SPIN VIBRATION STATES IN ^{120}Sn AND ^{208}Pb WITHIN $\Delta N = 0$ AND $\Delta N = 2$ OSCILLATOR SHELLS

By S. Ćwiok

Institute of Physics, Technical University of Warsaw*

and M. Wygonowska-Haensel

Medical Academy of Warsaw**

(Received November 3, 1973)

Energies, wave functions, and $M1$ electromagnetic transition widths have been calculated for ^{120}Sn and ^{208}Pb with the use of the Saxon-Woods potential and of "realistic" residual forces. The excitations within three neighbouring oscillator shells have been taken into account. Excited states $J^{\pi} = 1^{+}$ in the energy region 3.2 — 20.7 MeV have been obtained. They can be called spin vibration states, consisting mainly of excitations within one oscillator shell.

1. Introduction

It is well known that the spin-spin interaction term plays an important role in nuclear residual potential. Many experimental data may be accounted for only if one assumes this type of interaction. Using the spin-spin residual interaction one is able to explain the deviation of the values of magnetic moments from the Schmidt lines for odd-A nuclei [1, 2] and the decrease of probability of allowed Gamov-Teller β-decay [3, 4, 5]. In even-even nuclei the spin-spin residual interaction gives rise to excited states $J^{\pi} = 1^{+}$ characterized by a large value of reduced transition probability $B_{0^{+}\to 1^{+}}(M1)$ as compared with the single particle value. The excitation energy and structure of these states depend on the single particle level scheme which in turn is intimately connected with the shape of the nucleus.

In our previous paper [6] the excited states $J^{\pi} = 1^{+}$ in ^{56}Ni, ^{138}Ba, ^{140}Ce, ^{142}Nd, $^{114, 116, 118, 120, 122, 124}$Sn, and ^{208}Pb have been investigated. Tin isotopes are magic nuclei

---

* Address: Instytut Fizyki Politechniki Warszawskiej, Plac Jedności Robotniczej 1, 00-661 Warszawa, Poland.

** Address: Akademia Medyczna, Chałubińskiego 5, 02-004 Warszawa, Poland.

$Z = 50$. Barium, cerium and neodymium are magic with respect to neutron number $N = 82$. In the above-mentioned nuclei the subshells $50 < N, Z < 82$ are occupied to a certain degree. ^{56}Ni and ^{208}Pb are doubly magic. Excitations within one oscillator shell, neutron and proton, have been taken into consideration. The calculations were carried out in the realm of the RP and the TD approximations. Several excited states $J^\pi = 1^+$ were obtained in the energy region from 2.3 MeV to 12.4 MeV for the quoted nuclei.

It should be mentioned that experimental data on $J^\pi = 1^+$ states of this type are very scarce. We are able, however, to compare the 8.97 MeV state obtained for ^{138}Ba with that observed by Bechvarj et al. [7] for ^{136}Ba at energy 9.11 MeV. Theory and experiment yield a rather good agreement.

In paper [6] we considered excitations within one oscillator shell N. Our chief aim in the present paper is to study the properties of excited states $J^\pi = 1^+$ with $\Delta N = 2$ excitations taken into account.

2. *Theoretical description*

The calculation has been performed with the use of the random phase approximation (RPA) and the Tamm-Dankoff approximation (TDA).

In the RPA the vector of excited state has the form:

$$|E_n JM\rangle = Q^+(E_n JM)\,|RPA\rangle,$$

where $Q^+(E_n JM)$ is the phonon creation operator

$$Q^+(E_n JM) = \sum_{(j_1,j_2)\tau} [X^{(n)}_{j_1 j_2}(J^\pi, \tau) B^+_\tau(j_1 j_2; J^\pi M) -$$
$$- Y^{(n)}_{j_1 j_2}(J^\pi, \tau) B_\tau(j_1 j_2; \widetilde{J^\pi M})]. \tag{1}$$

