THE FIELD OF A CHARGE MOVING AT THE SPEED OF LIGHT

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Potential and corresponding electric and magnetic fields of a moving point-like charge are computed in several ways. The subtle limit of the charge velocity equal to the speed of light is taken either in the final formulas or in the equation of motion. The results are discussed.

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1. Prologue

It is a privilege and a pleasure for me to contribute to this Festschrift on the occasion of the ninetieth birthday of Professor Andrzej Białas. His work and diverse activities have greatly influenced numerous researchers of several generations, and I am one of them, even though I am from Warsaw not from Cracow.

Professor Białas has been a constant presence throughout my whole professional life. I was preparing my master's thesis when I came to Kraków in the spring of 1980 to discuss the Glauber model. Since then, I have had countless opportunities to listen to the Professor's lectures, to discuss physics with him, and argue on various topics. Our contacts had twice been of a more formal nature. Professor Białas was a reviewer of my habilitation thesis, his opinion was positive but… My thesis was composed of three parts. Professor Białas suggested throwing away the first and third parts. The second would be enough, he stated. Many years later, I was proud to serve as a referee for his honorary doctorate from Jan Kochanowski University in Kielce.

I have always admired the style of Professor Białas's lectures and research articles. The ideas behind them were usually surprisingly simple yet profound and the solutions remarkably elegant. And behind it all was a perfect understanding of physics. I have always tried to emulate that style and this contribution is an attempt to explain how I try to achieve it.

2. Introduction

In recent years, I have been mostly interested in the earliest phase of relativistic heavy-ion collisions, which is the least understood. At very early times, the system, which emerges in relativistic heavy-ion collisions, is strongly anisotropic, far from thermodynamic equilibrium, the energy density reaches its maximal values, and the dynamics is strongly non-Abelian. So, this phase is certainly of interest, it can significantly affect the subsequent evolution of the system and its final-state characteristics.

Several different approaches have been used to describe the earliest phase of relativistic heavy-ion collisions but the framework of the Color Glass Condensate effective theory [1–3], see also the review [4], is most promising. The theory is based on a separation of scales between hard valence partons and soft gluons. The former ones are treated as classical sources and the latter as classical fields. The phase of matter that exists at very early times is called a 'glasma', it is composed of coherent chromodynamic fields of large occupation numbers. The dynamics of the glasma fields is determined by the classical Yang–Mills equations with sources provided by the valence partons. Observables are calculated by performing a Gaussian averaging over color configurations of colliding nucleus.

Properties of the glasma have been studied for over two decades using more and more advanced numerical simulations but neither I nor my long-time collaborator, Margaret Carrington, enjoy numerical calculations. We decided to use the proper-time expansion method which is just designed to study the earliest phase of relativistic heavy-ion collisions. The method was proposed in [6] and further developed in [7–9]. It involves an expansion of the Yang–Mills equations in powers of the proper time which is treated as a small parameter. The dimensionless small parameter is the product of the proper time and saturation scale.

In a whole series of papers, which are reviewed in [10], we used the proper time expansion to describe the glasma from the earliest phase of relativistic heavy-ion collisions. We analyzed glasma characteristics like the energy density, longitudinal and transverse pressures, collective flow, and angular momentum. We also studied the role of the glasma in jet quenching, computing the collisional energy loss and transverse momentum broadening of hard probes.

The glasma fields are generated by the valence quarks of incoming nuclei which move against each other at the speed of light. However, one actually solves the sourceless Yang–Mills equations but the sources enter via the boundary condition. Thanks to them, the glasma fields smoothly change across the light cone into the chromodynamic fields of initial nuclei. Since the glasma fields are expanded in powers of the proper time and the Yang–Mills equations are solved iteratively, the glasma fields are ultimately expressed through the fields and their derivatives of the incoming nuclei. So, these initial fields play a key role in the calculations.

