

ANDREEV REFLECTION AND JOSEPHSON CURRENT AT HADRON–QUARK INTERFACE: A FIELD THEORETICAL APPROACH TO TRANSPORT PHENOMENA*

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In this manuscript, we show some interesting aspects of physics of interface. These are related to the phenomena occurring in the interior of neutron stars.

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1. Introduction

In nature, we have many boundaries and interfaces between different kinds of things. Then there are some interesting phenomena associated with those interfaces. We may say it in the following way. If we throw a ball at a wall, the ball might go through the wall. Or the ball reflected from the wall might not be identical to the original one, but become a new one. This new ball would carry the information about the wall. One of the goals in this kind of interest is to comprehend the transport phenomena in matter. Here, we especially focus on various phenomena happening at the interior of neutron stars (NS) and hypothetical quark stars (QS). The matter inside the neutron stars is strongly interacting and then can be described in terms of Quantum Chromodynamics (QCD). Thus, we need to know the phase structures of QCD. This is the issue of the QCD phase diagram [1].

It is a big challenge to explore the QCD phase diagram at nonzero temperature and density. In there, we expect various phases, phase transition curves, and even some critical points. Elucidating the QCD phase diagram

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is widely related to, for instance, physics of the early Universe after Big Bang, little bang known as quark–gluon plasma (QGP), and compact stars such as neutron stars.

However, when we try to investigate the QCD phase diagram, we face some difficulties, especially at nonzero chemical potential. One problem is the sign problem. We will encounter the problem in the evaluations of the path integral as well as the partition function. The other is that the system is strongly coupled, so that the perturbative method is not applicable. Due to these problems, it is hard to access the region at very low temperature and intermediate baryon density. The region is, however, very interesting because it is exactly the territory of neutron stars.

Recent observations of the mass and radius of neutron stars are quite important to pin down their internal structures. In particular, the NICER observations [2] made the simultaneous determination of the mass and radius of neutron star possible. Besides, there are recent observational developments of the gravitational waves from the binary neutron star mergers [3]. According to the observation of the tidal deformability, the matter inside neutron star should be neither too soft nor too hard. It is like a medium-boiled egg.

In this lecture, we are especially interested in the transport phenomena that would be expected to happen inside the neutron stars. Below, we will see several cases of the particle scatterings and transmissions at various interfaces. In Section 2, we introduce color superconductivity (CSC) and its important properties. In Section 3, we show the concept of quark–hadron continuity, which is relevant to physics of interface. In Section 4, we demonstrate some interesting examples of interfaces that would be associated with neutron stars. One is the Andreev reflection and the other is the Josephson tunneling current. In order to study them, we have developed various methods starting from a simple application using quantum mechanics to a new approach in terms of the non-equilibrium Green’s functions.

2. Color superconductivity

Color superconductivity is a possible phase which could be realized in the core of neutron stars [4]. The basic idea is that at very low temperature as well as very high density, the formation of quark–quark (diquark) pairing becomes energetically favorable so the QCD vacuum with non-vanishing diquark condensate is realized. The mechanism itself is very similar to that in the electron system, where an electron–electron pairing called the Cooper pairing exhibits superconductivity. The theory of superconductivity is described by the Bardeen–Cooper–Schrieffer (BCS) mechanism [5].

Unlike the case of electron superconductivity, however, in color superconductivity, since a quark has different degrees of freedom such as color and flavor, the pairing patterns become much more fruitful. In the two-flavor case (up and down quarks), we have the following pairing pattern called 2SC:

$$\Delta_{ab}^{ij} \equiv \langle \psi_a^i C \gamma_5 \psi_b^j \rangle = \Delta_{2\text{SC}} \epsilon^{ij} \epsilon_{abB}, \quad (1)$$

where i, j and a, b are the flavor and color indices, respectively. B shows the blue color. In the 2SC phase, only up and down quarks with red and green colors can be paired, and the blue quark does not participate in the pairing.

