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CHARGE TRANSFER FLUCTUATIONS IN THE NEUTRAL CLUSTER MODELS

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Charge transfer distribution in neutral cluster model of high-energy particle production is analyzed. Quigg and Thomas relation is derived for a realistic cluster decay distribution. The comparison with experimental data shows the agreement with the isotropic decay of clusters. Previous discrepancies are shown to be caused by neglecting the effects of non-zero initial charges.

1. *Introduction*

It was recently recognized [1–8] that a convenient way to study the correlations between positive and negative particles produced in the high-energy collisions is to investigate the charge transfer fluctuations. The data were shown to disagree with random distribution of charges suggesting some clustering of unlike charges in rapidity (local charge conservation). In particular, it was shown by Quigg and Thomas [2] that if the particles are produced by intermediate step of the neutral clusters, the charge fluctuations are related to the charge density and to the width of cluster decay spectrum in rapidity. This relation was found to be in qualitative agreement with existing data [7, 9].

In this paper we continue the analysis of charge fluctuations using realistic cluster decay distributions. We believe that this improvement allows for the quantitative analysis of the data and, in particular, makes possible the independent determination of the cluster width. We find that the obtained width is in agreement with the assumption of isotropic decay of the clusters. The discrepancies implied by previous analyses (see the discussion in Ref. [7]) were caused by neglecting the "leading charge" effects.

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In Section 2 we derive the Quigg-Thomas formula using a realistic cluster decay distribution. In Section 3 the corrections for leading charge (neglected in the neutral cluster model) are discussed. In Section 4 the comparison with experimental data is given. Our conclusions are listed in the last section.

2. *Charge transfer fluctuations in the neutral cluster models*

The definition of the charge transfer in the reaction

$$A+B \to C_1+\ldots+C_n \tag{2.1}$$

is illustrated in Fig. 1. The available rapidity region is divided into two parts by a cut at a given value of rapidity, say $\bar{y}$. The charge transfer $\Delta Q(\bar{y})$ is the difference between the net charge of the final state particles in one part (say $y < \bar{y}$) and the charge of initial particle belonging to this part:

$$\Delta Q(\bar{y}) = Q_f(y < \bar{y})-Q_i(y < \bar{y}). \tag{2.2}$$

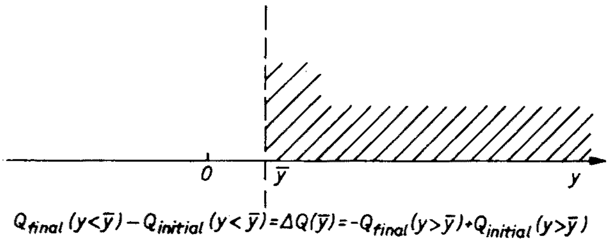


Fig. 1. Definition of the charge transfer

To see the relation of $\Delta Q(\bar{y})$ to correlations between positive and negative particles, let us note that for fixed $\bar{y}$ the shape of the distribution of $\Delta Q(\bar{y})$ is the same as that of the distribution of the difference of the number of positive and negative particles contained in the interval $y < \bar{y}$.

The simplest parameter for quantitative discussion of charge fluctuations is the dispersion $D^2_{\Delta Q}(\bar{y})$ of the distribution of $\Delta Q(\bar{y})$. It is defined as usual

$$D^2_{\Delta Q}(\bar{y}) = \sum_{\Delta Q} |\Delta Q(\bar{y})|^2 P(\Delta Q(\bar{y}))-\overline{\Delta Q(\bar{y})}^2, \tag{2.3}$$

where $P(\Delta Q(\bar{y}))$ is the probability of charge transfer ΔQ in the region $y < \bar{y}$. It is a matter of simple algebra to show that

$$D^2_{\Delta Q}(\bar{y}) = 2D^2_+(\bar{y})+2D^2_-(\bar{y})-D^2_{\text{ch}}(\bar{y}), \tag{2.4}$$

where the indices +, — and ch denote positive, negative and all charged particles, respectively.

