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INFORMATION FROM COHERENT PRODUCTION ON DEUTERIUM

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Possibilities of obtaining information from the coherent production process $\pi d \to (3\pi)d$ about the diffractive dissociation process on nucleons $\pi N \to (3\pi)N$ and the $(3\pi)-N$ elastic scattering are studied.

1. *Introduction*

A number of experiments concerning coherent production of hadrons on deuterium are now being done or proposed. Consequently, it seems interesting to investigate what information can be extracted from the coherent production data. Using Glauber's model and knowing the deuteron wave function it is possible to calculate the amplitude for the coherent production process

$$x+d \to x^*+d, \tag{1.1}$$

if the diffractive amplitudes for the scattering on nucleons

$$x+\mathrm{N} \to x+\mathrm{N}, \tag{1.2}$$

$$x+\mathrm{N} \to x^*+\mathrm{N}, \tag{1.3}$$

$$x^*+\mathrm{N} \to x^*+\mathrm{N} \tag{1.4}$$

are known. We stress the interest of the inverse problem: giving data on reaction (1.1) what can we learn about processes (1.3) and (1.4)? For process (1.4) scattering on nuclei is the only source of information. Experiments with beryllium and heavier nuclei, which are sensitive mainly to the value of the forward scattering amplitude, have shown [1] that the amplitudes for processes (1.4) and (1.2) nearly coincide, even if x^* is five-pion system. It would be of great interest to confirm this result on deuterium, where the scattering process is simpler. For process (1.3) data can be obtained directly, however, for lower

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energies there are serious background problems when one tries to extract the diffractive dissociation amplitudes. Therefore an additional source of information seems valuable.

In the present paper we concentrate on small momentum transfer $|t| \lesssim 0.16$ GeV^2 processes. For this range bubble chamber data are available. Also for such processes Glauber's approach is best justified. In the following section the basic formulae are given, and the available deuteron wave functions are discussed. In Section 3 we analyse the data for the process

$$\pi^- d \to \pi^- \pi^+ \pi^- d. \tag{1.5}$$

Section 4 contains our conclusions.

2. *Formalism*

When all the complications due to spins, isospins, and longitudinal momentum transfers are included, the formulae [2] obtained from Glauber's model for the amplitude of process (1.1) are quite complicated. Even for the simplest case $x = x^* = \pi$ it is hard to squeeze the full formulae [3, 4, 5] into one page. On the other hand by neglecting too many factors one risks a distortion of the results. In order to reduce the formula to a tractable form we propose the following simplifying assumptions.

1. Double isospin one exchange is neglected. The error of this approximation is of the order of the ratio of the charge exchange to the non charge exchange scattering cross sections for scattering on nucleons. Thus in general this approximation should be good at high energies.

2. For the recoil nucleon s-channel helicity conservation is assumed. This was found to hold for the isospin zero exchange part of the elastic πN scattering amplitude [6] and from the factorization property of the amplitudes it may be expected to hold for all diffractive processes. This assumption, of course, does not imply s-channel helicity conservation in the $x \to x^*$ process.

3. The longitudinal momentum transfer effects are neglected. This approximation should not be bad for high energy production processes on deuterium. It is known that the 3π and 5π coherent production cross-section on heavy nuclei increases with energy. This effect may be explained by the decrease of the longitudinal momentum transfer with growing energy [7]. For the production processes on deuterium a similar but much weaker effect should be present (cf. e. g. Ref. [8]). Our estimation shows that for the pion momenta larger than 15 GeV/c the correction to the integrated cross section does not exceed ten per cent.

4. Off-mass shell corrections in the double-scattering part of the production amplitude are neglected.

5. The diffractive amplitudes for processes (1.2)–(1.4) are parametrized in the form

$$f_j(t) = i\sqrt{A_j}\, e^{1/2 B_j t}, \tag{2.1}$$

where the subscript j labels the process, A_j and B_j are positive real constants and t is the four-momentum transfer squared. Assumption (2.1) could easily be refined, for the present data however, it seems sufficient.

With these assumptions the general formula given in [2] reduces to the simpler formula (cf. [9]):

$$\frac{d\sigma}{dt}(xd \to x^*d) = 4A_p\left\{\left[e^{B_p\frac{t}{2}}S_0\left(\frac{t}{4}\right)-D_{00}(t)\right]^2+\right.$$

$$\left.+\frac{1}{4}\left[e^{B_p\frac{t}{2}}S_2\left(\frac{t}{4}\right)-D_{20}(t)\right]^2+\frac{3}{4}\left[e^{B_p\frac{t}{2}}S_2\left(\frac{t}{4}\right)-D_{22}(t)\right]^2\right\}, \qquad (2.2)$$

where subscript p refers to process (1.3). Further

$$S_0(t) = \int_0^\infty [u^2(r)+w^2(r)]j_0(\sqrt{-t}\,r)dr, \qquad (2.3)$$

$$S_2(t) = \int_0^\infty \left[2u(r)w(r)-\frac{1}{\sqrt{2}}w^2(r)\right]j_2(\sqrt{-t}\,r)dr, \qquad (2.4)$$

where $u(r)$ and $w(r)$ are the s-wave and d-wave deuteron wave functions normalized by

$$\int_0^\infty [u^2(r)+w^2(r)]dr = 1 \qquad (2.5)$$

and $j_n(x)$ are the spherical Bessel functions. Using the subscripts x and x^* for processes (1.2) and (1.4) we have

