KINEMATIC CONSTRAINTS ON RHA $\bar{1}+0 \to 1/2+\bar{1}/2'$ REACTIONS

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The complete set of kinematic constraints on the regularized helicity amplitudes (RHA) for $\bar{1}+0 \to 1/2+\bar{1}/2'$ reactions is given. All interesting external mass configurations are covered and both parities of the reaction are taken into account.

1. Introduction

The question of kinematic singularities and zeros of two body helicity amplitudes and the existence of relationships between various two body helicity amplitudes at kinematic thresholds, pseudothresholds, and at $t = 0$ (t-square of c.m.s. energy), have received considerable attention in the past few years. The first problem — the kinematic singularity and zero structure of two body helicity amplitudes — has been investigated thoroughly from a number of points of view (see e.g. Refs [1–15]) since the original work of Hara [1] and Wang [2]. Now it seems the problem has been solved; one knows how to construct, in a quick and effective way, helicity amplitudes free from kinematic singularities and zeros (see e.g. Cohen-Tannoudji, Morel, and Navelet [4]). These regularized helicity amplitudes (RHA) are extensively used in Regge-pole models.

The problem of the existence of relationships between various helicity amplitudes has also been investigated extensively by many authors (see e.g. Refs [3–12, 16–30)]. The general method which has been formulated by Cohen-Tannoudji, Morel, and Navelet [4] consists in writing the crossing matrix for transversity amplitudes and looking for the singularities of its elements. This yields linear combinations of helicity amplitudes (and sometimes of derivatives of helicity amplitudes) that vanish for some special value of the t-variable. Concerning the case of all four external masses different, Cohen-Tannoudji, Morel and Navelet [4] have reached the conclusion that no constraint on helicity amplitudes can be found for vanishing c.m.-squared energy. However, in a Regge-pole model one is interested, first of all, in relationships between natural parity conserving amplitudes

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(NPCA). Taking into account the kinematic constraints on NPCA, one obtains some information either about possible relations between the residues of given Regge poles or about the existence of families of trajectories, depending on the ideas one may have about the question of evasion or conspiracy. The consequences of such relationships can propagate through interchannel and vertex factorizations to other channels (see e.g. Refs [29, 38]). We emphasize the importance of constraints on NPCA instead of constraints on helicity amplitudes, because, as has been noted by Frautschi and Jones [27] (for $\pi N \to \varrho\Delta$-type reactions) and by Ball, Frazer, and Jacob [30] (for $\gamma N \to VN$-type reactions), there are constraints on NPCA, which yield no constraints on the helicity amplitudes.

For the s-channel process $a+b \to c+d$ the differential cross section is

$$\frac{d\sigma}{dt} = \frac{1}{64sp_{\text{in}}^2} \cdot \frac{1}{(2s_a+1)(2s_b+1)} \sum_{\{\lambda\}} |{}^sM_{\lambda_a,\lambda_b}^{\lambda_c,\lambda_d}|^2, \tag{1}$$

where ${}^sM_{\lambda_a,\lambda_b}^{\lambda_c,\lambda_d}$ are the s-channel helicity amplitudes. In order to describe scattering in terms of exchanges it is more convenient to rewrite $d\sigma/dt$ and decay density matrix elements in terms of the t-channel amplitudes [31] (but see also Ref. [41]). The first obvious requirement in using the t-channel amplitudes is to incorporate the proper kinematic singularity and zero structure. This is done automatically in perturbation theory or with the use of invariant amplitudes. But in Regge-pole models with helicity amplitudes, the requirement must be explicitly imposed and this is usually imposed by multiplying the Regge-type term with the proper kinematic factor. However, this quite natural parametrization includes some disadvantages which can lead to incorrect inferences concerning, e.g., the dynamic behaviour of reduced residues. The examination of such formulae shows that if the kinematic singularity and zero structure is imposed but the constraints are not imposed then

a) assumed Mandelstam Analyticity of invariant amplitudes is violated, so the self-consistency of the model can be questioned;

b) the phenomenological expressions for the physical differential cross section and decay density matrix elements contain small t-dependent kinematic factors as denominators which may strongly influence results of the analysis of experimental data, at least for small $t \leqslant t_0$ ($|t_0|$ — the lowest physical $|t|$ for the s-channel process) (see e.g. Refs [6, 23]);

c) the expressions for $d\sigma/dt$ and decay density matrix elements are of the form which seems to be sensitive to details of a routine such as, e.g., the interpolation step, at least for small $t \leqslant t_0$.

Let us consider, e.g., the $p+K^- \to \Lambda+\varrho^0$ reaction. Using the formulae (17) and (B3) one can find that, e.g., the invariant amplitude A_5 (see next Sections for the explanation of the symbols) contains the term

$$\frac{F^2_{-\lambda,\lambda;2\lambda,0} - (\varphi_{34})^{-2}(4tpqz)F^1_{-\lambda,\lambda;2\lambda,0}}{t},$$

so if the constraint (18) is not imposed, the assumed analytic structure of A_5 is violated at $t = 0$, the point which, at least in principle (for $s \to \infty$), is the physical point of the s-channel. From the equations (B3) it follows that the Regge-type expression for the helicity amplitude $M_{0,0}^{\lambda,\lambda}$ will contain the factor Ψ_{34}^{-1} so the phenomenological differential cross section[1] will contain the factor

$$|(m_\Lambda - m_p)^2 - t|^{-1}.$$

The quantity $|(m_\Lambda - m_p)^2 - t|$ vanishes for $t = (m_\Lambda - m_p)^2 \approx 0.045\,(\mathrm{GeV}/c)^2$ which is near to $t = 0$. Therefore this sharply peaked kinematic factor governs the small t-behaviour of the Regge-type term and requires a kinematic "dip" in the corresponding reduced residue function in order to fit the experimental data. The presence of such small terms in the denominators of the phenomenological expressions for the differential cross section and decay density matrix elements is also highly undesirable as regards computational techniques. If interpolation steps are small enough (they have to be small!) then the presence of such terms can yield unpalatable, rapidly varying, reduced residues (for small $t \leqslant t_0$ at least). It is clear that all these disadvantages can be removed by building in the constraints on RHA in a Regge-type parametrization of the t-channel helicity amplitudes. The price one has to pay following this line — an additional smoothness of reduced residues — does not seem to be too high.

To the best of our knowledge the constraints on RHA for 1) ($\bar{1}+0 \to 1/2+\bar{1}/2'$, $\eta = +1$) reactions[2] and 2) ($\bar{1}+0 \to 1/2+\bar{1}/2'$, $\eta = -1$) — for all external masses different, are not available in the literature (constraints on the helicity amplitudes for $\bar{V}+P \to \mathrm{N}+\bar{\mathrm{N}}$ are already known, see, e.g., Refs [19, 24, 27]). The purpose of this paper is to present the complete set of constraints on RHA for $\bar{1}+0 \to 1/2+\bar{1}/2'$ reactions. All interesting mass configurations are covered, and both parities of the reaction are taken into account. No great originality is claimed; the paper is technical in nature. We have selected $\bar{1}+0 \to$ $\to 1/2+\bar{1}/2'$ reactions for considerations because

a) there is much experimental high energy data for $1/2'+0 \to 1/2+1$ reactions available at present, so one may readily apply our results for a fitting;

b) our personal interest in the problem of helicity conservation in processes like $\mathrm{N}+\pi \to \mathrm{N}+A_1$ where Regge-pole analysis seems to be very sensitive to such constraints.