Here $B^+(B)$ is an operator creating (annihilating) a pair of quasi-particles of total angular momentum J and parity π. The subscripts τ take the values p or n for proton or neutron respectively. The sum in (1) runs over quasi-particle orbits j_1 and j_2, such that $\varepsilon_{j_1} < \varepsilon_{j_2}$ where ε_j is the single-particle energy. The state described by the state vector $|\text{RPA}\rangle$ is the ground state and satisfies the equation

$$Q(E_n J^\pi M)\,|RPA\rangle = 0. \tag{2}$$

The amplitudes X and Y represent the contributions of two-quasi-particle configurations to a phonon state. The RPA method leads to equation [8]:

$$\begin{bmatrix} A - E * I, & B \\ -B \quad , & -A - E * I \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = 0. \tag{3}$$

Equation (3) and the normality condition completely define the amplitudes X and Y and the energy E of a phonon state. The integer number n runs over phonon states of the same total angular momentum J and parity π. The matrices A and B are real and symmetrical, and depend on the residual interaction as well as on the energies of two-quasi-particle states.

The TDA equations can be obtained from the RPA ones by setting $Y = 0$. The details of calculation using RPA and TDA methods can be seen in our previous work [6].

As the excited states under consideration bear the angular momentum and the parity $J^\pi = 1^+$ their electromagnetic properties are characterized by a reduced transition probability:

$$B_{0^+\to 1^+_n}(M1) = \frac{3}{4\pi}\left|\sum_{(j_1,j_2)\tau} \frac{1}{\sqrt{3}}[v_{j_1}(\tau)u_{j_2}(\tau)-u_{j_1}(\tau)v_{j_2}(\tau)]* \right.$$
$$\left. * \langle j_2(\tau)\,\|\bar{\mu}(\tau)\|\,j_1(\tau)\rangle\,[X^{(n)}_{j_1j_2}(J^\pi = 1^+,\tau) - Y^{(n)}_{j_1j_2}(J^\pi = 1^+,\tau)]\right|^2. \tag{4}$$

The states for which the contributions of various configurations are of the same sign as well as the same order have exceptionally large values of $B_{0^+\to 1^+}(M1)$. The ratio $K = B_{0^+\to 1^+}(M1)/B^g_{0^+\to 1^+}(M1)$ where

$$B^g_{0^+\to 1^+_n}(M1) = \frac{3}{4\pi}\sum_{(j_1,j_2)\tau}\left|\frac{1}{\sqrt{3}}[v_{j_1}(\tau)u_{j_2}(\tau)-u_{j_1}(\tau)v_{j_2}(\tau)]* \right.$$
$$\left. * \langle j_2(\tau)\,\|\bar{\mu}(\tau)\|\,j_1(\tau)\rangle\,[X^{(n)}_{j_1j_2}(J^\pi = 1^+,\tau) - Y^{(n)}_{j_1j_2}(J^\pi = 1^+,\tau)]\right|^2$$

can measure the degree of coherence of the state. We will also use the electromagnetic width $\Gamma_{0^+\to 1^+_n}(M1)$ connected with $B_{0^+\to 1^+_n}(M1)$ in the following way:

$$\Gamma_{0^+\to 1^+_n}(M1) = 1.21 * 10^{-2} * E_\gamma^3 B_{0^+\to 1^+_n}(M1). \tag{5}$$

$\Gamma(M1)$ in (5) is obtained in eV if one expresses the transition energy E_γ in MeV and $B(M1)$ in $(e\hbar/2m_pc)^2$ units. The electromagnetic transitions of type $M1$ obey the sum rule:

$$\langle \Psi_0|\,[\hat{\mu}_z,[H,\hat{\mu}_z]]\,|\Psi_0\rangle = \frac{8\pi}{3}\sum_n E_\gamma B_{0^+\to 1^+_n}(M1). \tag{6}$$

For $|\psi_0\rangle = |\mathrm{TDA}\rangle$ and $[\hat{\mu}_z, H] \cong [\hat{\mu}_z, H_{qp}]$ the left-hand side of Eq. (6) can be reduced to:

$$\frac{8\pi}{3}\sum_{(j_1,j_2)\tau}(e_{j_1}(\tau)+e_{j_2}(\tau)) * B^{qp}_{0^+\to 1^+\{(j_1,j_2)\tau\}}(M1),$$

where

$$B^{qp}_{0^+\to 1^+\{(j_1,j_2)\tau\}}(M1) = \frac{3}{4\pi}\left|\frac{1}{\sqrt{3}}\langle j_2(\tau)\|\bar{\mu}(\tau)\|j_1(\tau)\rangle * \right.$$
$$\left. *[v_{j_1}(\tau)u_{j_2}(\tau)-u_{j_1}(\tau)v_{j_2}(\tau)]\right|^2.$$

The sum rule (6) allows us to locate groups of states having especially large reduced transition probabilities.

