CENTRAL DIFFRACTIVE PRODUCTION

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The topological cross sections and some characteristics of the multiplicity distribution for central diffractive production via double pomeron exchange are discussed.

Recently experimental data concerning proton distributions and charged particles multiplicity have become available (for a review of the experimental situation see Ref. [1] and references therein). The data clearly show the existence of two competing mechanisms known as pionization and diffractive dissociation. However, the dynamical description of those mechanisms is to a large extent an open question and, in fact, numerous models have been proposed to account for them. Perhaps the most natural model for the diffractive dissociation consists in assuming the single pomeron exchange with the excitation of beam or target proton (Fig. 1a). This mechanism present already in the nova model is of course not unique and might be replaced or complemented by other term, as those shown in Fig. 1b and 1c (double diffractive dissociation and central diffractive production). We wish to concentrate our attention on the last mechanism. We are aware of the fact that convincing theoretical arguments exist against a large contribution of this mechanism at present energies. Those arguments cannot, however, give more than an estimate of the order of magnitude. Thus, the first aim of this paper might be to examine the properties of that mechanism in order to propose experimental tests which could directly measure its contribution at present energies[1]. Even, however, if this contribution is really very small, we believe that the mechanism in question may play quite important, if not dominant, role at higher but still experimentally accessible energies. This belief would be justified, e. g., if the coupling constant of the pomeron to proton with a large mass transfer were vanishing function of that transfer. This is, in fact, explicitly assumed in certain models of diffraction [3]. In that case contribution of the nova type (single or double) would become relatively small at higher energies and diffraction would be mainly due to the central diffractive production. As is seen from Fig. 1c we assume that the

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[1] An experimental estimate for pp collisions at 205 GeV/c one can find in [2].

central diffractive production (a) is due to a double exchange of factorizable Pomeron pole. Moreover we shall assume that (b) pomeron-pomeron collision is similar to the particle-pomeron collision (at least as far as $\langle n_{ch} \rangle$ is concerned). For definiteness we shall take pion-pomeron collision (two bosons) and use the reaction $\pi p \rightarrow Xp$ to get

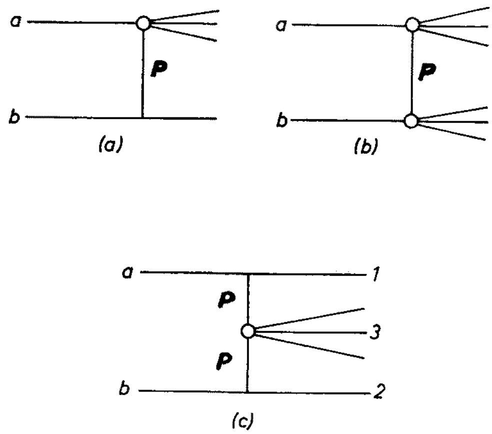

Fig. 1 a) Single diffractive excitation, b) double diffractive excitation,c) central diffractive production diagrams

the dependence $\langle n_{ch}(M_x) \rangle$ (where M_x is the pion-pomeron c. m. energy).

Consider then the diagram presented in Fig. 1c, and denote, as usual,

$$s = (p_a + p_b)^2, \quad s_i = (p_j + p_k)^2, \quad (i, j, k = 1, 2, 3)$$

$$t_1 = (p_a - p_1)^2, \quad t_2 = (p_b - p_2)^2, \quad M^2 = p_3^2$$

and let $m(\mu)$ denote proton (pion) mass. Given s there are still 5 independent kinematical variables, which can be chosen as M, s_1, s_2, t_1, t_2. Let $\varrho(M, s_1, s_2, t_1, t_2)$ denote mass distribution of the central fireball at given values of s_1, s_2, t_1, t_2. Then the contribution of the central diffraction production to the total cross section is given by

$$\sigma_{PP} = \int \varrho(M, s_1, s_2, t_1, t_2) dM ds_1 ds_2 dt_1 dt_2. \tag{1}$$

On the basis of the triple Regge approximation and the generalized optical theorem it is easy to find the asymptotic expression for ϱ in the kinematic region defined by the following conditions

$$s \gg M^2, \quad s_1 \gg M^2, \quad s_2 \gg M^2, \quad M \gg m, \quad t_1 \approx t_2 \approx 0. \tag{2}$$

This expression reads

$$\varrho(M, s_1, s_2, t_1, t_2) = \frac{\exp[2a(t_1+t_2)]}{s^2\sqrt{-\Delta}} \left(\frac{s_1}{M^2}\right)^{2\alpha_P(t_2)} \left(\frac{s_2}{M^2}\right)^{2\alpha_P(t_1)} (M^2)^{\alpha(0)+1/2}, \tag{3}$$

where $\exp(at)$ is the p–p–P vertex function, $\alpha_P(t) = 1 + 0.2\,t$ [4] and $\alpha(0)$ is the intercept of the leading Regge trajectory appearing as a dual equivalent of the sum over M^2. In the following we shall assume that $a \sim 4\ \text{GeV}^{-2}$.