It seems to us that for low spin reactions the generality of the C-T.M.N method [4] results in too tedious calculations with many peculiarities and subtlties which must be handled with great care (see e.g. Ref. [35]). So we have decided in favour of the use of invariant amplitudes which, with the kinematic structure exhibited explicitly, offers a more painless approach for low spin reactions.

2. *Kinematics*

We define now the notation which will be used all along this paper for the $\bar{1}+0 \to F_1+\bar{F}_2$ reaction (Fig. 1) where $\bar{1}$ stands for a spin 1 meson, 0 stands for a spin 0 meson, F_1 stands for a spin 1/2 baryon, $\bar{F}_2$ stands for a spin 1/2 antibaryon. We shall

[1] We do not discuss the behaviour of $d\sigma/dt$ when the variables (s, t) lie outside the physical region of the s-channel. We refer the reader to Refs [32–34] for a discussion of such case.

[2] The parity of the reaction η is the product of intrinsic parities of particles: $\eta = \eta_1\eta_2\eta_3\eta_4$.

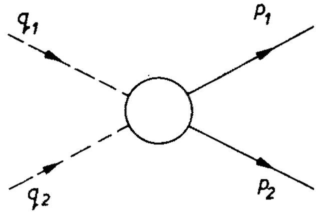

Fig. 1

call q_1, q_2, p_1, p_2 respectively, the four-momenta of spin 1 meson, spin 0 meson, baryon and antibaryon and μ_1, μ, M, m their masses. We define the (t, s, u) variable as follows:

$$t = (q_1+q_2)^2, \quad s = (q_1-p_1)^2, \quad u = (q_1-p_2)^2.$$

In the centre of mass frame we write the four vector as

$$(q_1^\nu) = (\omega_1, \vec{q}), \quad (q_2^\nu) = (\omega, -\vec{q}), \quad (p_1^\nu) = (E', \vec{p}), \quad (p_2^\nu) = (E, -\vec{p})$$

and

$$\vec{p}\vec{q} = pqz, \; z \equiv \cos\vartheta.$$

Finally it is worth noticing the useful identities[3] [4]:

$$\left.\begin{aligned} 2[(\omega_i \pm m_i)(\omega_j \pm m_j)]^{1/2} &= [1 \pm (m_i+m_j)t^{-1/2}]\Psi_{ij} \\ 2[(\omega_i \pm m_i)(\omega_j \mp m_j)]^{1/2} &= [1 \pm (m_i-m_j)t^{-1/2}]\varphi_{ij} \end{aligned}\right\} \begin{aligned} i &= 1,2 \leftrightarrow j = 2,1 \\ i &= 3,4 \leftrightarrow j = 4,3, \end{aligned}$$

where

$$\varphi_{ij} = [t-(m_i+m_j)^2]^{1/2},$$
$$\Psi_{ij} = [t-(m_i-m_j)^2]^{1/2};$$
$$m_1 \equiv \mu_1, m_2 \equiv \mu, m_3 \equiv M, m_4 \equiv m \quad \text{and} \quad \omega_2 \equiv \omega, \omega_3 \equiv E', \omega_4 \equiv E.$$

3. Helicity amplitudes, RHA and constraints on RHA

We expand the helicity amplitude $M_{\Lambda,0}^{\lambda^*,\lambda}$ on six following terms

$$M_{\Lambda,0}^{\lambda^*,\lambda} = \sum_{i=1}^{6} A_i \bar{u}(p, \lambda^*)\varepsilon_\nu^*(q, \Lambda)K_i^\nu v(p, \lambda), \tag{2}$$

where A_i are invariant amplitudes, K_i^ν kinematic covariants and λ^* (λ) stands for the helicity of baryon F_1 (antibaryon $\overline{F}_2$) respectively and Λ stands for the helicity of spin 1 meson. The explicit forms of the wave functions are given in Appendix A.

It is important that the kinematic covariants should be chosen in such a way that the decomposition (2) does not introduce kinematic singularities and zeros into the invariant amplitudes A_i. We have chosen the sets of kinematic covariants K_i^ν proposed by Scadron

[3] The expressions for ω_i, $\cos\vartheta$, $\sin\vartheta$, in terms of the (t, s, u) variables can be easily obtained, e. g., by the trivial change of the symbols: $s \leftrightarrow t$, in the corresponding formulae of Ref. [4] (p. 247).

and Jones [37] (see Table IV of their paper). These sets possess the required property that the invariant amplitudes A_i are free from the kinematic singularities and zeros. We refer the reader to Ref. [37] for the detailed discussion of this and related problems. We postulate, as usual, that the invariant amplitudes A_i satisfy Mandelstam Analyticity.

Since the explicit form of the kinematic covariants depends on the parity of reaction we separate the rest of this section into two parts. In the first part we consider $\eta = +1$ reactions, e.g., $\bar{A}_1+P \to \mathrm{N}+\bar{\mathrm{N}}$, and in the second we deal with $\eta = -1$ reactions, e.g., $\bar{V}+P \to \mathrm{N}+\bar{\mathrm{N}}$. In both parts we use the same set of symbols for the kinematic variables and the invariant and helicity amplitudes to avoid an orgy of indices. We hope it will not bring about a misinterpretation of our formulae. It is obvious that, in general, no physical connection exists between $\eta = +1$ and $\eta = -1$ reactions.

Part I: $\eta = +1$

Our choice leads to the following set of covariants $\{K_i^\nu\}$ [37]:

$$K_1^\nu = P^\nu, \qquad K_4^\nu = Q^\nu\hat{Q},$$
$$K_2^\nu = Q^\nu, \qquad K_5^\nu = \gamma^\nu,$$
$$K_3^\nu = P^\nu\hat{Q}, \qquad K_6^\nu = [\gamma^\nu, \hat{Q}], \tag{3}$$

where

$$2P^\nu = p_1^\nu - p_2^\nu, \qquad 2Q^\nu = q_2^\nu - q_1^\nu, \qquad \hat{Q} = \gamma^\nu Q_\nu.$$

After some algebraic manipulations, one obtains the helicity amplitudes in terms of invariant amplitudes A_i. The result is

$$2\lambda\mu_1 KM_{0,0}^{\lambda^*,\lambda} = \tfrac{1}{2}\{[2\omega_1 pz+q(E-E')]A_1 - qt^{1/2}A_2\}(\lambda^*+\lambda)pV^+ -$$
$$-\tfrac{1}{4}\{[2\tilde{\omega}_1 pz+q(E-E')]A_3 - qt^{1/2}A_4\}[(\lambda^*+\lambda)(\omega-\omega_1)pV^- + 2qYR^-] +$$
$$+A_5[(\lambda^*+\lambda)pqV^- - \omega_1 YR^-] + A_6[2q^2+\omega_1(\omega-\omega_1)]YR^+,$$
$$2\lambda\Lambda\sqrt{2}\,KM_{\Lambda,0}^{\lambda^*,\lambda} = (\lambda^*+\lambda)p^2xV^+A_1 - \tfrac{1}{2}px[(\lambda^*+\lambda)(\omega-\omega_1)pV^- + 2qYR^-]A_3 -$$
$$-2\lambda\{R^-A_5 - [2\Lambda pqV^+ + (\omega-\omega_1)R^+]A_6\}B, \tag{4}$$

where

$$x = \sin\vartheta,$$
$$B = (\lambda^*+\lambda)x - \Lambda(\lambda^*-\lambda)(1-2\lambda\Lambda z),$$
$$K = \left[\frac{4mM}{(E+m)(E'+M)}\right]^{1/2}, \qquad R^\pm = 1 \pm 4\lambda^*\lambda\frac{E-m}{E'+M},$$
$$V^\pm = \frac{2\lambda^*}{E'+M} \pm \frac{2\lambda}{E+m}, \qquad Y = 2\lambda(\lambda^*+\lambda)z - (\lambda^*-\lambda)x,$$

here and through the rest of the paper (barring Appendix A) the symbol Λ is taken to be (∓ 1).

