1$. This ratio remains smaller than 1.2 when the parameters of the Flatté-like functions are varied within reasonable ranges (see *e.g.* the compilation of couplings in Ref. [54]). We

thus consider the HRG approach by assigning 20% more charged over neutral kaons for both resonances $a_0^0(980)$ and $f_0(980)$. In all cases, the ratio R_K changes by 0.36% at large $\sqrt{s_{NN}}$. We also considered the ‘extreme scenario’ where $a_0^0(980)$ and $f_0(980)$ decay only into charged kaons. It gives an upper limit of the impact of $a_0^0(980)$ and $f_0(980)$ decays on R_K . There is no significant effect for $\sqrt{s_{NN}} \lesssim 10$ GeV, and a slight (1.84%) increase of R_K at high energies, see Fig. 3.

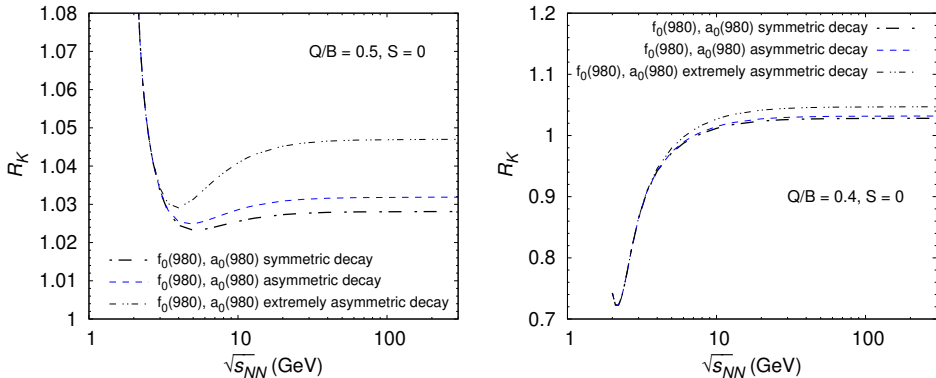


Fig. 3. Left: R_K as a function of $\sqrt{s_{NN}}$ calculated within HRG [47] for $Q/B = 0.5$ setting different branching ratios of $a_0(980)$ and $f_0(980)$ resonances (from down upwards): $a_0(980)$ and $f_0(980)$ with isospin-symmetric decays; $a_0(980)$ and $f_0(980)$ with realistic isospin-breaking branching ratios; $a_0(980)$ and $f_0(980)$ with $K\bar{K}$ mode going entirely into K^+K^- (unrealistic upper limit). Right: as in the left panel, but for $Q/B = 0.4$.

- (v) Mass differences of hadrons belonging to other isospin multiplets, as well as their decays into kaons, also break the charge symmetry and affect R_K . However, their impact is significantly smaller than the $\sim 4\%$ effect of ϕ decays; see Fig. 2.

7.2. Violation in the charm sector

Similar to kaons, exact charge-symmetry invariance predicts an equal production of charged mesons, D^+ and D^- , and neutral ones, D^0 and \bar{D}^0 . However, the production of charged D^\pm mesons in $h + A$ interactions is suppressed relative to D^0 and \bar{D}^0 [55]. A similar suppression was observed in nucleus–nucleus collisions at RHIC and at the LHC. For example, in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV (10–40% centrality), the STAR measurement finds mid-rapidity cross sections $d\sigma_{D^0}/dy \simeq 41 \mu\text{b}$ and $d\sigma_{D^\pm}/dy \simeq 18 \mu\text{b}$,

giving a yield ratio [56, 57]

$$R_D = \frac{\langle D^+ \rangle + \langle D^- \rangle}{\langle D^0 \rangle + \langle \bar{D}^0 \rangle} \approx 0.5. \quad (27)$$

At the LHC, the ALICE measurements of D^0 and D^+ production in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.024$ TeV show the ratio $\langle D^+ \rangle / \langle D^0 \rangle \approx 0.4\text{--}0.5$ [58], and it is compatible (within uncertainties) with the same ratio in $p + p$ interactions [59].