The single particle density in the cluster model [10] may be written as

$$\varrho(y) = \langle N(Y)\varrho_d(y, Y, m)\rangle, \tag{2.5}$$

where $N(Y)$ is the cluster density at rapidity Y and $\varrho_d(y, Y, m)$ is the cluster decay distribution

$$\varrho_d(y, Y, m) = \frac{m}{\sigma}\lambda(z, Y), \tag{2.6}$$

m is the number of particles in the cluster decay, σ is the half-width of the cluster and z is given by

$$z = \frac{y-Y}{\sigma}. \tag{2.7}$$

In the following σ is assumed to be rapidity independent. This is not quite obvious for diffractive events, where anisotropy and mass dependence of the width is expected in many models. We estimate, however, that for moderate energies the possible correction is rather small. Better data at NAL energies should be used to analyze this problem in more detail.

$\lambda(z, Y)$ describes the shape of the decay distribution and satisfies the normalization condition

$$\int_{-\infty}^{\infty} \lambda(z, Y)dz = 1. \tag{2.8}$$

Finally, $\langle \quad \rangle$ means the averaging over all parameters except y.

The charge fluctuations within one cluster can be calculated assuming the uncorrelated decay of a cluster.

The probability of k particles with rapidities $y < \bar{y}$ (from a cluster of m particles and rapidity Y) is given by the binomial distribution

$${}_1P_k = \binom{m}{k} p^k(1-p)^{m-k}, \tag{2.9}$$

where

$$p = \int_{-\infty}^{\frac{\bar{y}-Y}{\sigma}} \lambda(z, Y)dz = p(\bar{y}). \tag{2.10}$$

Then the dispersion is

$${}_1D^2(\bar{y}) = \overline{k^2} - \bar{k}^2 = mp(1-p), \tag{2.11}$$

and from (2.4) we get

$${}_1D^2_{\Delta Q}(\bar{y}) = m_{\text{ch}}p(1-p). \tag{2.12}$$

Since the clusters are assumed to be uncorrelated and neutral, their contributions to the total dispersion squared are additive. Thus we can write[1]

$$D^2_{\Delta Q}(\bar{y}) = \langle N(Y)m_{\text{ch}}\Lambda(\bar{z}, Y)\rangle, \tag{2.13}$$

[1] Formula (2.13) may be derived also by integrating the correlation function as given in Ref. [10]. The derivation presented here was proposed to us by Professor K. Zalewski. A more general treatment can be found in recent preprint by Baier and Bopp [6].

where

$$\Lambda(\bar{z}, Y) = \int_{-\infty}^{\bar{z}} \lambda(z, Y)dz \int_{\bar{z}}^{\infty} \lambda(z, Y)dz, \qquad \bar{z} = \frac{\bar{y} - Y}{\sigma}. \tag{2.14}$$

We notice that the structure of Eq. (2.13) is the same as that of Eq. (2.5), the only difference being that in Eq. (2.13) there is the function $\Lambda(\bar{z}, Y)$ instead of $\lambda(z, Y)$. Thus, to investigate the relation between $D^2_{\Delta Q}$ and ϱ we have to compare these two functions.

The relation between $\Lambda(z, Y)$ and $\lambda(z, Y)$ clearly depends on the shape of the cluster decay spectrum, i. e. the shape of $\lambda(z, Y)$. To obtain a realistic estimate, we have considered a Gaussian form

$$\lambda(z, Y) = \lambda(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}. \tag{2.15}$$