The expressions (4) contain all information of interest concerning kinematic singularity and zero structure of helicity amplitudes because the only singularities and zeros of A_i are dynamical ones. The relations between RHA and helicity amplitudes (HA) depend critically on external mass configurations in contrast to the relations (4) (HA versus A_i), which hold for any external mass configuration (provided $m_i \neq 0$). We focus our attention on the two-mass configurations ($\mu \neq \mu_1, m \neq M$) and ($\mu \neq \mu_1$, $m = M$) as the only ones important for applications. Using Tables (IV, VIII) given in Ref. [4] one easily obtains RHA in terms of HA and, for the convenience of the reader, the corresponding expressions are given in Appendix B. Combining the relations (4) and (B1, B2) (see Appendix B) one can get the regularized helicity amplitudes $F^i_{\{\lambda\}}$ in terms of invariant amplitudes A_i. The examination of such relations (i.e. $F^i_{\{\lambda\}}$ versus A_i) shows that for $t \to 0$, $t \to (m \mp M)^2$, and $t \to (\mu_1 \mp \mu)^2$ the relations among $F^i_{\{\lambda\}}$ exist, requiring only that invariant amplitudes A_i are finite at these points[4]. After some calculations one gets

I. $\mu_1 \neq \mu$, $M \neq m$.

$t \to 0$,

$$F^1_{-\lambda,\lambda;2\lambda,0} - (\Psi_{34})^{-2}(4tpqz)F^2_{-\lambda,\lambda;2\lambda,0} \sim t; \tag{5}$$

$t \to (M-m)^2$,

$$F^2_{\lambda,\lambda;\Lambda,0} - 2\lambda(M-m)t^{-1}F^2_{-\lambda,\lambda;2\lambda,0} \sim \Psi^2_{34}; \tag{6}$$

$t \to (\mu_1 \mp \mu)^2$,

$$\mu_1 F^1_{-\lambda,\lambda;0,0} - \lambda\sqrt{2}\,(t+\mu_1^2-\mu^2)t^{-1}F^1_{-\lambda,\lambda;2\lambda,0} \sim \varphi^2_{12}\Psi^2_{12}, \tag{7}$$

$$4\mu_1 F^2_{\lambda,\lambda;0,0} - \Lambda\sqrt{2}\,(4pqz)\,(t+\mu_1^2-\mu^2)F^1_{\lambda,\lambda;\Lambda,0} \sim \varphi^2_{12}\Psi^2_{12}; \tag{8}$$

$t \to (M+m)^2$,

$$4\lambda\mu_1 F^2_{\lambda,\lambda;0,0} - \mu_1[(m+M)\,(s-u)+(m-M)\,(\mu^2-\mu_1^2)]F^1_{-\lambda,\lambda;0,0} \sim \varphi^2_{34}, \tag{9}$$

$$\Lambda(m+M)F^1_{\lambda,\lambda;\Lambda,0} - F^1_{-\lambda,\lambda;2\lambda,0} \sim \varphi^2_{34}. \tag{10}$$

II. $\mu_1 \neq \mu$, $M = m$.

$t \to 0$,

$$2\lambda F^1_{-\lambda,\lambda;2\lambda,0} + m(\mu_1^2-\mu^2)F^2_{\lambda,\lambda,\Lambda,0} \sim t; \tag{11}$$

$t \to (\mu_1 \mp \mu)^2$,

$$\mu_1 F^1_{-\lambda,\lambda;0,0} - \lambda\sqrt{2}\,(t+\mu_1^2-\mu^2)t^{-1}F^1_{-\lambda,\lambda;2\lambda,0} \sim \varphi^2_{12}\Psi^2_{12}, \tag{12}$$

$$8\mu_1 F^2_{\lambda,\lambda;0,0} - \sqrt{2}\,(t+\mu_1^2-\mu^2)\,(s-u)\Lambda F^1_{\lambda,\lambda;\Lambda,0} \sim \varphi^2_{12}\Psi^2_{12}; \tag{13}$$

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[4] This may be shown by expressing A_i in terms of $F^i_{\{\lambda\}}$.

$t \to 4m^2$,

$$4\lambda\mu_1 F^1_{\lambda,\lambda;0,0} - \mu_1 m(s-u) F^1_{-\lambda,\lambda;0,0} \sim \varphi^2_{34}, \tag{14}$$

$$2m\Lambda F^1_{\lambda,\lambda;\Lambda,0} - F^1_{-\lambda,\lambda;2\lambda,0} \sim \varphi^2_{34}. \tag{15}$$

Eight relations (5)–(10) (or seven relations (11)–(15)) are just the constraints on RHA which have to hold in any Regge-type model[5] for $1/2'+0 \to 1/2+1$, ($\eta = +1$) reactions which pretends to be at least kinematically correct. The physical information one may extract from the relations (5)–(15) are similar in many respects to information one may obtain from relations (18)–(28), so we postpone the discussion of both cases to Section 4.

Part II: $\eta = -1$

One can get the set of the covariants $\{\tilde{K}^\nu_i\}$ for the ($\eta = -1$) reactions by multiplying each term of the set (3) by the γ_5-matrix[6] i.e.:

$$\tilde{K}^\nu_i = \gamma_5 K^\nu_i. \tag{16}$$

Starting from the set (16) one obtains HA[7] in terms of invariant amplitudes A_i:

$$\begin{aligned}
2\lambda\mu_1 KM^{\lambda^*,\lambda}_{0,0} &= -\tfrac{1}{2}\{[2\omega_1 pz + q(E-E')]A_1 - qt^{1/2}A_2\}(\lambda^*+\lambda)R^+ + \\
&+\tfrac{1}{4}\{[2\omega_1 pz + q(E-E')]A_3 - qt^{1/2}A_4\}[(\lambda^*+\lambda)(\omega-\omega_1)R^- + 2pqYV^-] - \\
&-A_5[(\lambda^*+\lambda)qR^- - \omega_1 pYV^-] - p[2q^2 + \omega_1(\omega-\omega_1)]YV^+A_6, \\
2\lambda\Lambda\sqrt{2}\,KM^{\lambda^*,\lambda}_{\Lambda,0} &= -(\lambda^*+\lambda)pxR^+A_1 + \tfrac{1}{2}p[(\lambda^*+\lambda)(\omega-\omega_1)R^- + 2pqYV^-]xA_3 + \\
&+2\lambda\{pV^-A_5 - [2\Lambda qR^+ + p(\omega-\omega_1)V^+]A_6\}B.
\end{aligned} \tag{17}$$