3. *Results and discussion*

Several excited states $J^\pi = 1^+$ have been obtained as the result for ^{120}Sn and ^{208}Pb. Their reduced transition probabilities $B_{0^+\to 1^+_n}$ and the sum rule are shown in Figs 1 and 2. Their structure, widths $\Gamma_{0^+\to 1^+_n}$, and values of $B_{0^+\to 1^+_n}(M1)/B^g_{0^+\to 1^+}(M1)$ are listed in Tables I and II for ^{120}Sn and ^{208}Pb respectively.

It can be observed that there are two groups of states: the states consisting of configurations belonging to one, neutron or proton, oscillator shell; and the states constructed of

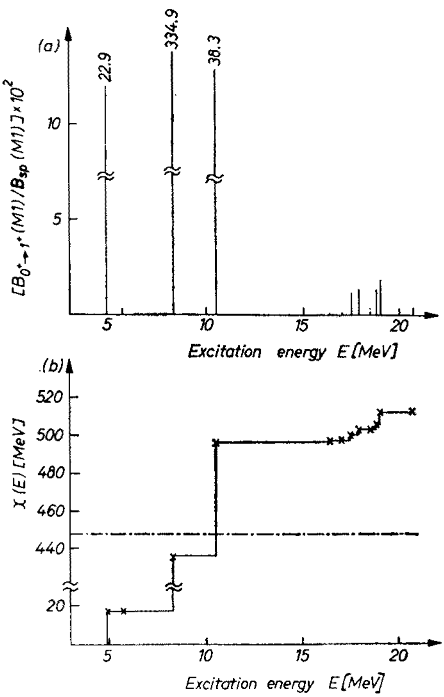

Fig. 1

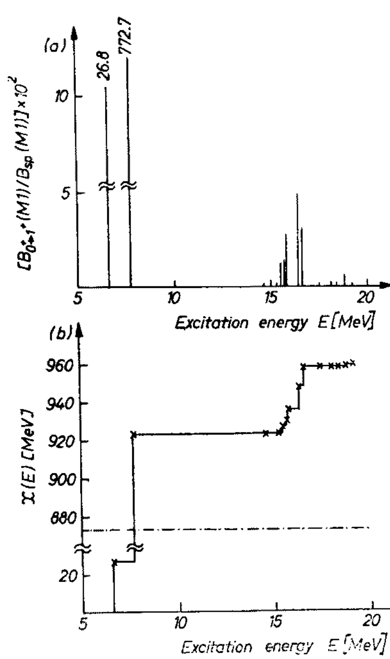

Fig. 2

Fig. 1. Excited states $J^\pi = 1^+$ in ^{120}Sn. a. Energies and widths of the excited states; b. Sum rule for $M1$ electromagnetic transitions. Solid line shows χ calculated in the frame of TDA, dash-and-dot line χ in the single-particle approximation

Fig. 2. Excited states $J^\pi = 1^+$ in ^{208}Pb. a. Energies and widths of the excited states; b. Sum rule for $M1$ electromagnetic transitions. Solid line shows χ calculated in the frame of DTA, dash-and-dot line χ in the single-particle approximation

configurations of $\Delta N = 2$. In the case of $\Delta N = 2$ the states have rather small transition probabilities. This is mainly due to the very small overlap of radial wave functions

$$\int_0^{R_0} R_{n_2 l_2 j_2}(r) R_{n_1 l_1 j_1}(r) dr.$$

However, in the case of ^{208}Pb the five states of the second group at the energy region from 15.5 MeV to 16.6 MeV have comparatively larger widths $\Gamma_{0^+\to 1^+_n}(M1)$, of the order