On the other hand, in the three-flavor case (up, down, and strange quarks), we have the following pairing pattern called color–flavor locking (CFL) [6]:

$$\Delta_{ab}^{ij} \equiv \langle \psi_a^i C \gamma_5 \psi_b^j \rangle = \Delta_{\text{CFL}} \epsilon^{ijI} \epsilon_{abI} = \Delta_{\text{CFL}} \left(\delta_a^i \delta_b^j - \delta_a^j \delta_b^i \right). \quad (2)$$

In the CFL phase, all the color and flavor degrees of freedom are equivalently paired and also those degrees of freedom are locked together through the Kronecker delta. This is the most symmetric ground state in color superconductivity. Apart from 2SC and CFL, lots of other types of condensates are proposed such as crystalline color superconductivity, color–spin locking (CSL), *etc.* In this paper, we simply consider the 2SC and CFL phases.

3. Quark–hadron continuity

As we discussed before, we could have various phases of QCD matter at low temperatures. Related to those phases, there is an interesting idea called Quark–Hadron continuity. This idea was originally proposed by Schäfer and Wilczek [7]. It states that the hadronic matter phase might be continuously connected to quark matter one with no phase transitions. Suppose that there is a chunk of sweet beans. They are considered hadrons. When they are mashed, they get into hadronic matter, which is partially homogeneous and partially inhomogeneous. When they are mashed more and more, eventually they get into quark matter, that is completely homogeneous. Since both nuclear and quark matters are originally sweet beans, they are essentially identical.

The idea of quark–hadron continuity consists of two stages. One is the continuity of the symmetry-breaking patterns. In the hadronic phase, chiral symmetry is broken by the chiral condensate and the U(1) baryon symmetry is broken by baryon superfluidity. On the other hand, we consider the CFL phase as the quark matter side. In CFL, both the chiral and U(1) baryon symmetries are broken by the diquark condensate. Since the gauge symmetry is not a genuine symmetry, the symmetry-breaking patterns of the

hadronic and CFL phases are exactly the same. Then, we have another continuity associated with the elementary excitations. In the hadronic phase, we have pseudoscalar mesons (such as pions), vector mesons (such as rho mesons), and baryons. On the other side, in the CFL phase, we have pions corresponding to the pions in the hadronic matter, the massive gluons through the Meissner effect that corresponds to the vector mesons in the hadronic matter, and massive quarks having the CFL gap that corresponds to the baryons in the hadronic matter.

In [8], based on the Ginzburg–Landau study, a concrete model realizing the quark–hadron continuity was constructed. The model involves both chiral and diquark fields and the interaction between those fields, which is possible through the U(1) axial anomaly. As a result, the possible existence of a new critical point at moderate density and a very low temperature region was proposed. This is called the anomaly-induced critical point.

Furthermore, the continuity of the elementary excitation using the QCD sum rule as well as the Ginzburg–Landau method was studied [9, 10]. Besides, the continuity of topological excitations such as superfluid vortices, that was not touched in the original study of Schäfer and Wilczek, has been investigated [11]. The idea of quark–hadron continuity is still a hypothesis, but it would be interesting to see if the situation is realized inside the neutron star. Whether we have the sharp interface between different phases or not will be an important and interesting issue. Therefore, in the next section, let us consider some interesting phenomena if we have various interfaces.

4. Physics of hadron–quark interface

4.1. Andreev reflection in condensed matter physics

As examples of physics of interface, let us here consider the Andreev reflection and the Josephson effect. They are originally studied in the condensed matter system and now we try to apply the idea into the hadron–quark system.

The Andreev reflection is an interesting phenomenon due to the proximity of different phases [12]. In order to demonstrate this, let us start with the Bogoliubov–de-Gennes equations [13]

$$i\partial_t \begin{pmatrix} \psi_\uparrow(t, \vec{r}) \\ \psi_\downarrow^\dagger(t, \vec{r}) \end{pmatrix} = \begin{pmatrix} -\frac{\nabla^2}{2m} - E_F & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & \frac{\nabla^2}{2m} + E_F \end{pmatrix} \begin{pmatrix} \psi_\uparrow(t, \vec{r}) \\ \psi_\downarrow^\dagger(t, \vec{r}) \end{pmatrix}. \quad (3)$$