Combining relations (17) and (B3), (B4) (see Appendix B) one can obtain the regularized helicity amplitudes $F^i_{\{\lambda\}}$ in terms of invariant amplitudes and likewise for the $\eta = +1$ case one can show that the relations among $F^i_{\{\lambda\}}$ exist. After some calculations one obtains the following constraints on $F^i_{\{\lambda\}}$:

I. $\mu_1 \neq \mu$, $\quad M \neq m$.

$t \to 0$,

$$F^2_{-\lambda,\lambda;2\lambda,0} - (\varphi_{34})^{-2}(4tpqz)F^1_{-\lambda,\lambda;2\lambda,0} \sim t; \tag{18}$$

$t \to (M-m)^2$,

$$\mu_1 F^1_{\lambda,\lambda;0,0} + \lambda\mu_1[(m-M)(s-u) + (m+M)(\mu^2-\mu_1^2)]F^2_{-\lambda,\lambda;0,0} \sim \Psi^2_{34}, \tag{19}$$

$$F^2_{-\lambda,\lambda;2\lambda,0} + \Lambda(m-M)F^2_{\lambda,\lambda;\Lambda;0} \sim \Psi^2_{34}; \tag{20}$$

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[5] We use the term "Regge-type model" as a synonym for the expression "the formulation of a Regge pole and cut model in terms of the t-channel helicity amplitudes". For the other point of view see e. g. Ref. [41].

[6] We work in a representation where $\gamma_5^2 = 1$ (see Ref. [39]).

[7] For the determination of the T matrix see Ref. [37].

$t \to (\mu_1 \mp \mu)^2$,

$$4\lambda\mu_1 F^2_{-\lambda,\lambda;0,0} - \sqrt{2}\, t^{-1}(t+\mu_1^2-\mu^2) F^2_{-\lambda,\lambda;2\lambda,0} \sim \varphi_{12}^2 \Psi_{12}^2, \tag{21}$$

$$4\mu_1 F^1_{\lambda,\lambda;0,0} - \Lambda \sqrt{2}\,(t+\mu_1^2-\mu^2)\,(4pqz) F^2_{\lambda,\lambda;\Lambda,0} \sim \varphi_{12}^2 \Psi_{12}^2; \tag{22}$$

$t \to (M+m)^2$,

$$2\lambda F^1_{\lambda,\lambda;\Lambda,0} - (m+M) t^{-1} F^1_{-\lambda,\lambda;2\lambda,0} \sim \varphi_{34}^2. \tag{23}$$

II. $\mu_1 \neq \mu, \quad M = m.$

$t \to 0$,

$$\mu_1 F^1_{\lambda,\lambda;0,0} - m\lambda(\mu_1^2-\mu^2) F^2_{-\lambda,\lambda;0,0} \sim t, \tag{24}$$

$$\Lambda(\mu_1^2-\mu^2) F^2_{\lambda,\lambda;\Lambda,0} - 2m\varphi_{12}^2 \Psi_{12}^2 (\varphi_{34})^{-2} F^1_{-\lambda,\lambda;2\lambda,0} \sim t; \tag{25}$$

$t \to (\mu_1 \mp \mu)^2$,

$$4\lambda\mu_1 F^2_{-\lambda,\lambda;0,0} - \sqrt{2}\,(t+\mu_1^2-\mu^2) F^2_{-\lambda,\lambda;2\lambda,0} \sim \varphi_{12}^2 \Psi_{12}^2, \tag{26}$$

$$4\mu_1 F^1_{\lambda,\lambda;\Lambda,0} - \Lambda(2)^{-1/2}(t+\mu_1^2-\mu^2)\,(s-u) F^2_{\lambda,\lambda;\Lambda,0} \sim \varphi_{12}^2 \Psi_{12}^2; \tag{27}$$

$t \to 4m^2$,

$$2\lambda t F^1_{\lambda,\lambda;\Lambda,0} - m F^1_{-\lambda,\lambda;2\lambda,0} \sim \varphi_{34}^2. \tag{28}$$

From relations (24), (25) one easily obtains well known results of Högaasen and Salin [19] for $VP \to N\overline{N}$-type reactions (see also Eqs (2.3) and (2.4) of Ref. [27]).

Before we discuss the content of the constraints on $F^i_{\{\lambda\}}$ it seems worthwhile to make two remarks about the amplitudes $F^i_{\{\lambda\}}$.

1. It is worth noting that for the $(m_3 \neq m_4)$ mass case there exists a formal "substitution rule" that may be used for the verification of a part of our calculation.

Let us denote the RHA for the two reactions

$$\eta = -1,\ (1, m_1, \lambda_1) + (0, m_2, 0) \to (1/2, m_3, \lambda_3) + (\overline{1}/2', m_4, \lambda_4)$$

and

$$\eta = +1,\ (1, m_1, \lambda_1) + (0, m_2, 0) \to (1/2, m_3, \lambda_3) + (\overline{1}/2', m_4, \lambda_4),$$

by

$$F^i_{\lambda_3,\lambda_4;\lambda_1,0}(-1; m_1, m_2; m_3, m_4) \quad \text{and} \quad F^i_{\lambda_3,\lambda_4;\lambda_1,0}(+1; m_1, m_2; m_3, m_4)$$

respectively.

Now if we replace every symbol A_i in an expression for $F^i_{\{\lambda\}}(-1, m_1, m_2, m_3, m_4)$ (i.e. F_i versus A_i) by a corresponding symbol for the $\eta = +1$ reaction (i.e. $A_i(\eta = -1) \to$ $\to A_i(\eta = +1)$) then the following "substitution rule" holds:

$$i2\lambda_4(-1)^{\lambda+\mu+|\lambda|-1} F^k_{\lambda_3,\lambda_4;\lambda_1,0}(-1; m_1, m_2; -m_3, m_4) \to F^l_{\lambda_3,\lambda_4;\lambda_1,0}(+1; m_1, m_2; m_3, m_4),$$

where

$$\lambda = \lambda_1, \quad \mu = \lambda_3 - \lambda_4, \quad i(-8mM)^{1/2} = +(8mM)^{1/2}, \quad k \neq l, \quad k, l = 1, 2.$$

We emphasize that this formal rule, which has no dynamical content, exists for the $(m_3 \neq m_4)$ mass case only.