Using Eqs (2.14) and (2.15) it is possible to calculate $\Lambda(z, Y)$ which is then a function of z only. In Fig. 2 $\Lambda(z)$ and $\lambda(z)$ are compared. It is seen that their shapes are not very different from each other. Consequently, to a reasonable approximation we can write

$$\Lambda(z) = \alpha\lambda(z), \tag{2.16}$$

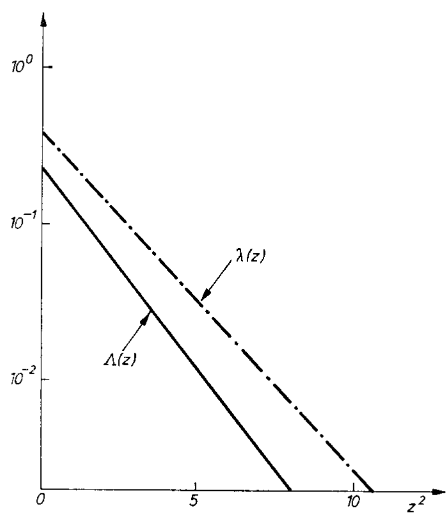


Fig. 2. Comparison of the functions $\Lambda(z)$ (Eq. (2.14)) and $\lambda(z)$ (Eq. (2.15))

where

$$\alpha = \int_{-\infty}^{\infty} \Lambda(z)dz \simeq 0.62. \tag{2.17}$$

Taking into account Eqs (2.5), (2.6), (2.13), (2.14), and (2.16) we thus obtain

$$D^2_{\Delta Q}(\bar{y}) = \alpha\sigma\varrho_{\text{ch}}(\bar{y}). \tag{2.18}$$

This is our final result. It differs from that of Quigg and Thomas only by a slightly different value of the coefficient α due to the different cluster decay spectrum. Thus we have proven that the Quigg-Thomas relation is approximately valid also for the Gaussian decay distribution of clusters. Since such distributions are consistent with isotropic decay [10] and with ISR correlation data [11–17], we think that our calculation gives a realistic evaluation of the neutral cluster model predictions. In the next sections we compare this prediction with the experimental data.

3. *Effect of initial charges*

The model discussed deals with neutral clusters only. Thus to compare its predictions with data we have to exclude the effects of non-zero charges introduced by the initial particles ("leading charges"). Since these charges can be displaced along the rapidity axis, additional charge transfer across the investigated boundary may occur. In general, the observed charge transfer may be always decomposed into two parts. The first part, coming from the fluctuations of produced charge, is described by the neutral cluster model, whereas the displacement of the initial charges is responsible for the second part

$$\Delta Q = Q_f - Q_i = Q_{\text{prod}} + Q_{\text{lead}} - Q_i = Q_{\text{prod}} + \Delta Q_{\text{lead}}. \tag{3.1}$$

To proceed further, let us assume that the two parts are independent from each other, i. e. the probability of a given partition of the produced charge does not depend on the position of the "leading" charge in the rapidity axis. Thus the probability for a given charge transfer k may be written as

$$P_k = \sum_{n=-\infty}^{\infty} p_{k-n} q_n, \tag{3.2}$$

where p_n denotes the probability of charge transfer n for the produced charge and q_n the probability of charge transfer n for the leading charges. Since the average net charge produced in any part of the phase-space is zero, the first probability distribution must be symmetric

$$p_n = p_{-n}. \tag{3.3}$$

On the other hand, q_n is non-zero only for few low values of n determined by values of the initial charges. Consequently, Eq. (3.2) reduces the number of independent probabilities and may therefore be tested experimentally. As will be noted in the next section, the resulting fits are quite satisfactory for all the cases, for which the data are good enough to perform the fit. This suggests the validity of Eq. (3.2), at least for moderate rapidities. We will use this equation to correct the observed dispersion of charge for the effect of leading charges, writing

$$D^2(y) = D^2_{\text{prod}}(y) + D^2_{\text{lead}}(y). \tag{3.4}$$

If we assume that the two leading charges are independent (which seems to be very reasonable at high energies) the correction term $D^2_{\text{lead}}(y)$ can be computed according to

$$D^2_{\text{lead}}(y) = \alpha_{\text{L}}(1-\alpha_{\text{L}}) + \alpha_{\text{R}}(1-\alpha_{\text{R}}), \tag{3.5}$$

where

$$\alpha_i = \int_{-\infty}^{y} \varrho_i(y')dy', \tag{3.6}$$

and $\varrho_{L,R}$ denote the densities of the left and right leading charge, respectively. At very high energies they should be completely separated from each other, at the conventional accelerator energies more sophisticated methods are necessary to separate them. The rough estimates, however, can be given also for the reactions measured at accelerator energies. Thus it is possible, in principle, to estimate $D^2_{\text{prod}}(y)$ from existing data.