Notice that the condition $\Delta \leqslant 0$ defines the physical region. It is easy to perform the integration over t_1 and t_2. Denoting

$$A = a+\alpha' \ln \frac{s_1}{M^2}, \qquad B = a+\alpha' \ln \frac{s_2}{M^2} \tag{4}$$

and assuming that $m^2 \ll s$ and $s_i \ll s$ we get

$$\varrho(M, s_1, s_2) = \frac{s_1^2 s_2^2 (M^2)^{\alpha(0)+1/2}}{2s^3 M^8 (A+B)} \exp\left[-\frac{2AB}{A+B}\left(\frac{s_1 s_2}{s} - M^2\right)\right]. \tag{5}$$

To get the distribution of central fireball mass at a given s one now has to integrate (5) over the Dalitz region. In the triple Regge limit this region has a simplified shape given by the formula

$$(s_1+s_2-s-M^2)(s_1 s_2 - sM^2) = 0. \tag{6}$$

Unfortunately the integration cannot be performed analytically. It is, however, easy to find its leading term for $s \gg M^2$. It reads

$$\varrho(M, s) = \frac{\ln\left(\frac{s}{M^2}\right)(M^2)^{\alpha(0)}}{4aM^3\left(2a+\alpha' \ln \frac{s}{M^2}\right)}. \tag{7}$$

We have now two possibilities. If pomeron trajectory has a slope different from 0 ($\alpha' \neq 0$), then for $s \gg M^2$

$$\varrho(M, s) \to \frac{(M^2)^{\alpha(0)}}{4a\alpha' M^3} \sim M^{2\alpha(0)-3}. \tag{8}$$

If, on the contrary, this trajectory were flat then

$$\varrho(M, s) \to M^{2\alpha(0)-3} \ln s. \tag{9}$$

We shall assume the first possibility. If the triple pomeron coupling does not vanish [5], we should assume that our fireball is dual to pomeron and put $\alpha(0) = 1$. In that case we get

$$\varrho(M, s) \to \frac{1}{M}. \tag{8a}$$

It is easy to see that integrating (7) over M from M_{th} to $\sqrt{s}$ (assuming $\alpha(0) = 1$) we get the following expression as a contribution to the σ_{tot} coming from the central diffractive production

$$\sigma_{\text{CDP}} = \frac{1}{8a\alpha'}\left[\ln \frac{s}{M_{\text{th}}^2} - \frac{2a}{\alpha'} \ln\left|1+\frac{\alpha'}{2a} \ln \frac{s}{M_{\text{th}}^2}\right|\right] \tag{10}$$

Taking into account that single pomeron exchange (diffractive excitation of the beam or target particle) gives contribution growing with the energy like $\ln(\ln s)$ [6] and two reggeon exchange (central pionization) keeps the σ_{tot} constant, one may conjecture that the relative importance of the mechanism discussed by us may become greater for larger though not asymptotic energy.

This argument, of course, should be treated with caution for at least three reasons. Firstly, the mass distribution cannot behave in all the region (from M_{th} to $\sqrt{s}$) as given by the triple Regge formula, since this formula is supposed to be valid only for M being far from both the limits. Secondly, as follows from the paper by Finkelstein and Kajantie [7] when adding a new link in a multipomeron chain we change the asymptotic behaviour of the contribution to σ_{tot} by a factor $\ln(\ln s)$, what is clearly not fulfilled in our case. Thirdly, one could expect some absorptive corrections to appear which may change the asymptotic behaviour to the contributions to σ_{tot}. All those objections, however, are not necessarily fully convincing. One could argue that it is the M^{-1} term in (7) which determines the asymptotic behaviour and that this behaviour does not essentially change even if we multiply (7) by reasonable factors making it to vanish at M_{th} and $\sqrt{s}$ (at least for some of such factors). The results obtained by Finkelstein and Kajantie are not immediately applicable in our case as in that paper fixed (and small) masses have been assumed to be produced in the multiperipheral chain so that there is no obvious contradiction here. Finally it is very hard to say at what energy possible absorptive corrections start to influence visibly the s-behaviour of the amplitudes. Taking into account all those arguments and counterarguments one realizes that the situation is confused and that there is a possibility of CDP dominance (or, at least, importance) at a certain, perhaps limited, energy region, certainly higher than this which is accessible to day.