2. Define the natural parity amplitude $f^N_{\{\lambda\}}$

$$f^N_{\lambda_3,\lambda_4;\lambda_1,\lambda_2} = \hat{M}^{\lambda_3,\lambda_4}_{\lambda_1,\lambda_2} + N\eta_1\eta_2(-1)^{M+\mu+v-S}\hat{M}^{\lambda_3,\lambda_4}_{-\lambda_1,-\lambda_2}, \tag{29}$$

where $N = \mp 1$ is the naturality, $M = \max\{|\bar{\lambda}|, |\mu|\}$, $S = s_1+s_2$, $v = 0$ for integer S and $v = 1/2$ for half-odd-integral S (for the explanation for other symbols see Appendix B). For the cases we deal with, the expression (29) takes the form

$$f^N_{\lambda_3,\lambda_4;\lambda_1,0} = \hat{M}^{\lambda_3,\lambda_0}_{\lambda_1,0} - N\eta_1\eta_2(-1)^{M+\mu}\hat{M}^{\lambda_3,\lambda_4}_{-\lambda_1,0}.$$

If follows from the definitions of amplitudes $F^i_{\{\lambda\}}$ (see Appendix B) that they are proportional to corresponding natural parity, amplitudes $f^{N_i}_{\{\lambda\}}$

$$F^i_{\lambda_3,\lambda_4;\lambda_1,0} = R_i f^{N_i}_{\lambda_3,\lambda_4;\lambda_1,0}$$

where R_i is a kinematic factor that can be determined from the relations between the amplitudes $F^i_{\{\lambda\}}$ and amplitudes $\hat{M}^{\lambda_3,\lambda_4}_{\lambda_1,0}$. From now on we will call the amplitude $F^i_{\{\lambda\}}$ a regularized natural (unnatural) parity amplitude RNPA(RUNPA) if $N_i = +1$ $(N_i = -1)$ respectively. Using Table I one can easily show that in the $(\eta = \mp 1, \eta_1\eta_2 = +1)$ case

TABLE I

RHA versus naturality N

$\eta = +1$			$\eta = -1$		
	$\eta_1\eta_2 = +1$	$\eta_1\eta_2 = -1$		$\eta_1\eta_2 = +1$	$\eta_1\eta_2 = -1$
$F^1_{-\lambda,\lambda;0,0}$	RUNPA $(N = -1)$	RNPA $(N = +1)$	$F^1_{\lambda,\lambda;0,0}$	RUNPA $(N = -1)$	RNPA $(N = +1)$
$F^2_{\lambda,\lambda;0,0}$	RUNPA $(N = -1)$	RNPA $(N = +1)$	$F^2_{-\lambda,\lambda;0,0}$	RUNPA $(N = -1)$	RNPA $(N = +1)$
$F^1_{\lambda,\lambda;\Lambda,0}$	RUNPA $(N = -1)$	RNPA $(N = +1)$	$F^1_{\lambda,\lambda;\Lambda,0}$	RNPA $(N = +1)$	RUNPA $(N = -1)$
$F^2_{\lambda,\lambda;\Lambda,0}$	RNPA $(N = +1)$	RUNPA $(N = -1)$	$F^2_{\lambda,\lambda;\Lambda,0}$	RUNPA $(N = -1)$	RNPA $(N = +1)$
$F^1_{-\lambda,\lambda;2\lambda,0}$	RUNPA $(N = -1)$	RNPA $(N = +1)$	$F^1_{-\lambda,\lambda;2\lambda,0}$	RNPA $(N = +1)$	RUNPA $(N = -1)$
$F^2_{-\lambda,\lambda;2\lambda,0}$	RNPA $(N = +1)$	RUNPA $(N = -1)$	$F^2_{-\lambda,\lambda;2\lambda,0}$	RUNPA $(N = -1)$	RNPA $(N = +1)$

only, RUNPA $(N_i = -1)$ contribute to the helicity amplitudes with $\lambda_1 = 0$ (i.e. spin 1 meson helicity 0-states), whereas in the $(\eta = \mp 1, \eta_1\eta_2 = -1)$ case only RNPA$(N_i = +1)$ contribute to the helicity amplitudes $M^{\lambda_3,\lambda_4}_{0,0}$. Both RNPA and RUNPA contribute to the helicity amplitudes $M^{\lambda_3,\lambda_4}_{\Lambda,0}$ (i.e. spin 1 meson helicity $\mp$1-states) in each of the cases[8].

4. *Discussion of results*

1. Since each of the amplitudes $F^i_{\{\lambda\}}$ can be decomposed into amplitudes of definite signature σ

$$2F^i_{\{\lambda\}} = F^{i,+}_{\{\lambda\}} + F^{i,-}_{\{\lambda\}},$$

[8] This is the obvious generalization of well known results for $\bar{V}+P \to N+\bar{N}$ reactions.

(the $\mp$ superscripts to the amplitudes F refer to the value of σ) then our constraints, which are exact properties of RHA, are in the framework of a Regge-pole model constraints on reduced residues and trajectory functions. If the amplitudes $F^i_{\{\lambda\}}$ are dominated by Regge-poles, different poles will dominate different RNPA(RUNPA). The amplitudes $F^0_{-\lambda,\lambda;2\lambda,0}$ and $F^2_{-\lambda,\lambda;2\lambda,0}$ are the amplitudes of different naturalities (see Table I) so, e.g., the constraints (5) and (18) are of a nontrivial nature.

2. Neither the relation (5) nor the relation (18) imposes a constraint on the helicity amplitudes $M^{\lambda_3,\lambda_4}_{\lambda_1,0}$. This is in agreement with the general conclusions of Cohen-Tannoudji, Morel and Navelet [4] about the lack of conspiracy between the helicity amplitudes at $t = 0$ for all external mass different.

3. The examination of our constraints shows that in general there is no continuity in terms of the masses of external particles. However, if one supposes that $F^i_{\{\lambda\}}$ are smooth functions of external masses then one observes a different pattern. Let us consider, e.g., the relations (5) and (6). From these relations it follows that

$$(m-M)^{-1}F^2_{-\lambda,\lambda;2\lambda,0} = [(\mu_1^2-\mu^2)(m+M)]^{-1}F^1_{-\lambda,\lambda;2\lambda,0} \quad \text{at} \quad t = 0, \tag{30}$$

and

$$(m-M)^{-1}F^2_{-\lambda,\lambda;2\lambda,0} = -2\lambda F^2_{\lambda,\lambda;\Lambda,0} \quad \text{at} \quad t = (M-m)^2. \tag{31}$$

Now, if one supposes that $\lim_{m\to M} (m-M)^{-1}F^2_{-\lambda,\lambda;2\lambda,0}$ exists, then it is easy to see that the relations (11)–(15) are the limits of the relations (5)–(10) as the baryon mass difference is put to zero. Consider now relations (18)–(23). One can find again that if $\lim_{m\to M} (m-M)^{-1}F^2_{-\lambda,\lambda;2\lambda,0}$ exists, then the relations (24)–(28) are the limits of the relations (18)–(23) as $m \to M$[9].

If one accepts such additional assumptions then our constraints are practically indifferent to small mass differences so the mass difference between proton and neutron does not change drastically the structure of kinematic constraints on RHA and, as a consequence, does not change the dynamics.

Taking into account that $(m_\Lambda - m_p)^2 \approx 0.045\,(\mathrm{GeV}/c)^2$ one may also expect that

$$(\mu_1^2-\mu^2)(m+M)^{-1}F^1_{-\lambda,\lambda;2\lambda,0} \approx -\Lambda F^2_{\lambda,\lambda;\Lambda,0} \quad \text{at} \quad t = t_1,\ t_1 \approx 0 \tag{32}$$

for the $\varrho^0 + K^- \to \Lambda + \bar{p}$ reaction
and

$$2\lambda[(\mu_1^2-\mu^2)(m+M)]^{-1}F^1_{-\lambda,\lambda;2\lambda,0} \approx -F^2_{\lambda,\lambda;\Lambda,0} \quad \text{at} \quad t = t_2,\ t_2 \approx 0 \tag{33}$$

for the $A_1^0 + K^- \to \Lambda + \bar{p}$ reaction.