Since the incoming nuclei move at the speed of light, the associated potentials are rather peculiar. I decided to derive them on my own to really understand the problem, to be able to present it in a clear elegant way, as Andrzej Białas always does. I started with the electromagnetic potential generated by a point-like charge moving at the speed of light. I had been teaching classical electrodynamics for years, so I did not expect any difficulties. However, the problem turned out to be quite subtle, and solving it took me a lot of effort. Well, imitating the Białas style is not an easy task. I am sure the problem is discussed in the literature, but I could not find a suitable publication. Since the result is apparently known, I have not included its derivation in my publications on glasma. However, let me present it here as it documents my effort to understand the problem.

3. Problem

I consider a point-like charge q moving along the axis z with the velocity $\boldsymbol{v} = (0, 0, v)$. At time $t = 0$, the charge is at $x = y = z = 0$. The charge density and current are

$$\rho(t, \boldsymbol{r}_\perp, z) = q\, \delta^{(2)}(\boldsymbol{r}_\perp)\, \delta(z - vt)\,, \qquad \boldsymbol{j}(t, \boldsymbol{r}_\perp, z) = q\, \boldsymbol{v}\, \delta^{(2)}(\boldsymbol{r}_\perp)\, \delta(z - vt)\,, \tag{1}$$

where $\boldsymbol{r}_\perp \equiv (x, y)$. The four-current is written as

$$j^\mu(t, \boldsymbol{r}_\perp, z) = q\, v^\nu \delta^{(2)}(\boldsymbol{r}_\perp)\, \delta(z - vt)\,, \tag{2}$$

where $v^\nu \equiv (1, 0, 0, v)$.

The potential generated by the four-current (2) can be found in several ways, see *e.g.* the classical textbook [11]. A problem is how and at what stage of the calculation to take the limit $v = 1$.

3.1. Performing Lorentz transformation

Presumably, the simplest way to get the potential of a charge moving with a constant velocity is to perform the Lorentz transformation of the Coulomb potential which in the charge rest frame is

$$A'^\mu\left(t', \boldsymbol{r}'_\perp, z'\right) = g^{\mu 0} \frac{q}{4\pi} \frac{1}{\sqrt{\boldsymbol{r}'^2_\perp + z'^2}}\,. \tag{3}$$

The potential in the laboratory frame $A^\mu(t,\boldsymbol{r}_\perp,z)$ is obtained by performing the Lorentz transformation

$$\begin{cases} t' = \gamma(t - vz)\,, \\ \boldsymbol{r}'_\perp = \boldsymbol{r}_\perp\,, \\ z' = \gamma(z - vt)\,, \end{cases} \qquad \begin{cases} t = \gamma\left(t' + vz'\right)\,, \\ \boldsymbol{r}_\perp = \boldsymbol{r}'_\perp\,, \\ z = \gamma\left(z' + vt'\right)\,, \end{cases} \tag{4}$$

where $\gamma \equiv (1-v^2)^{-1/2}$. I use the Heaviside–Lorentz units with $c=1$ which are usually used in quantum field theory. Transforming the potential (3), one finds

$$\begin{cases} A^0(t,\boldsymbol{r}_\perp,z) = \frac{q}{4\pi}\frac{\gamma}{\sqrt{\boldsymbol{r}_\perp^2+\gamma^2(z-vt)^2}}\,, \\ \boldsymbol{A}_\perp(t,\boldsymbol{r}_\perp,z) = 0\,, \\ A^z(t,\boldsymbol{r}_\perp,z) = \frac{q}{4\pi}\frac{\gamma\, v}{\sqrt{\boldsymbol{r}_\perp^2+\gamma^2(z-vt)^2}}\,. \end{cases} \tag{5}$$

One observes that the potentials (3) and (5) obey the Lorentz gauge condition $\partial'_\mu A'^\mu(x') = 0$ or $\partial_\mu A^\mu(x) = 0$, respectively.