After this general discussion we wish to examine a little bit closer possible physical consequences of the proposed mechanism at finite energies. One could look at two things: (a) inclusive distributions of protons and (b) multiplicity distribution of the produced pions. Unfortunately very little is known, as we have stressed above, about the numerical contribution of the central diffractive production to the total picture of the collision. This leaves too much freedom to the model and forces us to make our discussion rather qualitative than quantitative.

To get proton distributions consider formula (5). In this formula the exponential function falls rapidly when the argument goes further and further from the kinematic border of the region given by the equation $s_1 s_2 = sM^2$. Taking this into account we can insert, instead of the exponential, a delta function of the same argument.

Denoting the rest of the ϱ function by $\tilde{\varrho}$ we get

$$\varrho(s_1, s_2) = \int dM \delta\left[-\frac{2AB}{A+B}\left(\frac{s_1 s_2}{s} - M^2\right)\right] \tilde{\varrho}(M, s_1, s_2) =$$

$$= \left[8 s_1 s_2 \left(a+\alpha' \ln \frac{s}{s_1}\right) \left(a+\alpha' \ln \frac{s}{s_2}\right)\right]^{-1}. \tag{11}$$

As is easily seen this distribution factorizes in s_1 and s_2. Introducing Feynman variables x_1 and x_2 by the formulae

$$s_1 = s(1-x_1), \qquad s_2 = s(1+x_2), \tag{12}$$

we get

$$\varrho(x_1, x_2) \approx f(x_1) f(-x_2), \tag{13}$$

where

$$f(x) = [(1-x)(a-\alpha' \ln|1-x|)]^{-1}. \tag{14}$$

It is clear that $\varrho(x_1, x_2)$ has the largest value in the region $x_1 \approx 1$, $x_2 \approx -1$. This should be contrasted with the case of single pomeron exchange where either $x_1 \approx 1$ or $x_2 \approx -1$ regions are populated the most (the sum and not the product of the two regions). Looking at the single proton distribution one would not be able to see it, and it is necessary to measure two proton correlations in order to have an idea of the numerical importance of the proposed model.

Let us come now to the examination of the multiplicities. We start with a comment that whereas the proton x distribution gives us the idea of the behaviour of the function $\varrho(M, s_1, s_2)$ at large M (where it falls down like $1/M$), the multiplicities will be sensitive to the shape of this function for smaller M where $\varrho(M)$ has its maximum. This region corresponds to x very close to ± 1 which are not very easily accessible from experimental point of view. The data for single proton distribution show roughly the behaviour of $\varrho(x)$ of the type $(1-x)^{-1}$ for $x \lesssim 0.98$. This value corresponds to $M^2 \gtrsim 0.02\, s$ ($M \gtrsim 0.14\, \sqrt{s}$) and it is in that (at least) region where $\varrho(M) \sim 1/M$. For smaller M values the data are still lacking and we are forced to make some plausible guesses concerning the $\varrho(M)$ function in that region. Firstly notice that in the present energy region, where $s \lesssim 10^3$ GeV^2 and if M^2 is of the order of few GeV^2, the $\ln s/M^2$ is of the order of 5 (even for M^2 at the threshold this logarithm is about 9) and when multiplied by α' is does not become greater than $2a$. This means that when dealing with the data in the presently accessible energy region (and even higher up to, say, 10^{15} GeV) one is far from the limit (8) of the formula (7). So, we should use the formula (7) rather than (8) for numerical estimations of CDP. Moreover, we have one more condition to impose on that function, namely, that at the threshold (which corresponds to two pion masses) it behaves like $\sqrt{1-4\mu^2/M^2}$. What we need is the interpolating formula. It seems that there are two possible classes of such formulae. One of them corresponds to the maximum of $\varrho(M)$ at fixed M (not moving with s) and the second one to the position of the maximum of $\varrho(M)$ moving like $M_{\max} = k \cdot \sqrt{s}$ to larger M when s is growing.