Since the amplitudes $F^1_{-\lambda,\lambda;2\lambda,0}$ and $F^2_{\lambda,\lambda;\Lambda,0}$ are the amplitudes of different naturalities, the relations (32), (33) are of a nontrivial nature.

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[9] The definitions of some amplitudes $F^i_{\{\lambda\}}$ (see Appendix B) differ by the factor 2 for $(m \neq M)$ and $(m = M)$ cases and this has to be taken into account in order to obtain the exact limiting form of constraints as $m \to M$.

4. It is worth noting that in the framework of a Regge-pole model each of the terms involved in a constraint that belongs to the sets (5)–(15) or (18)–(23) simulates the same type of asymptotic behaviour. This is not the case for the ($\eta = -1$, $\mu_1 \neq \mu$, $M = m$) mass configuration; the relation (24) constraints the two amplitudes — $F^1_{\lambda,\lambda;0,0}$ and $F^2_{-\lambda,\lambda;0,0}$ — having a different type of asymptotic behaviour (see also the relation (27)).

5. Each of the terms involved in the constraints is a kinematically regular function of the vector-meson mass μ_1. Nevertheless it is a hard task (if possible at all) to obtain a limit form of our constraints in the limit of vanishing mass of the spin 1 particle. The reason is very simple; for $\mu_1 = 0$ the (1, 2)-particle's threshold and pseudothreshold kinematic singularity locations coincide with the dynamic singularity location — a pole at $t = \mu^2$. It seems to us that owing to the constraints on RHA, one cannot obtain RHA for the $1^- + 0^- \to 1/2^+ + \bar{1}/2^+$ reaction (here 1^- stands for photon) from RHA for the corresponding massive vector-meson reaction by setting the mass of vector particle equal to zero and simply dropping out the helicity amplitudes $M^{\lambda_3,\lambda_4}_{0,0}$. It is not enough to assume an additional smooth vector-meson mass dependence of the regularized helicity amplitudes, but one also has to check that limiting forms (if they exists at all) of all constraints obtained by such a limit procedure, correspond to the forms one can get by a direct eduction of constraints on RHA for the reaction with photon.

6. We have tacitly assumed that there are no poles or resonant states at thresholds and pseudothresholds. Otherwise the constraints at thresholds and pseudothresholds would have to be modified.

7. If $s = u$ then the definitions of some $F^i_{\{\lambda\}}$ have to be modified. This, of course, will also change some constraints. One cannot automatically use the results of Cohen-Tannoudji, Morel, and Navelet [4], e. g., for the ($\eta = +1$, $m_1 \neq m_2$, $m_3 = m_4$, $s_3 = s_4$) case (see Table VIII) of their paper) when $s = u$. They have considered the most singular case, e. g., $\cos \vartheta_t \sim (pq)^{-1}$ at thresholds and the pseudothreshold. This is not the case if $s = u$; if $m_1 = m_2$ or $m_3 = m_4$ then $\cos \vartheta_t = 0$ on the $s = u$ line. Neither explicit expression for RHA nor constraints on them in the $s = u$ case are given in this paper because they are obviously not of interest at present.

8. Sometimes it is more convenient to discuss the physical content of the constraints in terms of the reduced helicity amplitudes $\hat{M}^{\lambda_3,\lambda_4}_{\lambda_1,\lambda_2}$ instead of RHA (see e. g. Ref. [42]). Following this traditional line we discuss below the implications of the constraints at $t = 0$ in terms of NPCA for the two important (for applications) mass configurations.

A) $\mu_1 \neq \mu$, $\quad M \neq m$.

In each of the amplitudes ($\hat{M}^{-\lambda,\lambda}_{2\lambda,0} \mp \hat{M}^{-\lambda,\lambda}_{-2\lambda,0}$) the kinematic factor allows a maximum singularity t^{-1} at $t = 0$. So the $t = 0$ constraint can be satisfied either by having the contribution of each natural parity amplitude vanish separately like t, or by conspiracy among coefficients of the singularities.

B) $\mu_1 \neq \mu$, $\quad M = m$.

The $t = 0$ constraints can be satisfied either by having the contribution of each NPCA vanish separately like t, or each of the two NPCA can retain the singular $t^{-1/2}$ behaviour at $t = 0$. In this last case the two terms in the equations must approach the same constant.

C)

An example of a solution of the constraints at $t = 0$ is given in Tables II and III. We have not written down examples of solution for other mass configurations because they are obviously of academic interest at present.

TABLE II

Amplitudes for $(\mu_1 \neq \mu, M \neq m)$ mass configuration

Amplitudes	$\bar{\lambda}$	μ	Dominant parity $\eta_1\eta_2 = +1$	Dominant parity $\eta_1\eta_2 = -1$	Kinematic factor	Extra factor in no conspiracy case
$\hat{M}^{-\lambda,\lambda}_{2\lambda,0}+\hat{M}^{-\lambda,\lambda}_{-2\lambda,0}$	2λ	-2λ	$(-1)^{J+1}$	$(-1)^{J}$	t^{-1}	t
$\hat{M}^{-\lambda,\lambda}_{2\lambda,0}-\hat{M}^{-\lambda,\lambda}_{-2\lambda,0}$	2λ	-2λ	$(-1)^{J}$	$(-1)^{J+1}$	t^{-1}	t
$\hat{M}^{\lambda,\lambda}_{\Lambda,0}+\hat{M}^{\lambda,\lambda}_{-\Lambda,0}$	Λ	0	$(-1)^{J}$	$(-1)^{J+1}$	$t^{-1/2}$	
$\hat{M}^{\lambda,\lambda}_{\Lambda,0}-\hat{M}^{\lambda,\lambda}_{-\Lambda 0}$	Λ	0	$(-1)^{J+1}$	$(-1)^{J}$	$t^{-1/2}$	
$\hat{M}^{-\lambda,\lambda}_{0,0}$	0	-2λ	$(-1)^{J+1}$	$(-1)^{J}$	$t^{-1/2}$	
$\hat{M}^{\lambda,\lambda}_{0,0}$	0	0	$(-1)^{J+1}$	$(-1)^{J}$	1	