The corresponding electric ($\boldsymbol{E} = -\nabla A^0 - \dot{\boldsymbol{A}}$) and magnetic ($\boldsymbol{B} = \nabla \times \boldsymbol{A}$) fields are

$$\begin{cases} \boldsymbol{E}_\perp(t,\boldsymbol{r}_\perp,z) = \frac{q}{4\pi}\frac{\gamma\boldsymbol{r}_\perp}{\left(\boldsymbol{r}_\perp^2+\gamma^2(z-vt)^2\right)^{3/2}}\,, \\ E^z(t,\boldsymbol{r}_\perp,z) = \frac{q}{4\pi}\frac{\gamma(z-vt)}{\left(\boldsymbol{r}_\perp^2+\gamma^2(z-vt)^2\right)^{3/2}}\,, \end{cases} \tag{6}$$

$$\begin{cases} \boldsymbol{B}_\perp(t,\boldsymbol{r}_\perp,z) = \frac{q}{4\pi}\frac{\gamma\boldsymbol{v}\times\boldsymbol{r}_\perp}{\left(\boldsymbol{r}_\perp^2+\gamma^2(z-vt)^2\right)^{3/2}}\,, \\ B^z(t,\boldsymbol{r}_\perp,z) = 0\,. \end{cases} \tag{7}$$

When $v \to 1$ and $z \neq t$, the four-potential (5) is

$$\begin{cases} A^0(t,\boldsymbol{r}_\perp,z) = \frac{q}{4\pi}\frac{1}{|z-t|}\,, \\ \boldsymbol{A}_\perp(t,\boldsymbol{r}_\perp,z) = 0\,, \\ A^z(t,\boldsymbol{r}_\perp,z) = \frac{q}{4\pi}\frac{1}{|z-t|}\,, \end{cases} \tag{8}$$

and the electric and magnetic fields (6) and (7) vanish. The same result is found when the fields are obtained from the potential (8). When we first put $z = t$ in the formulas (5) and (6), (7) and then take the limit $v \to 1$, the potential components A^0 and A^z become infinite and so do the fields $\boldsymbol{E}_\perp$ and $\boldsymbol{B}_\perp$. The fields E^z and B^z again vanish. So, in the limit $v = 1$, the potential (5) is pure gauge everywhere except the plane $t = z$.

3.2. Solving Maxwell equations with Green's function

The electromagnetic field generated by the four-current $j^\nu(x)$ can be found as a solution of the Maxwell equations

$$\partial_\mu F^{\mu\nu}(x) = j^\nu(x)\,, \tag{9}$$

where the position four-vector is $x = (t, \boldsymbol{r}_\perp, z)$ and the strength tensor reads $F^{\mu\nu}(x) = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x)$.

Choosing the Lorentz gauge $\partial_\mu A^\mu(x) = 0$, the Maxwell equations (9) read

$$\Box A^\nu(x) = j^\nu(x)\,, \tag{10}$$

with $\Box \equiv \partial_\mu \partial^\mu$.

The general solution of Eq. (10) is

$$A^\mu(x) = A_0^\mu(x) + \int \mathrm{d}^4x' D_\mathrm{R}\left(x - x'\right) j^\nu\left(x'\right)\,, \tag{11}$$

where $A_0^\mu(x)$ solves the homogeneous equation $\Box A^\mu(x) = 0$ and the retarded Green's function obeys the equation

$$\Box D_\mathrm{R}(x) = \delta^{(4)}(x)\,. \tag{12}$$

The explicit solution of Eq. (12), which satisfies the retarded initial condition ($D_\mathrm{R}(x) = 0$ for $t < 0$), is found as

$$D_\mathrm{R}(x) = -\int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{\mathrm{e}^{-ikx}}{k^2 + i\,\mathrm{sgn}(k_0)0^+} = \frac{\Theta(t)}{4\pi|\boldsymbol{x}|}\left[\delta(t - |\boldsymbol{x}|) - \delta(t + |\boldsymbol{x}|)\right]\,. \tag{13}$$

Since $D_\mathrm{R}(x)$ is non-zero only for $t \geq 0$, the last term can be ignored and the Green's function equals

$$D_\mathrm{R}(x) = \frac{\Theta(t)\,\delta(t - |\boldsymbol{x}|)}{4\pi|\boldsymbol{x}|} = \frac{1}{2\pi}\,\Theta(t)\,\delta\left(x^2\right)\,, \tag{14}$$

which is the formula (12.133) of [11].