Following the same reasoning as in the kaon case, this is a manifestation of charge-symmetry breaking. However, in the charm case, there is a simple explanation for the breaking. The charged vector resonance $D^*(2010)^+$ decays into $D^+\pi^0$ and $D^0\pi^+$ with branching ratios of $30.7 \pm 0.5\%$ and $67.7 \pm 0.5\%$, respectively. Their ratio is close to 0.5, as expected from isospin-symmetry invariance. On the other hand, the resonance $D^*(2007)^0$ decays into $D^0\pi^0$ with a branching ratio of $64.7 \pm 0.9\%$, but the decay into its charged partner, $D^-\pi^+$, is forbidden because it is below the threshold. Exact isospin symmetry would imply a 1 : 2 ratio between $D^0\pi^0$ and $D^-\pi^+$ decays. The effect is qualitatively similar to $\phi(1020)$ resonance decay in the case of kaons. Quantitatively, the impact of D^* decays on the charged-to-neutral D ratio is much larger, since the resonance $D^*(2007)^0$ does not decay at all into D^+ . Moreover, this resonance has a significant ($35.3 \pm 0.9\%$) branching ratio to $D^0\gamma$, which also increases neutral D meson production w.r.t. charged D mesons. Similar considerations hold for the charge-conjugated states $D^*(2010)^+$ and $\bar{D}^*(2007)^0$. Summarising, neutral D mesons are significantly favoured (by a factor of about two) over charged ones.

In Fig. 4, we show the HRG result for R_D . At high energies, the model yields $R_D \simeq 0.45$, in good agreement with the experimental value $R_D \simeq 0.5$. This agreement indicates that the HRG framework provides a reliable baseline description for charmed mesons. The fact that the same framework fails to reproduce the charged-to-neutral kaon ratio highlights the non-trivial nature of the R_K puzzle.

We conclude this section by stressing that the known effects violating charge symmetry in strong interactions cannot explain the observed violation of charge symmetry between charged and neutral kaons. However, an even larger violation of the charged symmetry measured for D mesons can be explained by an asymmetry in D^* meson decays.

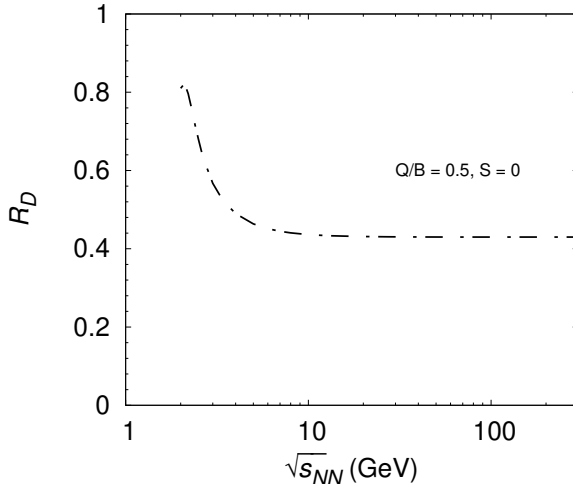


Fig. 4. Ratio R_D within HRG for $Q/B = 0.5$ and including resonances up to $D^*(2010)$. Note, using $Q/B = 0.4$ leads to negligible numerical differences.

8. Closing remarks

In this work, triggered by the novel experimental findings of Ref. [1] and discussion therein, we have investigated the ratio R_K of charged-to-neutral kaon yields in nucleus–nucleus collisions at high energies, both within the general theoretical QCD framework and within the Hadron Resonance Gas model.

In the limit of exact ud -flavour symmetry, the R_K ratio is unity if the initial state of nuclei has an equal number of protons and neutrons ($Q/B = 1/2$). Here, we have didactically clarified the origin of the result, presented a formal proof, and discussed differences to the Shmushkevich method.

The known charge-symmetry violation effects included in HRG yield $R_K \approx 1.03$ at high collision energies. Many experimental results correspond to collisions of heavy nuclei with more neutrons than protons ($Q/B \simeq 0.4$). This imbalance reduces R_K significantly only at low collision energies, where the ratio is lower than one. We thus conclude that the known charge-symmetry-violating effects cannot explain experimental results on nucleus–nucleus collisions, for which R_K is mostly within the 1.1–1.2 range.

The evidence for unexpectedly large CSV in the kaon sector is well established. It is, however, important that new experimental tests take place in the future. Measurements of the charged-to-neutral kaon ratio in collisions of an equal number of protons and neutrons would reduce interpretation uncertainties. In this respect, the NA61/SHINE Collaboration initiated an

investigation of the R_K ratio in oxygen–oxygen collisions. In addition, measurements of R_K in deuterium–deuterium interactions, the lightest nuclei with $I = 0$ and $Q/B = 1/2$ would also be important.