TABLE III

Amplitudes for $(\mu_1 \neq \mu,\ M = m)$ mass configuration*

	Amplitudes	$\bar{\lambda}$	μ	Dominant parity $\eta_1\eta_2 = +1$	Dominant parity $\eta_1\eta_2 = -1$	Kinematic factor	Extra factor in no conspiracy case
$\eta = -1$	$\hat{M}^{\lambda,\lambda}_{0,0}$	0	0	$(-1)^{J+1}$	$(-1)^{J}$	$t^{-1/2}$	t
	$\hat{M}^{-\lambda,\lambda}_{0,0}$	0	-2λ	$(-1)^{J+1}$	$(-1)^{J}$	$t^{-1/2}$	t
	$\hat{M}^{\lambda,\lambda}_{\Lambda,0}-\hat{M}^{\lambda,\lambda}_{-\Lambda,0}$	Λ	0	$(-1)^{J+1}$	$(-1)^{J}$	$t^{-1/2}$	t
	$\hat{M}^{-\lambda,\lambda}_{2\lambda,0}-\hat{M}^{-\lambda,\lambda}_{-2\lambda,0}$	2λ	-2λ	$(-1)^{J}$	$(-1)^{J+1}$	$t^{-1/2}$	t
	$\hat{M}^{\lambda,\lambda}_{\Lambda,0}+\hat{M}^{\lambda,\lambda}_{-\Lambda,0}$	Λ	0	$(-1)^{J}$	$(-1)^{J+1}$	1	
	$\hat{M}^{-\lambda,\lambda}_{2\lambda,0}+\hat{M}^{-\lambda,\lambda}_{-2\lambda,0}$	2λ	-2λ	$(-1)^{J+1}$	$(-1)^{J}$	1	
$\eta = +1$	$\hat{M}^{-\lambda,\lambda}_{2\lambda,0}+\hat{M}^{-\lambda,\lambda}_{-2\lambda,0}$	2λ	-2λ	$(-1)^{J+1}$	$(-1)^{J}$	$t^{-1/2}$	t
	$\hat{M}^{\lambda,\lambda}_{\Lambda,0}+\hat{M}^{\lambda,\lambda}_{-\Lambda,0}$	Λ	0	$(-1)^{J}$	$(-1)^{J+1}$	$t^{-1/2}$	t
	$\hat{M}^{\lambda,\lambda}_{\Lambda,0}-\hat{M}^{\lambda,\lambda}_{-\Lambda,0}$	Λ	0	$(-1)^{J+1}$	$(-1)^{J}$	1	
	$\hat{M}^{-\lambda,\lambda}_{2\lambda,0}-\hat{M}^{-\lambda,\lambda}_{-2\lambda,0}$	2λ	-2λ	$(-1)^{J}$	$(-1)^{J+1}$	1	
	$\hat{M}^{\lambda,\lambda}_{0,0}$	0	0	$(-1)^{J+1}$	$(-1)^{J}$	1	
	$\hat{M}^{-\lambda,\lambda}_{0,0}$	0	-2λ	$(-1)^{J+1}$	$(-1)^{J}$	1	

* The solution for the $(\eta = -1, \eta_1\eta_2 = +1)$ case is exactly that which has been obtained by Frautschi and Jones [27] (see Table I of their paper) through a different method.

9. In practice the constraints at the pseudothresholds are more important than the ones at thresholds because of the proximity of these pseudothresholds to the physical s-channel region. For the $\varrho^{\circ}K^{-} \to \Lambda\bar{p}$ the $(\Lambda, \bar{p})$ pseudothreshold is at $t = 0.045$ $(\mathrm{GeV}/c)^2$. It is closeness to $t = 0$ and the degree of the kinematic singularity that demands careful attention to the constraints at this pseudothreshold.

5. *Summary*

We have given the complete set of constraints on RHA for the two important (in applications) mass configurations for $\bar{1}+0 \to 1/2+\bar{1}/2'$ reactions (no "chance coincidences" of masses, e. g. $(m_1 \mp m_2)^2 = (m_3 \mp m_4)^2$, were taken into account). These constraints can be used as the remedy for disadvantages of the Regge-type parametrization discussed in Section 1, and they can also serve, e. g., as a guide in selecting approximate models for reduced residues. We also discussed the question of the continuity of the constraints in terms of masses of external particles. We have found that under some assumptions about the external mass dependence of the regularized helicity amplitudes (see Section 4) our constraints are practically indifferent to small mass difference, e. g., between proton and neutron. It has been also pointed out that owing to small mass difference between Λ-hyperon and proton the constraints at $t = 0$ and at $t = (m_\Lambda - m_p)^2$ lead to nontrivial approximate relations between the amplitudes $F^1_{-\lambda,\lambda;2\lambda,0}$ and $F^2_{\lambda,\lambda;\Lambda,0}$ for the $\varrho(A_1)+K \to \Lambda+\bar{p}$ reactions (see relations (32)–(33)). An example of a solution of the constraints at $t = 0$ has also been given (see Tables II and III) for the two most important mass configurations. The constraints are given in a form which seems to be a convenient one for applications in the framework of a Regge-pole model for the reactions under consideration. We hope that the results of the paper will stimulate phenomenologists (see Ref. [43] for the explanation of this term) to use the constraints in fitting procedures. It seems to us that by giving the explicit form of constraints we make the question of taking them into account in a Regge-pole model for $1/2'+0 \to 1/2+1$ reactions a somewhat less difficult task than is usually expected (see Ref. [41], p. 413).

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APPENDIX A

Wave functions

$$\mu_1 \varepsilon_\nu^{(0)} = (q, \omega_1 x, 0, \omega_1 z), \qquad x \equiv \sin\vartheta,$$

$$\sqrt{2}\, \varepsilon_\nu^{(\pm 1)} = (0, \mp z, -i, \pm x), \qquad z \equiv \cos\vartheta,$$

$$\varepsilon_\alpha^{(\Lambda')}(q) g^{\alpha\beta} \varepsilon_\beta^{+(\Lambda)}(q) = -\delta_{\Lambda',\Lambda}, \qquad g_{00} = 1 = -g_{kk}, \quad k = 1, 2, 3,$$

$$\chi_{\lambda^*}(\vec{n}_1) = 2\lambda^* \begin{pmatrix} \frac{1}{2}-\lambda^* \\ \\ \frac{1}{2}+\lambda^* \end{pmatrix}, \qquad \chi_\lambda(\vec{n}_2) = \begin{pmatrix} \frac{1}{2}+\lambda \\ \\ \frac{1}{2}-\lambda \end{pmatrix},$$

$$\vec{n}_i = \vec{p}_i/p, \qquad \chi^{+}_{\lambda_1}(\vec{n}_i)\chi_{\lambda_2}(\vec{n}_i) = \delta_{\lambda_1,\lambda_2},$$

$$u(p, \lambda^*) = \left(\frac{E'+M}{2M}\right)^{1/2} \begin{pmatrix} \chi_{\lambda^*}(\vec{n}_1) \\ \dfrac{2\lambda^* p}{E'+M}\,\chi_{\lambda^*}(\vec{n}_1) \end{pmatrix},$$

$$v(p, \lambda) = 2\lambda\left(\frac{E+m}{2m}\right)^{1/2} \begin{pmatrix} -\dfrac{2\lambda p}{E+m}\,\chi_{-\lambda}(\overleftarrow{n}_2) \\ \chi_{-\lambda}(\vec{n}_2) \end{pmatrix},$$

$$\bar{u}(p, \lambda^*)u(p, \bar{\lambda}^*) = \delta_{\lambda^*,\bar{\lambda}^*}, \qquad \bar{v}(p, \lambda)v(p, \lambda') = -\delta_{\lambda,\lambda'}.$$

See also Refs [36, 37, 39, 40].