Substituting the Green's function (14) and the current (2) into Eq. (11), one finds

$$A^\mu(x) = \frac{qv^\mu}{4\pi} \int\limits_{-\infty}^{t} \mathrm{d}t' \frac{\delta\left(t - t' - \sqrt{\boldsymbol{r}_\perp^2 + (z - vt')^2}\right)}{\sqrt{\boldsymbol{r}_\perp^2 + (z - vt')^2}}\,, \tag{15}$$

where the solution of the homogeneous equation is ignored. To take the integral over t', one has to solve the equation

$$t - t' - \sqrt{\boldsymbol{r}_\perp^2 + (z - vt')^2} = 0 \tag{16}$$

under the condition $t \geq t'$.

If $v = 1$, the solution of Eq. (16) is

$$t' = \frac{t^2 - \boldsymbol{r}_\perp^2 - z^2}{2(t - z)}\,, \tag{17}$$

and the condition $t \geq t'$ is satisfied for $t > z$ but not for $t < z$.

For $v = 1$, the potential (15) equals

$$A^\mu(x) = \frac{qv^\mu}{4\pi} \int\limits_{-\infty}^{t} \mathrm{d}t'\, \frac{\delta\left(t - t' - \sqrt{\boldsymbol{r}_\perp^2 + (z - t')^2}\right)}{\sqrt{\boldsymbol{r}_\perp^2 + (z - t')^2}} = \frac{qv^\mu}{4\pi} \frac{\Theta(t - z)}{t - z}\,. \tag{18}$$

The potential (18) agrees with the transformed potential (8) for $t > z$ but differs for $t < z$. The difference results from the retarded initial condition, as the potential (18) is nonzero only behind the charge.

To obtain the result for $v = 1$, one can also proceed differently. One first solves Eq. (16) for $v < 1$ and finds two solutions which can be written as

$$t'_\pm = \gamma^2 \left(t - zv \pm \sqrt{(vt - z)^2 + (1 - v^2)\,\boldsymbol{r}_\perp^2} \right). \tag{19}$$

When $v = 0$ is put in Eq. (16), there is one solution $t' = t - \sqrt{\boldsymbol{r}_\perp^2 + z^2}$ of the equation. However, there are two solutions $t'_\pm = t \pm \sqrt{\boldsymbol{r}_\perp^2 + z^2}$ provided by Eq. (19) with $v = 0$. It obviously results from squaring the square root in Eq. (16). So, the solution t'_+ must be eliminated. One also checks that the condition $t'_- \leq t$ is always satisfied.

The potential (5) computed with the solution t'_- is

$$A^\mu(x) = \frac{q}{4\pi} \frac{v^\mu}{\sqrt{(1 - v^2)\boldsymbol{r}_\perp^2 + (z - vt)^2}}\,, \tag{20}$$

and it agrees with the potential (5) obtained by means of the Lorentz transformation.

In the limit $v \to 1$, the potential (20) becomes

$$A^\mu(x) = \frac{q}{4\pi} \frac{v^\mu}{|t - z|}\,, \tag{21}$$

which differs from formula (18). It shows that the limit $v \to 1$ is not unique and the results depend on how it is taken.

3.3. Solving Maxwell equations with Fourier transformation

Using the Fourier transformation, which is defined as

$$f(\omega, \boldsymbol{k}) = \int \mathrm{d}t \int \mathrm{d}^3 r \; \mathrm{e}^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})} f(t, \boldsymbol{r}) \,, \tag{22}$$

$$f(t, \boldsymbol{r}) = \int \frac{\mathrm{d}\omega}{2\pi} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \; \mathrm{e}^{-i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})} f(\omega, \boldsymbol{k}) \,, \tag{23}$$

the transformed charge density and current (1) are

$$\rho(\omega, \boldsymbol{k}) = 2\pi\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\, q \,, \qquad \boldsymbol{j}(\omega, \boldsymbol{k}) = 2\pi\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\, q\boldsymbol{v} \,. \tag{24}$$