In the case of $\Delta(\vec{r}) = 0$, equations (3) decouple from each other

$$i\psi_\uparrow(t, \vec{r}) = \left(-\frac{\nabla^2}{2m} - E_F \right) \psi_\uparrow(t, \vec{r}),$$

$$i\dot{\psi}_{\downarrow}^*(t, \vec{r}) = \left(\frac{\nabla^2}{2m} + E_F \right) \psi_{\downarrow}^*(t, \vec{r}). \quad (4)$$

Then, the plane wave solutions,

$$\phi \equiv \begin{pmatrix} \psi_{\uparrow}(t, \vec{r}) \\ \psi_{\downarrow}^*(t, \vec{r}) \end{pmatrix} = \begin{pmatrix} f \exp(-iEt + i\vec{q}\vec{r}) \\ g \exp(-iEt + i\vec{q}\vec{r}) \end{pmatrix}, \quad (5)$$

lead to the dispersion relations

$$\begin{aligned} E &= \epsilon_q && \text{for particles,} \\ E &= -\epsilon_q && \text{for holes,} \end{aligned} \quad (6)$$

where $\epsilon_q = \vec{q}^2/2m - E_F$ is a measure of the kinetic energy with respect to the Fermi level and E describes the energy excitation above (below) the Fermi energy for particles (holes). The above physical picture can be treated as a simple description of the conductors. The energy spectrum contains particles and holes in the vicinity of the Fermi surface which can be excited easily for the negligible cost of energy.

Let us now consider equations (3) with $\Delta(\vec{r}) = \text{const.}$, which gives us the model of a superconductor. Then the wave function is given as

$$\begin{aligned} \phi &= D \begin{pmatrix} \sqrt{\frac{1}{2}(1 + \xi/E)} \exp(i\delta/2) \\ \sqrt{\frac{1}{2}(1 - \xi/E)} \exp(-i\delta/2) \end{pmatrix} \exp(i\vec{q}_+\vec{r} - iEt) \\ &+ F \begin{pmatrix} \sqrt{\frac{1}{2}(1 - \xi/E)} \exp(i\delta/2) \\ \sqrt{\frac{1}{2}(1 + \xi/E)} \exp(-i\delta/2) \end{pmatrix} \exp(i\vec{q}_-\vec{r} - iEt), \end{aligned} \quad (7)$$

where $E^2 = \epsilon_q^2 + |\Delta|^2$ and $\xi = \sqrt{E^2 - |\Delta|^2}$. δ is a phase of the gap parameter. D and F are some constants. The momenta of the excitations are given by the relations

$$\frac{\vec{q}_{\pm}^2}{2m} = E_F \pm \xi. \quad (8)$$

Let us consider the interface between the conductor and superconductor. If the interface is sharp, the gap function Δ changes from zero (conductor) to a constant (superconductor) discontinuously. This is exactly the one-dimensional potential problem in quantum mechanics. Then, we can match the wave functions at the interface. As a result, we obtain the probability of the reflection of the hole instead of the electron. This is the Andreev reflection.

4.2. Andreev reflection in QCD

Let us now apply the idea of the Andreev reflection into QCD [14–16]. The effective Hamiltonian describing the quark interaction with the diquark condensate (quark Cooper pairing) can be written at the mean field level [14, 15]

$$H = \int d^3x \left[\sum_{a,i} \psi_a^{i\dagger} (-i\vec{\alpha} \cdot \nabla - \mu) \psi_a^i + \sum_{a,b,i,j} \Delta_{ab}^{ij} \psi_a^{iT} C \gamma_5 \psi_b^j + \text{h.c.} \right], \quad (9)$$

where ψ_a^i is the Dirac spinor with a, b and i, j being color and flavor indices, respectively. C is the charge conjugation matrix and μ the quark chemical potential. From the detailed structure of the gap matrix, one finds that for instance, a red up quark (u_R) is Andreev-reflected as a green down hole (d_G^H) or a blue strange hole (s_B^H). The Andreev reflection of quarks is summarized in Table 1.

Table 1. Andreev reflection of quarks.