The preceding discussion shows that the proportionality between $D^2(y)$ and $\varrho(y)$ predicted by Eq. (2.18) applies in fact to D^2_{prod} and ϱ_{prod}, since the neutral cluster picture may be valid only after excluding the leading charges. In fact, ϱ_{lead} was subtracted from data in the previous analyses [7, 9]. No correction for D^2 was made however.

To point out the necessity of the proposed correction let us imagine the case when $\varrho_L(y)$ is very sharply peaked around some value of y. Since the integral of ϱ_L is one, the maximal value of the function should be then very high. On the other hand, the contribution of left charge to the dispersion as given by

$$D^2_L(y) = \int_{-\infty}^{y} \varrho_L(y')dy' \int_{y}^{\infty} \varrho_L(y')dy' \tag{3.7}$$

never exceeds 1/4. In general, $D^2_L(y)$ and $\varrho_L(y)$ can have the similar shape for a rather special form of ϱ_L only (approximately Gaussian). Since the shape observed experimentally is different (and depends significantly on the kind of initial particle), the proportionality between D^2 and ϱ in the regions where the leading charges contribute may be accidental only. Of course, at very high energies the central region is expected to be free from leading charge contributions and no corrections should be necessary. To check the predictions of the neutral cluster model at the conventional accelerator energies we have, however, to exclude the leading charge effects, since they are present practically in the full phase--space. This will be shown in the discussion of experimental data given in the next section.

4. *Comparison with experimental data*

In the previous analyses [9, 7] approximate proportionality between $D^2(y)$ and $\varrho_{\text{prod}}(y)$ was found. The proportionality coefficient was, however, surprisingly large, suggesting the width of cluster in rapidity of the order 1.2–1.3 in disagreement with the isotropic decay found at ISR energies. Thus we face now two important questions:

(*i*) Is $D^2_{\text{prod}}(y)$ proportional to $\varrho_{\text{prod}}(y)$ with the same (or better) accuracy as $D^2(y)$?

(*ii*) If so, is the width of cluster estimated from the proportionality coefficient compatible with isotropic decay of clusters?

To answer these questions we performed the analysis of the published K^-p data at 10 and 16 GeV/c [9] and pp data at 12 and 24 GeV/c [4, 18]. For the K^-p data the densities $\varrho_{L,R}$ necessary to calculate D^2_{prod} were estimated [9]. We have used the same technique to separate $\varrho_{L,R}$ for the pp case. The result is shown in Fig. 3.

The calculated $D^2_{\text{prod}}(y)$ are shown in Fig. 4, and compared with corresponding densi-

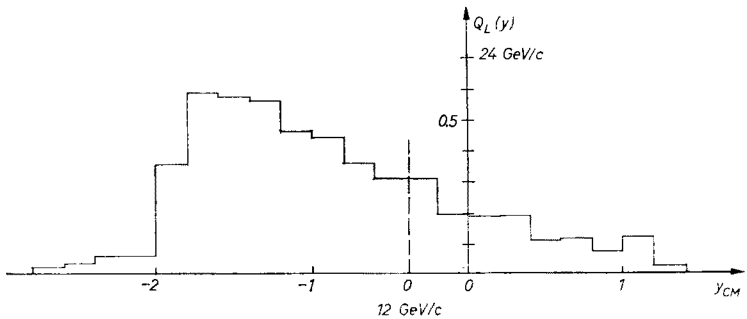
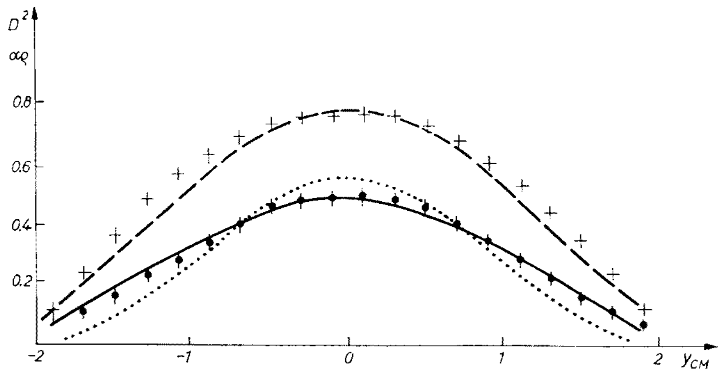