Another interesting system concerns pion–nucleon interactions, in particular $\pi^+ + C$ and $\pi^- + C$. Previous data on $\pi^- + C$ have shown that $R_K \simeq 1.2$ [60], in line with nucleus–nucleus collisions. Charge symmetry implies that the average R_K obtained by combining both measurements should be close to unity.

On the theoretical side, different directions can be taken. Concerning the HRG approach, one may argue that isospin-breaking contributions have not yet been included. This scenario seems, however, improbable: the higher the mass of a resonance, the smaller its multiplicity. Moreover, such resonances should exhibit a large, unexplained isospin-symmetry-breaking decay to account for the experimentally measured R_K .

One may also consider models that parametrise isospin-breaking processes and fit their parameters to the data. Along this line, the UrQMD approach has been modified by including violation of isospin symmetry [62]: the string breaking is parametrized as three times more probable to produce a $\bar{u}u$ pair than a $\bar{d}d$ one; in this way, the ratio R_K as well as BESIII data on kaon productions in e^+e^- scattering can be correctly described. Similarly, the quark coalescence model of Ref. [61] has been used to fit to R_K data, implying an excess of u/d quarks, and to make predictions such as the $p/n \simeq 1.2$ and $\Sigma^+/\Sigma^- \simeq 1.4$ ratios.

It is expected that, in the near future, both experimental and theoretical advances will help clarify the origin of the isospin kaon anomaly.

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REFERENCES

- [1] NA61/SHINE Collaboration (F. Giacosa, M. Gorenstein, R. Poberezhniuk, S. Samanta), «Evidence of isospin-symmetry violation in high-energy collisions of atomic nuclei», *Nat. Commun.* **16**, 2849 (2025).

- [2] Y. Balkova, T. Šuša, «News on strangeness production from the NA61/SHINE experiment», [arXiv:2508.14998 \[nucl-ex\]](#).
- [3] W. Heisenberg, «Über den Bau der Atomkerne. I (On the structure of atomic nuclei)», *Z. Physik* **77**, 1 (1932).
- [4] E. Wigner, «On the Consequences of the Symmetry of the Nuclear Hamiltonian on the Spectroscopy of Nuclei», *Phys. Rev.* **51**, 106 (1937).
- [5] D.D. Warner, M.A. Bentley, P.V. Isacker, «The role of isospin symmetry in collective nuclear structure», *Nat. Phys.* **2**, 311 (2006).
- [6] N. Kemmer, «The particle aspect of meson theory», *Proc. R. Soc. Lond. A* **173**, 91 (1939).
- [7] H. Yukawa, «On the Interaction of Elementary Particles I», *Proc. Phys. Math. Soc. Jap.* **17**, 48 (1935).
- [8] N. Kemmer, «Isospin», *J. Phys. Colloq.* **43**, 359 (1982).
- [9] R. Webb, «Spin's nuclear sibling», *Nat. Phys* **4 (Supl. 1)**, S8 (2008).
- [10] M. Gell-Mann, «A schematic model of baryons and mesons», *Phys. Lett.* **8**, 214 (1964).
- [11] G. Zweig, «An SU₃ model for strong interaction symmetry and its breaking. Version 2», in: D.B. Lichtenberg, S.P. Rosen (Eds.) «Developments in the Quark Theory of Hadrons», *Hadronic Press, Inc., Nonantum, Massachusetts, USA 1980*, pp. 22–101.
- [12] H. Fritzsch, M. Gell-Mann, H. Leutwyler, «Advantages of the color octet gluon picture», *Phys. Lett. B* **47**, 365 (1973).
- [13] M.S. Lukashov, Y.A. Simonov, «New ideas in nonperturbative QCD — I», [arXiv:2406.05584 \[hep-ph\]](#).
- [14] Particle Data Group (S. Navas *et al.*), «Review of Particle Physics», *Phys. Rev. D* **110**, 030001 (2024).
- [15] J.L. Kneur, A. Neveu, « $\Lambda_{\overline{MS}}^{\text{QCD}}$ from renormalization group optimized perturbation», *Phys. Rev. D* **85**, 014005 (2012).
- [16] P.B. Pal, «An Introductory Course of Particle Physics», *CRC Press*, 2014.
- [17] J.F. Donoghue, E. Golowich, B.R. Holstein, «Dynamics of the Standard Model, Vol. 2», *Cambridge University Press*, 2014, available at [Open Access publication on Cambridge Core](#).
- [18] G. 't Hooft, «How instantons solve the U(1) problem», *Phys. Rep.* **142**, 357 (1986).
- [19] M.R. Pennington, «Swimming with Quarks», *J. Phys.: Conf. Ser.* **18**, 1 (2005).
- [20] J.M. Alarcón, J. Martin Camalich, J.A. Oller, «The chiral representation of the πN scattering amplitude and the pion–nucleon sigma term», *Phys. Rev. D* **85**, 051503 (2012).
- [21] D.J. Gross, S.B. Treiman, F. Wilczek, «Light-quark masses and isospin violation», *Phys. Rev. D* **19**, 2188 (1979).