Incident quark	Reflected hole
u_R	d_G^H or s_B^H
u_G	d_R^H
u_B	s_R^H
d_R	u_G^H
d_G	u_R^H or s_B^H
d_B	s_G^H
s_R	u_B^H
s_G	d_B^H
s_B	u_R^H or d_G^H

In [17], an extension of the above studies has been done. The question is how the single-quark scattering problem is extended to the multi-quark one. This is, in general, a very hard problem and, so far, we have no clear answer. Thus, we made the following assumption: quarks in the hadronic phase are not interacting with each other, but they should always be in the color-singlet states. This is the constituent quark picture. Then hadrons are defined as a superposition of every single quark. Thus, if a hadron hits the hadron/CSC interface, each quark inside the hadron is Andreev-reflected and the only color-singlet combination is left out of the reflected quarks.

In the case of the hadron/2SC interface, the Andreev reflection of s quark from the hadronic side does not occur because it cannot make any Cooper pairing in the 2SC phase. Moreover, a blue quark with any flavor cannot make any Cooper pairing in the 2SC phase. The results for the case of pions are summarized in Table 2.

Table 2. Andreev reflection of pions at the hadron/2SC interface.

Incident particle	Incident (color)	Reflected (color)	Reflected particle
π^+	π_R^+	$(\pi_G^-)^H$	π^+
	π_G^+	$(\pi_R^-)^H$	
	π_B^+	\times	
π^-	π_R^-	$(\pi_G^+)^H$	π^-
	π_G^-	$(\pi_R^+)^H$	
	π_B^-	\times	
π^0	π_R^0	$(\pi_G^0)^H$	π^0
	π_G^0	$(\pi_R^0)^H$	
	π_B^0	\times	

4.3. A field theoretical approach to transport phenomena: Tunneling current

In the previous subsection, we studied the Andreev reflection in QCD. This is, however, not enough to describe the system which would be realized in neutron stars. One problem is, as was already noted, how to dynamically treat the confinement of quarks. The other is that the system is relativistic and in non-equilibrium. In this section, we introduce the tunneling transport at the hadron–quark interface within the Schwinger–Keldysh approach [18].

Let us consider the hybrid model consisting of relativistic baryons and quarks. The Hamiltonian of relativistic fermions is given by

$$H = H_B + H_Q + H_T, \quad (10)$$

where $H_{B(Q)}$ and H_T are the baryon (quark) Hamiltonian and the tunneling Hamiltonian, respectively. Then the tunneling current operator is defined as

$$\hat{I} = -\frac{d}{dt}\rho_B = i[\rho_B, H], \quad (11)$$

where

$$\rho_B = \sum_B B_{\mathbf{K}\sigma_B\tau_B}^\dagger B_{\mathbf{K}\sigma_B\tau_B} \quad (12)$$

is the baryon number-density operator. $B_{\mathbf{K}\sigma_B\tau_B}^{(\dagger)}$ and $\bar{B}_{\mathbf{K}\sigma_B\tau_B}^{(\dagger)}$ are the annihilation (creation) operators of a baryon and an anti-baryon with spin σ_B and isospin τ_B .

Under the non-equilibrium condition, we are interested in the two-time expectation values of the tunneling current given by

$$\begin{aligned} \langle \hat{I}(t, t') \rangle &= \sum_{B, Q_1, Q_2, Q_3} \mathcal{T}_{(\mathbf{K}, \sigma_B, \tau_B, \{\mathbf{k}_i, \sigma_i, \tau_i, a_i\})} \varepsilon_{a_1 a_2 a_3} \\ &\times \left\langle T_C \left[\hat{S}_C B_{\mathbf{K}\sigma_B\tau_B}^{(\dagger(H))} (t) q_{\mathbf{k}_1 \sigma_1 \tau_1 a_1}^{(H)} (t') q_{\mathbf{k}_2 \sigma_2 \tau_2 a_2}^{(H)} (t') q_{\mathbf{k}_3 \sigma_3 \tau_3 a_3}^{(H)} (t') \right] \right\rangle + \text{h.c.}, \quad (13) \end{aligned}$$