TABLE I

Some properties of states $J^{\pi} = 1^+$ in ^{120}Sn and their structure

E [MeV]	$B_{0^+\to 1^+}$	K	$\Gamma_{0^+\to 1^+}(M1)$ [W. u.]	Structure	X
3.20	0.00	1.55	0.00	$3s_{1/2}-2d_{3/2}(n)$	−1.00
8.33	6.00	1.02	3.35	$1g_{9/2}-1g_{7/2}(n)$	0.12
				$1g_{9/2}-1g_{7/2}(p)$	1.00
10.52	0.69	2.00	0.38	$1g_{9/2}-1g_{7/2}(n)$	0.99
				$1g_{9/2}-1g_{7/2}(p)$	−0.11
17.54	0.02	1.32	0.01	$1f_{5/2}-2f_{7/2}(n)$	−0.67
				$2p_{3/2}-3p_{3/2}(n)$	−0.60
				$2p_{3/2}-3p_{1/2}(n)$	0.10
				$2p_{1/2}-3p_{1/2}(n)$	0.42
17.92	0.02	1.62	0.01	$1f_{5/2}-2f_{7/2}(n)$	0.66
				$2p_{3/2}-3p_{3/2}(n)$	−0.52
				$2p_{3/2}-3p_{1/2}(n)$	0.44
				$2p_{1/2}-3p_{3/2}(n)$	0.22
				$2p_{1/2}-3p_{1/2}(n)$	0.21
				$1d_{3/2}-2d_{5/2}(p)$	0.13
18.52	0.00	1.36	0.00	$1f_{5/2}-3p_{3/2}(n)$	1.00
18.76	0.02	1.36	0.01	$2p_{3/2}-3p_{3/2}(n)$	0.20
				$2p_{3/2}-3p_{1/2}(n)$	0.60
				$1d_{3/2}-2d_{5/2}(p)$	−0.76
				$2s_{1/2}-3s_{1/2}(p)$	−0.11
19.00	0.03	2.32	0.02	$1f_{5/2}-2f_{7/2}(n)$	−0.25
				$2p_{3/2}-3p_{3/2}(n)$	0.30
				$2p_{3/2}-3p_{1/2}(n)$	0.64
				$2p_{1/2}-3p_{3/2}(n)$	−0.15
				$1d_{3/2}-2d_{5/2}(p)$	0.59
				$2s_{1/2}-3s_{1/2}(p)$	0.26
20.65	0.00	2.56	0.00	$1d_{3/2}-2d_{5/2}(p)$	−0.24
				$2s_{1/2}-3s_{1/2}(p)$	0.96

The parameters K and X are defined in Section 2.

of 10^{-2} W. u. For the last of these states, 16.56 MeV, the ratio $K = B_{0^+\to 1^+_n}(M1)/B^g_{0^+\to 1^+_n}(M1)$ equals 3.9, which suggests its collective structure.

In ^{120}Sn states of the second group appear less distinctly; this also applies to the case of $\Delta N = 0$. They are shifted to the higher energy region, and on the average have the strength of the electromagnetic $M1$ transition smaller by half than that of ^{208}Pb.

It can be seen from Tables I and II that mixing of configurations $\Delta N = 0$ and $\Delta N = 2$ is small. For example, the energy of 7.82 MeV state in ^{208}Pb takes the value 7.73 MeV when the configurations $\Delta N = 2$ are taken into account, the width $\Gamma_{0^+\to 1^+}(M1)$ changes value from 8.05 W. u. to 7.73 W. u.

The sum rule in the case of ^{208}Pb as well as of ^{120}Sn is practically saturated with $\Delta N = 0$ states. The $\Delta N = 2$ states give 17 MeV and 37 MeV, or 3.4% and 3.8%, contri-