- [22] IHEP-IISN-LAPP Collaboration (F. Binon *et al.*), «The isospin-violating decay $\eta' \rightarrow 3\pi^0$ », *Phys. Lett. B* **140**, 264 (1984).
- [23] M. Gazdzicki, O. Hansen, «Hadron production in nucleon–nucleon collisions at 200 GeV/c — A compilation», *Nucl. Phys. A* **528**, 754 (1991).
- [24] T. Nakano, K. Nishijima, «Charge Independence for V-particles», *Prog. Theor. Phys.* **10**, 581 (1953).
- [25] M. Gell-Mann, «The interpretation of the new particles as displaced charge multiplets», *Nuovo Cim.* **4**, 848 (1956).
- [26] F. Halzen, A.D. Martin, «Quarks and Leptons: An Introductory Course in Modern Particle Physics», *John Wiley & Sons*, 1984.
- [27] S. Lenzi, M. Bentley, «Test of Isospin Symmetry Along the $N = Z$ Line», in: J. Al-Khalili, E. Roeckl (Eds.) «The Euroschool Lectures on Physics with Exotic Beams, Vol. III», Lecture Notes in Physics, Vol. 764, *Springer, Berlin, Heidelberg* 2009, pp. 57–98.
- [28] Hadron Spectrum Collaboration (J.J. Dudek, R.G. Edwards, P. Guo, C.E Thomas), «Toward the excited isoscalar meson spectrum from lattice QCD», *Phys. Rev. D* **88**, 094505 (2013).
- [29] P. Maris, C.D. Roberts, «Dyson–Schwinger Equations: A Tool for Hadron Physics», *Int. J. Mod. Phys. E* **12**, 297 (2003).
- [30] T. Hatsuda, T. Kunihiro, «QCD phenomenology based on a chiral effective Lagrangian», *Phys. Rep.* **247**, 221 (1994).
- [31] F. Giacosa, P. Kovács, S. Jafarzade, «Ordinary and exotic mesons in the extended Linear Sigma Model», *Prog. Part. Nucl. Phys.* **143**, 104176 (2025).
- [32] S. Godfrey, N. Isgur, «Mesons in a relativized quark model with chromodynamics», *Phys. Rev. D* **32**, 189 (1985).
- [33] W. Lucha, F.F. Schoberl, D. Gromes, «Bound states of quarks», *Phys. Rep.* **200**, 127 (1991).
- [34] C.S. Fischer, «Infrared properties of QCD from Dyson–Schwinger equations», *J. Phys. G: Nucl. Part. Phys.* **32**, R253 (2006).
- [35] P. Kovács, G. Wolf, N. Weickgenannt, D.H. Rischke, «Phenomenology of isospin-symmetry breaking with vector mesons», *Phys. Rev. D* **109**, 096007 (2024).
- [36] J.F. Donoghue, A.F. Perez, «Electromagnetic mass differences of pions and kaons», *Phys. Rev. D* **55**, 7075 (1997).
- [37] G. 't Hooft, «Symmetry Breaking Through Bell–Jackiw Anomalies», *Phys. Rev. Lett.* **37**, 8 (7 1976).
- [38] R.D. Pisarski, F. Wilczek, «Remarks on the chiral phase transition in chromodynamics», *Phys. Rev. D* **29**, 338 (1984).
- [39] F. Giacosa, S. Jafarzade, R.D. Pisarski, «Anomalous interactions between mesons with nonzero spin and glueballs», *Phys. Rev. D* **109**, L071502 (2024).
- [40] F. Giacosa, A. Koenigstein, R.D. Pisarski, «How the axial anomaly controls flavor mixing among mesons», *Phys. Rev. D* **97**, 091901 (2018).