APPENDIX B

RHA versus HA

Part I: $\eta = +1$

I. $\mu_1 \neq \mu, \quad m \neq M.$

$$\begin{aligned}
F^1_{-\lambda,\lambda;0,0} &= 2t^{1/2}(\Psi_{34})^{-1}\hat{M}^{-\lambda,\lambda}_{0,0},\\
F^2_{\lambda,\lambda;0,0} &= 2\varphi_{12}\Psi_{34}\varphi_{34}\hat{M}^{\lambda,\lambda}_{0,0},\\
F^1_{\lambda,\lambda;\Lambda,0} &= t^{1/2}(\Psi_{34})^{-1}(\hat{M}^{\lambda,\lambda}_{\Lambda,0} - \hat{M}^{\lambda,\lambda}_{-\Lambda,0}),\\
F^2_{\lambda,\lambda;\Lambda,0} &= t^{1/2}(\varphi_{12}\Psi_{12}\varphi_{34})^{-1}(\hat{M}^{\lambda,\lambda}_{\Lambda,0} + \hat{M}^{\lambda,\lambda}_{-\Lambda,0}),\\
F^1_{-\lambda,\lambda;2\lambda,0} &= t(\Psi_{34})^{-1}(\hat{M}^{-\lambda,\lambda}_{2\lambda,0} + \hat{M}^{-\lambda,\lambda}_{-2\lambda,0}),\\
F^2_{-\lambda,\lambda;2\lambda,0} &= t(\varphi_{12}\Psi_{12}\varphi_{34})^{-4}(\hat{M}^{-\lambda,\lambda}_{2\lambda,0} - \hat{M}^{-\lambda,\lambda}_{-2\lambda,0}).
\end{aligned} \tag{B1}$$

II. $\mu_1 \neq \mu, \quad m = M.$

$$\begin{aligned}
F^1_{-\lambda,\lambda;0,0} &= 2\hat{M}^{-\lambda,\lambda}_{0,0},\\
F^2_{\lambda,\lambda;0,0} &= \varphi_{12}\Psi_{12}\varphi_{34}\hat{M}^{\lambda,\lambda}_{0,0},\\
F^1_{\lambda,\lambda;\Lambda,0} &= \hat{M}^{\lambda,\lambda}_{\Lambda,0} - \hat{M}^{\lambda,\lambda}_{-\Lambda,0},\\
F^2_{\lambda,\lambda;\Lambda,0} &= 2t^{1/2}(\varphi_{12}\Psi_{12}\varphi_{34})^{-1}(\hat{M}^{\lambda,\lambda}_{\Lambda,0} + \hat{M}^{\lambda,\lambda}_{-\Lambda,0}),\\
F^1_{-\lambda,\lambda;2\lambda,0} &= t^{1/2}(\hat{M}^{-\lambda,\lambda}_{2\lambda,0} + \hat{M}^{-\lambda,\lambda}_{-2\lambda,0}),\\
F^2_{-\lambda,\lambda;2\lambda,0} &= 2(\varphi_{12}\Psi_{12}\varphi_{34})^{-1}(\hat{M}^{-\lambda,\lambda}_{2\lambda,0} - \hat{M}^{-\lambda,\lambda}_{-2\lambda,0}).
\end{aligned} \tag{B2}$$

Part II. $\eta = -1$

I. $\mu_1 \neq \mu, \quad m \neq M.$

$$F^1_{\lambda,\lambda;0,0} = 2\Psi_{12}\varphi_{12}\Psi_{34}\hat{M}^{\lambda,\lambda}_{0,0},$$

$$\begin{aligned}
F^2_{-\lambda,\lambda;0,0} &= 2t^{1/2}(\varphi_{34})^{-1}\hat{M}_{0,0}^{-\lambda,\lambda},\\
F^1_{\lambda,\lambda;\Lambda,0} &= t^{1/2}(\Psi_{12}\varphi_{12}\Psi_{34})^{-1}(\hat{M}_{\Lambda,0}^{\lambda,\lambda}+\hat{M}_{-\Lambda,0}^{\lambda,\lambda}),\\
F^2_{\lambda,\lambda;\Lambda,0} &= t^{1/2}(\varphi_{34})^{-1}(\hat{M}_{\Lambda,0}^{\lambda,\lambda}-\hat{M}_{-\Lambda,0}^{\lambda,\lambda}),\\
F^1_{-\lambda,\lambda;2\lambda,0} &= t(\Psi_{12}\varphi_{12}\Psi_{34})^{-1}(\hat{M}_{2\lambda,0}^{-\lambda,\lambda}-\hat{M}_{-2\lambda,0}^{-\lambda,\lambda}),\\
F^2_{-\lambda,\lambda;2\lambda,0} &= t(\varphi_{34})^{-1}(\hat{M}_{2\lambda,0}^{-\lambda,\lambda}+\hat{M}_{-2\lambda,0}^{-\lambda,\lambda}).
\end{aligned} \tag{B3}$$

II. $\mu_1 \neq \mu, \quad m = M.$

$$\begin{aligned}
F^1_{\lambda,\lambda;0,0} &= 2t^{1/2}\Psi_{12}\varphi_{12}\hat{M}_{0,0}^{\lambda,\lambda},\\
F^2_{-\lambda,\lambda;0,0} &= 4t^{1/2}(\varphi_{34})^{-1}\hat{M}_{0,0}^{-\lambda,\lambda},\\
F^1_{\lambda,\lambda;\Lambda,0} &= (\Psi_{12}\varphi_{12})^{-1}(\hat{M}_{\Lambda,0}^{\lambda,\lambda}+\hat{M}_{-\Lambda,0}^{\lambda,\lambda}),\\
F^2_{\lambda,\lambda;\Lambda,0} &= 2t^{1/2}(\varphi_{34})^{-1}(M_{\Lambda,0}^{\lambda,\lambda}-M_{-\Lambda,0}^{\lambda,\lambda}),\\
F^1_{-\lambda,\lambda;2\lambda,0} &= t^{1/2}(\Psi_{12}\varphi_{12})^{-1}(\hat{M}_{2\lambda,0}^{-\lambda,\lambda}-\hat{M}_{-2\lambda,0}^{-\lambda,\lambda}),\\
F^2_{-\lambda,\lambda;2\lambda,0} &= 2(\varphi_{34})^{-1}(\hat{M}_{2\lambda,0}^{-\lambda,\lambda}+\hat{M}_{-2\lambda,0}^{-\lambda,\lambda}).
\end{aligned} \tag{B4}$$

The reduced amplitudes $\hat{M}_{\lambda_1,\lambda_2}^{\lambda_3,\lambda_4}$ are defined as usual

$$\hat{M}_{\lambda_1,\lambda_2}^{\lambda_3,\lambda_0} = M_{\lambda_1,\lambda_2}^{\lambda_3,\lambda_0}\cos^{-|\bar{\lambda}+\mu|}\vartheta/2\,\sin^{-|\bar{\lambda}-\mu|}\vartheta/2$$

where λ_i is the helicity of i-th particle, e. g. $\lambda_1 = 0, \Lambda$, $\lambda_2 = 0$, $\lambda_3 = \lambda^*$, $\lambda_4 = \lambda$, $\lambda = \pm 1/2$. It is worth noting parity conservation condition for $F^i_{\{\lambda\}}$ [4]:

$$F^i_{\lambda_3,\lambda_4;\lambda_1,\lambda_2} = \eta(-1)^{\sum\limits_i(s_i+\lambda_i)}F^i_{-\lambda_3,-\lambda_4;-\lambda_1,-\lambda_2}.$$

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