where

$$\hat{S}_C = \exp \left[-i \int_C dt'' H_T^{(H)}(t'') \right] \quad (14)$$

is the S -matrix operator with respect to the tunneling Hamiltonian $H_T^{(H)}(t) = e^{i(H_B+H_Q)t} H_T e^{-i(H_B+H_Q)t}$ in the interaction representation. The annihilation (creation) operators $q_{\mathbf{k}\sigma\tau a}^{(\dagger)}$ and $\bar{q}_{\mathbf{k}\sigma\tau a}^{(\dagger)}$ of a quark and an anti-quark involve the spin σ , flavor (or isospin) τ , and color a . $\mathcal{T}_{(\mathbf{K}, \sigma_B, \tau_B, \{\mathbf{k}_i, \sigma_i, \tau_i, a_i\})}$ is the tunneling amplitudes of a color-singlet baryon to three quarks and that of a color-singlet anti-baryon to three anti-quarks across the interface. The time integration contour C in Eq. (14) is called the Keldysh contour [19] that runs from $t'' = -\infty$ to $t'' = \infty$ (forward branch C_-) and then returns back to $t'' = -\infty$ (backward branch C_+) as shown in Fig. 1.

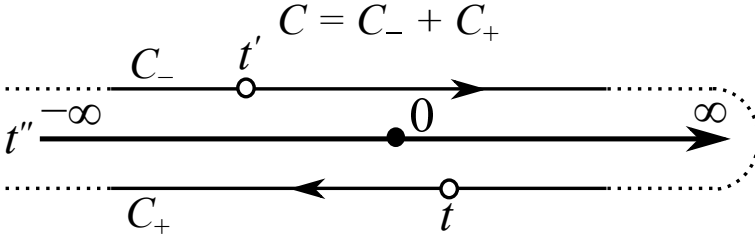


Fig. 1. The Keldysh contour $C = C_- + C_+$ along the t'' axis in Eq. (14), consisting of a forward branch C_- and a backward one C_+ . Two time parameters t and t' in $\langle \hat{I}(t, t') \rangle$ and $\langle \hat{F}(t, t') \rangle$ locate on C_+ and C_- , respectively.

In [18], the perturbative expansion of the S -matrix has been performed up to the second order and the following DC Josephson tunneling current was eventually obtained after some steps:

$$I_{\text{DC}} = -4 \sum_B \sum_{Q_1, Q_2, Q_3 \rightarrow \infty} \int \frac{d\omega}{2\pi} \mathcal{T}^2 \left[\text{Re} G_{Q, \{\mathbf{k}_i, \sigma_i, \tau_i, a_i\}}^{(12)\text{ret}}(\omega) \text{Im} G_{B, \mathbf{K}, \sigma_B, \tau_B}^{(21)\text{ret}}(\omega) f(\omega) \right. \\ \left. + \text{Im} G_{Q, \{\mathbf{k}_i, \sigma_i, \tau_i, a_i\}}^{(12)\text{ret}}(\omega) \text{Re} G_{B, \mathbf{K}, \sigma_B, \tau_B}^{(21)\text{ret}}(\omega) f(\omega) \right] \sin(\Delta\phi). \quad (15)$$

Here, $f(\omega)$ is the Fermi distribution function and $G^{(21)\text{ret}}(\omega)$ and $G^{(12)\text{ret}}(\omega)$ are the retarded Green's functions. $\Delta\phi$ is the phase difference of the gap parameters between the hadron and quark sides. For future perspectives, it is worthwhile to examine the Andreev reflection in this method. This will be our forthcoming work [20].

In this paper, we introduced the important properties of interfaces. This itself is a very general perspective, but we are especially interested in the interior of neutron stars, where various phases exist and there are many interfaces. Associated with it, we showed color superconductivity as a ground state of high-density quark matter. Then the idea of quark–hadron continuity, which provides a new insight into high-density QCD world was presented. Finally, we showed some studies related to physics of interface such as the Andreev reflection. These studies are still insufficient to treat the relativistic and non-equilibrium system. Thus, we outlined a new approach based on quantum field theory. As a concrete demonstration, we have shown the computation of the Josephson current. One of the goals in this kind of study is to comprehend some phenomena happening inside the neutron star such as the pulsar glitch and the cooling of neutron stars.

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