- [41] NA61/SHINE Collaboration (K. Grebieskow *et al.*), «Report from the NA61/SHINE experiment at the CERN SPS», Technical report, CERN, Geneva, 2025.
- [42] I.M. Shmushkevich, «On deduction of relations between cross sections that arise from the hypothesis of isotopic invariance», *Dokl. Akad. Nauk SSSR* **103**, 235 (1955).
- [43] N. Dushin, I.M. Shmushkevich, «On the relations between cross sections which result from the hypothesis of isotopic invariance», *Dokl. Akad. Nauk SSSR* **106**, 801 (1956).
- [44] G. Pinski, A.J. MacFarlane, E.C.G. Sudarshan, «Shmushkevich’s Method for a Charge-Independent Theory», *Phys. Rev.* **140**, B1045 (1965).
- [45] C.G. Wohl, «Isospin relations by counting», *Am. J. Phys.* **50**, 748 (1982).
- [46] M. Thomson, «Modern Particle Physics», *Cambridge University Press*, New York 2013.
- [47] V. Vovchenko, H. Stoecker, «Thermal-FIST: A package for heavy-ion collisions and hadronic equation of state», *Comput. Phys. Commun.* **244**, 295 (2019).
- [48] S.A. Bass *et al.*, «Microscopic models for ultrarelativistic heavy ion collisions», *Prog. Part. Nucl. Phys.* **41**, 255 (1998).
- [49] M. Bleicher *et al.*, «Relativistic hadron–hadron collisions in the ultra-relativistic quantum molecular dynamics model», *J. Phys. G: Nucl. Part. Phys.* **25**, 1859 (1999).
- [50] M. Bleicher, E. Bratkovskaya, «Modelling relativistic heavy-ion collisions with dynamical transport approaches», *Prog. Part. Nucl. Phys.* **122**, 103920 (2022).
- [51] S.M. Flatté, «On the nature of 0^+ mesons», *Phys. Lett. B* **63**, 228 (1976).
- [52] V. Baru *et al.*, «Flatté-like distributions and the $a_0(980)/f_0(980)$ mesons», *Eur. Phys. J. A* **23**, 523 (2005).
- [53] F. Giacosa, A. Okopińska, V. Shastry, «A simple alternative to the relativistic Breit–Wigner distribution», *Eur. Phys. J. A* **57**, 336 (2021).
- [54] J.-J. Wu, B. S. Zou, «Study $a_0^0(980) - f_0(980)$ mixing from $a_0^0(980) \rightarrow f_0(980)$ transition», *Phys. Rev. D* **78**, 074017 (2008).
- [55] S. Frixione, M.L. Mangano, P. Nason, G. Ridolfi, «Heavy-Quark Production», *Adv. Ser. Direct. High Energy Phys.* **15**, 609 (1998).
- [56] STAR Collaboration (J. Vanek), «Measurement of D^\pm meson production in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV with the STAR experiment», *PoS (HardProbes2020)*, 065 (2021).
- [57] STAR Collaboration (G. Xie), «Measurement of D^0 Meson Production and Azimuthal Anisotropy in Au+Au Collisions at $\sqrt{s_{NN}} = 200$ GeV», *Nucl. Part. Phys. Proc.* **289–290**, 209 (2017).
- [58] ALICE Collaboration (S. Acharya *et al.*), «Measurement of D^0 , D^+ , D^{*+} and D_s^+ production in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV», *J. High Energy Phys.* **2018**, 174 (2018).

- [59] ALICE Collaboration (S. Acharya *et al.*), «Prompt D^0 , D^+ , and D^{*+} production in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV», *J. High Energy Phys.* **2022**, 174 (2022).
- [60] NA61/SHINE Collaboration (H. Adhikary *et al.*), «Measurement of hadron production in π^- – C interactions at 158 and 350 GeV/ c with NA61/SHINE at the CERN SPS», *Phys. Rev. D* **107**, 062004 (2023).
- [61] F. Giacosa, M. Rohrmoser, «Isospin kaon anomaly and its consequences», *Eur. Phys. J. C* **85**, 1058 (2025).
- [62] T. Reichert, J. Steinheimer, M. Bleicher, «Explanation of the observed violation of isospin symmetry in relativistic nucleus–nucleus reactions», *Phys. Lett. B* **873**, 140170 (2026).