Since the equation of the potential in the Lorentz gauge is

$$\Box A^\mu(t, \boldsymbol{r}) = j^\mu(t, \boldsymbol{r}) = 2\pi\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\, q v^\mu \,, \tag{25}$$

where $v^\mu \equiv (1, \boldsymbol{v})$, the Fourier transform potential equals

$$A^\mu(\omega, \boldsymbol{k}) = -\frac{j^\mu(\omega, \boldsymbol{k})}{\omega^2 - \boldsymbol{k}^2 + i\omega 0^+} \,, \tag{26}$$

where the infinitesimally small imaginary element $i\omega 0^+$ is included to guarantee the retarded boundary condition. Using the charge density and current (24), the potential is

$$A^\mu(t, \boldsymbol{r}) = -q\, v^\mu \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{\mathrm{e}^{i(\boldsymbol{r} - \boldsymbol{v}t)\cdot\boldsymbol{k}}}{(\boldsymbol{k}\cdot\boldsymbol{v})^2 - \boldsymbol{k}^2 + i\boldsymbol{k}\cdot\boldsymbol{v}0^+} \,, \tag{27}$$

where the trivial integral over ω is taken. One observes that if $\boldsymbol{v}^2 < 1$, the denominator of the integrand does not vanish for any $\boldsymbol{k}$ and the infinitesimal imaginary element can be ignored. However, it is important when the limit $v = 1$ is taken.

If $\boldsymbol{v}^2 < 1$ and the infinitesimal imaginary element is ignored, the integral over the wave vector in Eq. (27) is elementary in cylindrical coordinates and after some work, one finds the special case of the Liénard–Wichert potential

$$A^\mu(t, \boldsymbol{r}) = \frac{1}{4\pi} \frac{q v^\mu}{\left(\boldsymbol{R}^2 - (\boldsymbol{R}\times\boldsymbol{v})^2\right)^{1/2}} \,, \tag{28}$$

where $\boldsymbol{R} \equiv \boldsymbol{r} - \boldsymbol{v}t$. The corresponding electric and magnetic fields are

$$\boldsymbol{E}(t, \boldsymbol{r}) = \frac{q}{4\pi} \frac{\left(1 - \boldsymbol{v}^2\right)\, \boldsymbol{R}}{\left(\boldsymbol{R}^2 - (\boldsymbol{R}\times\boldsymbol{v})^2\right)^{3/2}} \,, \tag{29}$$

$$\boldsymbol{B}(t, \boldsymbol{r}) = \frac{q}{4\pi} \frac{\left(1 - \boldsymbol{v}^2\right)\, \boldsymbol{v}\times\boldsymbol{R}}{\left(\boldsymbol{R}^2 - (\boldsymbol{R}\times\boldsymbol{v})^2\right)^{3/2}} \,. \tag{30}$$

The limit $v = 1$ is, however, problematic.

3.4. Taking $v=1$ limit in the Maxwell equations

The Maxwell equation (10) with the four-current (2) in the limit of $v=1$ reads

$$\Box A^\mu(x) = q\, v^\mu \delta^{(2)}(\boldsymbol{r}_\perp)\,\delta(z-t)\,, \tag{31}$$

where $v^\mu \equiv (1,0,0,1)$. Performing the Fourier transformation of Eq. (31), one finds

$$A^\mu(k) = -q\, v^\mu \frac{2\pi\delta(k_0-k_z)}{k_0^2-k_z^2-\boldsymbol{k}_\perp}\,, \tag{32}$$

which transformed back to the coordinate space gives

$$A^\mu(x) = q\, v^\mu\, \delta(z-t) \int \frac{\mathrm{d}^2\boldsymbol{k}_\perp}{(2\pi)^2}\, \frac{\mathrm{e}^{i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp}}{\boldsymbol{k}_\perp^2}\,. \tag{33}$$

The integral is divergent in the infrared domain. So, the potential (33) is infinite like that one obtained from formula (5) putting $t=z$ and taking the limit $v\to 1$. The divergence occurs because the three-dimensional integral is effectively changed into the two-dimensional. One also observes that formula (33) does not reproduce the potential (8) valid for $t\neq z$. However, since the potential (8) is pure gauge, the difference is not physically significant.