TABLE II

Some properties of states $J^{\pi} = 1^{+}$ in ^{208}Pb and their structure

E [MeV]	$B_{0^+\to 1^+}(M1)$	K	$\Gamma_{0^+\to 1^+}(M1)$ [W. u.]	Structure	X
6.63	0.48	0.07	0.27	$1i_{13/2}-1i_{11/2}(n)$	0.61
				$1h_{11/2}-1h_{9/2}(p)$	0.81
7.73	13.9	1.97	7.73	$1i_{13/2}-1i_{11/2}(n)$	−0.80
				$1h_{11/2}-1h_{9/2}(p)$	0.60
15.46	0.02	1.16	0.01	$1g_{7/2}-2g_{9/2}(n)$	−0.48
				$2d_{5/2}-3d_{5/2}(n)$	−0.51
				$2d_{3/2}-3d_{5/2}(n)$	0.15
				$2d_{3/2}-4s_{1/2}(n)$	0.10
				$2d_{3/2}-3d_{3/2}(n)$	−0.66
				$3s_{1/2}-4s_{1/2}(n)$	−0.19
15.68	0.02	1.51	0.01	$1g_{7/2}-2g_{9/2}(n)$	−0.14
				$2d_{3/2}-3d_{3/2}(n)$	0.24
				$3s_{1/2}-3d_{3/2}(n)$	−0.85
15.76	0.05	1.29	0.03	$1g_{7/2}-2g_{9/2}(n)$	0.67
				$2d_{3/2}-3d_{5/2}(n)$	0.22
				$2d_{3/2}-4s_{1/2}(n)$	0.18
				$2d_{3/2}-3d_{3/2}(n)$	−0.39
				$3s_{1/2}-4s_{1/2}(n)$	−0.15
				$1f_{5/2}-2f_{7/2}(p)$	0.16
16.35	0.09	1.38	0.05	$2d_{5/2}-3d_{5/2}(n)$	−0.32
				$2d_{3/2}-3d_{3/2}(n)$	0.27
				$1f_{7/2}-2f_{7/2}(p)$	0.17
				$1f_{5/2}-2f_{7/2}(p)$	0.88
16.56	0.07	3.96	0.04	$1g_{7/2}-2g_{9/2}(n)$	0.24
				$1g_{7/2}-2g_{7/2}(n)$	−0.10
				$2d_{5/2}-3d_{5/2}(n)$	−0.64
				$2d_{5/2}-3d_{3/2}(n)$	0.28
				$2d_{3/2}-3d_{5/2}(n)$	0.20
				$2d_{3/2}-3d_{3/2}(n)$	0.46
				$3s_{1/2}-4s_{1/2}(n)$	−0.19
				$1f_{7/2}-2f_{7/2}(p)$	−0.13
				$1f_{5/2}-2f_{7/2}(p)$	−0.38
17.47	0.00	4.04	0.00	$1g_{7/2}-3d_{5/2}(n)$	1.00
19.22	0.01	2.30	0.01	$1g_{7/2}-2g_{7/2}(n)$	−0.12
				$2d_{5/2}-2g_{7/2}(n)$	0.18
				$1f_{7/2}-1h_{9/2}(p)$	−0.97

The parameters K and X are defined in Section 2.

butions to the sum rule for ^{120}Sn and ^{208}Pb respectively. Let us point out that the single-particle evaluation of the left-hand side of formula (6) (448 MeV for ^{120}Sn and 873 MeV for ^{208}Pb in TDA) suggests strong electromagnetic transitions of type $M1$ in the case of lead. This is due to the completely filled $1i_{13/2}$ and empty $1i_{11/2}$ neutron shells. It applies also to the $1h_{11/2}$ and $1h_{9/2}$ proton shells.

In conclusion we can say that the excited states $J^\pi = 1^+$ in spherical medium and heavy nuclei are spin vibrations within one oscillator shell. Such states contribute most to the strength of electromagnetic $M1$ transitions. Because of their large width $\Gamma_{0^+ \to 1^+}(M1)$, for example 7.73 W. u. in the case of ^{208}Pb, they can manifest themselves in various nuclear reactions, such as the radiative capture of nucleons.

The DTA and RPA methods give similar results, which is a feature of finite range forces [8]. For this reason we present the results obtained by means of the RPA only.

REFERENCES

[1] Z. Bochnacki, S. Ogaza, *Nucl. Phys.* **69**, 186 (1965).
[2] A. Bohr, B. R. Mottelson, *Nuclear Structure*, Benj. Inc. 1969, v. I, p. 336.
[3] J. Fujita, K. Ikeda, *Nucl. Phys.* **67**, 145 (1965).
[4] J. Halbleib, R. A. Sørensen, *Nucl. Phys.* **A98**, 542 (1967).
[5] S. I. Gabrakov, A. A. Kuliev, *preprint* JINR, P4-5003, Dubna 1970.
[6] S. Ćwiok, M. Wygonowska, *Acta Phys. Pol.* **B4**, 233 (1973).
[7] F. Bechvarj et al., *preprint* JINR, P3-3696, Dubna 1968.
[8] M. Jean, *Proc. of International School of Physics Enrico Fermi*, 1969.