Accordingly we have tried to find the prediction of the model for two functions

$$\varrho_1(M, s) = \varrho(M, s)\sqrt{1-\frac{4\mu^2}{M^2}}\exp\left(-\frac{\beta}{M}\right) \tag{15}$$

and

$$\varrho_2(M, s) = \varrho(M, s)\sqrt{1-\frac{4\mu^2}{M^2}}\exp\left(-\frac{\gamma\sqrt{s}}{M}\right), \tag{16}$$

where $\varrho(M, s)$ is given by (7). Taking $\beta = 2.5$ GeV we fixed position of the maximum of $\varrho_1(M, s)$ at about $M_{\max} = 1.5$ GeV almost s independently. For the second function $\varrho_2(M, s)$ we take $\gamma = 0.18$ to get $M_{\max} = 0.12\sqrt{s}$. This function has fixed position of the maximum in the variable M^2/s which corresponds to the scaling form $\varrho(x)$ for protons (maximum $\varrho(x)$ at fixed $x = 0.98$).

According to our assumption (b) pions resulting from the decay of the central cluster with fixed M are produced independently so that we can assume Poisson law for the multiplicity distribution. To calculate the topological mass sections we have then the following formula:

$$\sigma_{n_-}^{1,2}(s) = \int dM \varrho_{1,2}(M, s) \frac{\exp(-\langle n_- \rangle) \langle n_- \rangle^{n_-}}{n_-!}. \tag{17}$$

This formula contains unknown so far quantity $\langle n_- \rangle$ (n_- denotes number of π^-'s). We assume consistently with (b) that this quantity depends only on the total energy accessible, which in our case is equal to M. To guess the shape of this dependence we consider the experimental data for the process [8]

$$\pi^- p \to (n\pi)p,$$

at 205 GeV/c concerning the pion multiplicity as a function of the total mass of the system M. This distribution can be fitted by the following function

$$\langle n_{ch}(M) \rangle = 1 + 1.1(M - \mu)^{2/3}, \tag{18}$$

for M from 1 to 14 GeV approximately. This function is chosen in such a way as to fulfill the normalization condition at the threshold (at $M = \mu$ we have one π^-). Exponent 2/3 is in agreement with the predictions of the model considered in Ref. [9]. Taking over the coefficients from above formula to our case we assume

$$\langle n_{ch}(M) \rangle = \tfrac{4}{3} + 1.1(M - 2\mu)^{2/3}, \tag{19}$$

where a similar normalization condition has been accepted at the threshold ($M = 2\mu$; we have either $2\pi^0$, $\pi^+\pi^-$, or $\pi^-\pi^+$ with equal weights in the isospin 0 state). This gives us

$$\langle n_-(M) \rangle = 0.67 + 0.55(M - 2\mu)^{2/3}. \tag{20}$$

With this function we enter formula (17) to get $\sigma_{n_-}(s)$ and other measurable quantities, amongst them modal N_0 and median N_d values of multiplicity distributions,

$$N_0 = 2k + 2 + \ln\left(\frac{\sigma_{k+1}}{\sigma_{k-1}}\right) \Big/ \ln\left(\frac{\sigma_k^2}{\sigma_{k+1}\sigma_{k-1}}\right), \tag{21}$$

where σ_{k-1}, σ_k, σ_{k+1} are three largest values of σ_{n_-}, and

$$N_d = 2j + 2 + \left(\sum_{i=j+1}^{\infty} \sigma_i - \sum_{i=0}^{j} \sigma_i\right) / \sigma_j, \tag{22}$$

where j is the smallest value of n for which the condition

$$\sum_{i=0}^{n} \sigma_i \geqslant \sum_{i=n+1}^{\infty} \sigma_i$$

is fulfilled.

Numerical calculations were performed for two types of functions ϱ_1 and ϱ_2 in the range of energy from $s = 200$ to $s = 2000$ GeV^2. Our results are presented in Figs 2, 3, 4 and 5. As we see, both functions ϱ_1 and ϱ_2 give growing $\langle n_{ch} \rangle$, N_0 and N_d. However, if the maximum of ϱ is kept at a fixed M (ϱ_1 case) the growth is very slow, whereas for

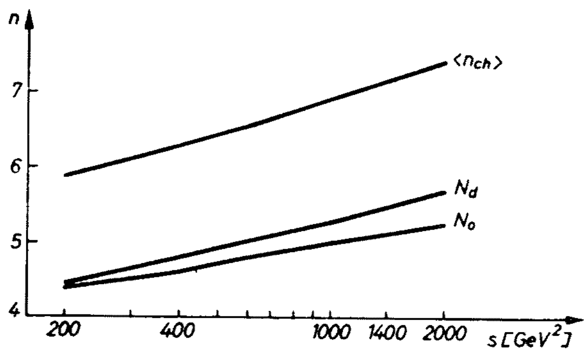

Fig. 2. Average charged multiplicities $\langle n_{ch} \rangle$ modal N_0 and median N_d values as a function of s in the ϱ_1 case