Fig. 3. Distribution of the left leading charge $\varrho_L(y)$ for the pp interaction at 12 and 24 GeV/c



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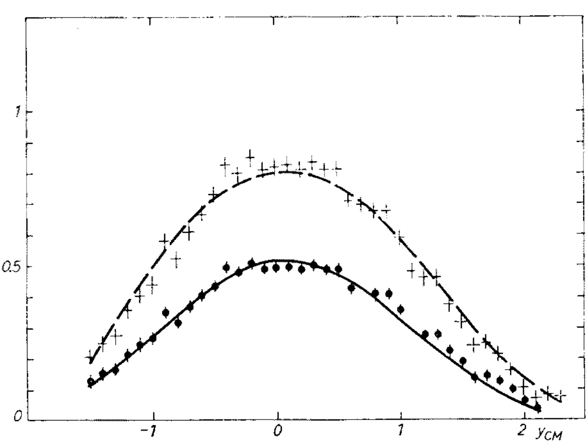

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Fig. 4 a) Dispersion of produced charge $D^2_{\text{prod}}(y)$ (black dots) and unccorected dispersion $D^2(y)$ (crosses) compared with suitably normalized density of produced charged particles $\varrho_{\text{prod}}(y)$ (full and broken line, respectively) for 24 GeV/c pp data. Prediction from random charge distribution is also shown as dotted line. b) Dispersion of produced charge $D^2_{\text{prod}}(y)$ (black dots) and uncorrected dispersion $D^2(y)$ (crosses) compared with suitably normalized density of produced charged particles $\varrho_{\text{prod}}(y)$ (full and broken line, respectively) for 16 GeV/c K^-p data ties. As we can see, for the pp case the proportionality is definitely better than observed between uncorrected $D^2(y)$ and $\varrho_{\text{prod}}(y)$ (shown also in Fig. 4). For the K^-p case the situation is less clear, since the proportionality holds well both for D^2 and D^2_{prod}.

The proportionality coefficient leads, for the pp case, to the cluster width of 0.82, in good agreement with the isotropic decay [10, 20]. The value for K^-p case is 0.87.

To show the effect of clustering, the curve obtained for random distribution of charges

$$D^2_{\text{prod}}(y) = \langle n\rangle^{-1}_{\text{prod}} \int_{-\infty}^{y} \varrho(y')dy' \int_{y}^{\infty} \varrho(y')dy'$$

is also shown for the pp case. It disagrees clearly with the data.

For completeness, let us note that the full charge exchange distribution at $y = 0$ in discussed data as well as the preliminary analysis of the 8 and 16 GeV/c π^+p data [21] seem to agree with Eq. (3.2) supporting our assumptions. The 200 GeV/c pp data [19] do not include the y dependence; the value of proportionality coefficient in (2.18) at $y = 0$ agrees with lower energy data within very large errors. Note that in this case the correction for leading charges is expected to be smaller.

5. *Conclusions*

Our conclusions can be summarized as follows:

(*i*) The Quigg-Thomas relation between the dispersion of the charge transfer distribution and single charge particle density holds in the neutral cluster model with realistic cluster decay distribution.

(*ii*) The cluster width estimated from analysis of the charge transfer fluctuations is compatible with that required by assumption of isotropic cluster decay.

(*iii*) The discrepancies found in previous analyses [7] are caused by the leading charge effects which are not taken into account in the neutral cluster model. After removing these effects the model with neutral isotropic clusters describes correctly the observed charge fluctuations.

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