To compute the integrals in Eq. (33), one regularizes it by including the mass parameter m in the integrand's denominator and finds

$$A^\mu(x) = q\, v^\mu\, \delta(z-t) \int \frac{\mathrm{d}^2\boldsymbol{k}_\perp}{(2\pi)^2}\, \frac{\mathrm{e}^{i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp}}{\boldsymbol{k}_\perp^2+m^2} = \frac{q\, v^\mu}{2\pi}\,\delta(z-t)\, K_0(m|\boldsymbol{r}_\perp|)\,, \tag{34}$$

where $K_n(x)$ is the n^{th} Macdonald function. Then, the longitudinal electric and magnetic fields vanish, while the transverse are

$$\boldsymbol{E}_\perp(t,\boldsymbol{r}_\perp,z) = \frac{q\,m}{2\pi}\,\frac{\boldsymbol{r}_\perp}{|\boldsymbol{r}_\perp|}\,\delta(z-t)\,K_1(m|\boldsymbol{r}_\perp|)\,, \tag{35}$$

$$\boldsymbol{B}_\perp(t,\boldsymbol{r}_\perp,z) = -\frac{q\,m}{2\pi}\,\frac{\boldsymbol{r}_\perp\times\hat{\boldsymbol{z}}}{|\boldsymbol{r}_\perp|}\,\delta(z-t)\,K_1(m|\boldsymbol{r}_\perp|)\,, \tag{36}$$

where $\hat{\boldsymbol{z}}\equiv(0,0,1)$ and the identity $K_0'(z) = -K_1(z)$ has been used. One can remove the regulator m by taking the limit $m\to 0$ as $K_1(z)\approx 1/z$ for $1/z \gg 1$. Thus, one finds the desired formulas of the transverse fields

$$\boldsymbol{E}_\perp(t,\boldsymbol{r}_\perp,z) = \frac{q}{2\pi}\,\frac{\boldsymbol{r}_\perp}{\boldsymbol{r}_\perp^2}\,\delta(z-t)\,, \tag{37}$$

$$\boldsymbol{B}_\perp(t,\boldsymbol{r}_\perp,z) = \frac{q}{2\pi}\,\frac{\hat{\boldsymbol{z}}\times\boldsymbol{r}_\perp}{\boldsymbol{r}_\perp^2}\,\delta(z-t)\,. \tag{38}$$

It is interesting to note that the integrals over z of $\boldsymbol{E}_\perp(t, \boldsymbol{r}_\perp, z)$ given by Eqs. (6) and (37) are equal to each other. The same holds for $\boldsymbol{B}_\perp(t, \boldsymbol{r}_\perp, z)$. Indeed, the integration of the function (6) gives

$$\int\limits_{-\infty}^{\infty} \mathrm{d}z\, \boldsymbol{E}_\perp(t, \boldsymbol{r}_\perp, z) = \frac{q}{2\pi} \frac{\boldsymbol{r}_\perp}{\boldsymbol{r}_\perp^2}\,. \tag{39}$$

This result shows that formulas (6) and (7) of the transverse fields for $v < 1$ can be treated as a representation of the delta-like formulas (37) and (38). Therefore, the results (37) and (38) can be inferred directly from formulas (6) and (7).

4. Epilogue

A few years ago, Andrzej Białas asked me to write a short piece about Andrzej Sołtan, one of the initiators of nuclear research in Poland and the founder of Warsaw nuclear physics. The article entitled «Andrzej Sołtan — Physicist by Vocation» appeared in *PAUza*, the journal of the Polish Academy of Arts and Sciences. Andrzej Białas once told me that he dreamed of playing basketball, and studying physics was just an obstacle. Nevertheless, the term 'physicist by vocation' describes him well, I think. My article about Andrzej Sołtan ends: 'Polish science owes him much'. These words fully apply to Andrzej Białas.

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