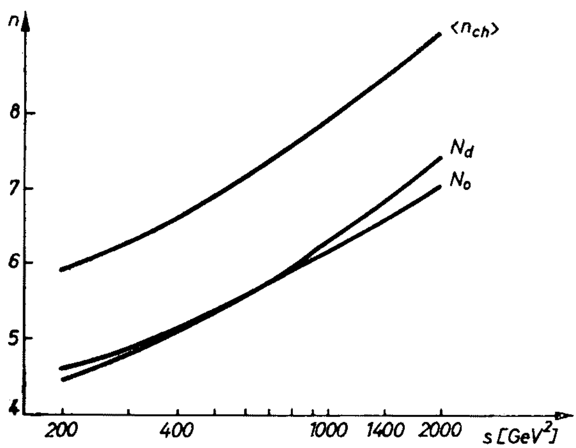

Fig. 3. Average charged multiplicities $\langle n_{ch} \rangle$ modal N_0 and median N_d values as a function of s [GeV^2] in the ϱ_2 case

ϱ_2 we get increase of the three quantities comparable to the experimental data [10] (which include of course, predominantly, the pionization component). We conclude that it is very probable that the dip between the pionization and diffractive component [11] in the data may not appear or it will show at quite high energies only. As for the topological cross section we notice that in the ϱ_1 case we get a picture usually expected for the diffraction, that is, we get growing σ_n for all n (Fig. 4). It turns out, however, that it is rather easy to get σ_n behaving as functions of the energy like in the pionization case, which can be observed in the ϱ_2 case (Fig. 5) and one can obtain the same result σ_n for single diffractive excitation. It means that perhaps we cannot separate diffractive and pionization component on the basis of energy dependence of the topological cross sections. It could be more easily done by looking at rapidity distribution of produced pions. We expect that pions

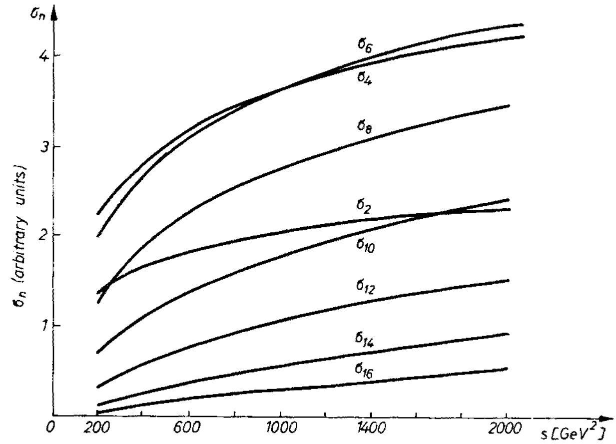

Fig. 4. Topological cross sections for various total charged multiplicities n ($= 2n_- + 2$) as a function of s in the ϱ_1 case

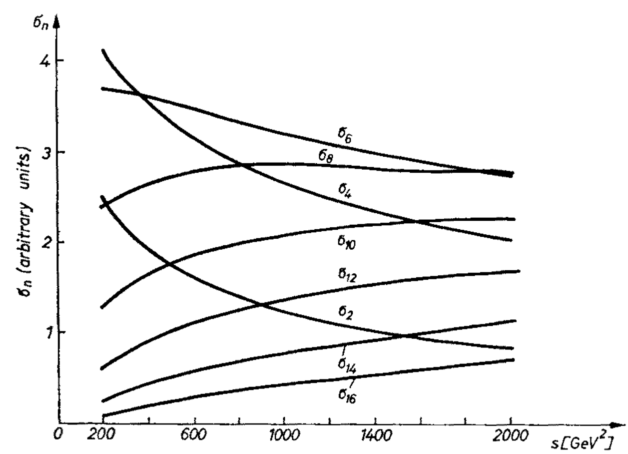

Fig. 5. Topological cross sections for various total charged multiplicities n as a function of s in the ϱ_2 case

emerging from the central cluster will be produced in the central rapidity region and will be separated in rapidity from throughgoing protons.

Concluding we point out the following observations:

(1) The role of the central diffractive production may increase at higher energies,

(2) The two proton correlations and rapidity distributions of produced pions present

the most sensitive test to discover the mechanism discussed in our paper even before it eventually becomes dominant,

(3) The exact shape of $\varrho(M, s)$ for not asymptotic M is very important for the pion multiplicities and related quantities.

In particular, making very natural assumptions concerning this functions, one can get a picture quite similar to that given by pionization.

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