$$D_{mn}(t) = \frac{1}{16\pi}\int_0^\infty d\tau S_m(\tau)\left\{\sigma_x \exp\left[\frac{1}{2}(B_p+B_x)\left(-\tau+\frac{1}{4}t\right)\right]I_n\left(\frac{B_p-B_x}{2}\sqrt{-t\tau}\right)+\right.$$

$$\left.+\sigma_{x^*}\exp\left[\frac{1}{2}(B_p+B_{x^*})\left(-\tau+\frac{1}{4}t\right)\right]I_n\left(\frac{B_p-B_{x^*}}{2}\sqrt{-t\tau}\right)\right\}d\tau, \qquad (2.6)$$

where $I_n(x)$ are the modified Bessel functions and σ_x and σ_{x^*} are the xN and x^*N total cross sections.

In order to evaluate expression (2.2) it is necessary to know besides the amplitudes (2.1) the form factors $S_0(t)$ and $S_2(t)$. We tried the soft core and the hard core functions given by Reid [10], Humberston's wave function quoted in Ref. [3], and the best (third) of the analytic approximations given by Moravcsik [11] for the Gartenhaus wave function [12]. The differential cross sections for elastic pd scattering at incident momentum 60.8 GeV/c was calculated for each set of wave functions and compared with the experimental data from Ref. [13] (Fig. 1). In the region of interest ($|t| \lesssim 0.16$ GeV^2) the results for the first three of the sets of deuteron wave functions coincide within about one per cent while Moravcsik's function gives results higher by up to four per cent. In the following calculations Reid's functions were used. The slope parameter $B = 12$ GeV^{-2}, the same for the proton-proton and proton-neutron elastic scattering amplitude was chosen, and the parameter A was calculated from $\sigma_{pp}^{tot} = 38.5$ mb; the two numbers being taken from Refs [14] and [15], respectively. We checked that changing these parameters within their errors does not effect significantly our results. In our calculation we have neglected Coulomb effects, the real parts of the proton-nucleon amplitudes and the possible contribution of inelastic shadowing. The Coulomb effects are expected to be important only

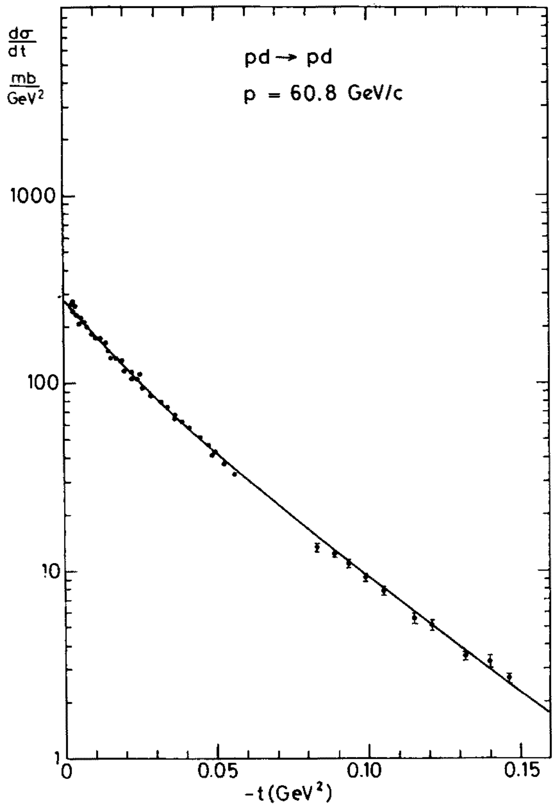

Fig. 1. Comparison of the calculated and measured cross section for elastic pd scattering at 60.8 GeV/c

for very small momentum transfers $|t| < 0.01$ GeV^2. Neglected real parts can give at most two per cent positive correction to the differential cross section while the inelastic shadow effects tend to diminish slightly the cross section (cf. Ref. [16]).

3. *Analysis of the process* $\pi^- d \to \pi^-\pi^+\pi^- d$

For the process

$$\pi^- d \to \pi^-\pi^+\pi^- d \tag{3.1}$$

at 15 GeV/c preliminary data of the Seattle-Berkeley collaboration are available [17]. These data include an unnormalized momentum transfer distribution $d\sigma/dt$ and a mass distribution $d\sigma/dm_{3\pi}$ for 1836 fitted events. In our calculation of $d\sigma/dt$ for reaction (3.1) we use the following parameters of the elastic πN scattering:

$$B_\pi = 8.5\ \mathrm{GeV}^{-2}, \qquad \sigma_\pi = 24\ \mathrm{mb}. \tag{3.2}$$

.These values correspond to high-energy elastic πp scattering (cf. e. g. Refs [15], [18], [19]), where diffraction is believed to dominate.

A relatively clean sample of diffractive $\pi^- p \to (\pi^-\pi^+\pi^-)p$ events was obtained at 205 GeV/c by the Berkeley-NAL collaboration [20]. The authors estimate the integrated cross section for this process as

$$(330 \pm 55)\ \mu\mathrm{b}, \tag{3.3}$$

but do not give the slope parameter B_p. The slope parameters at 16 GeV/c for five mass intervals of the (3π) system was obtained in Ref. [21]. In order to reduce the non dif-

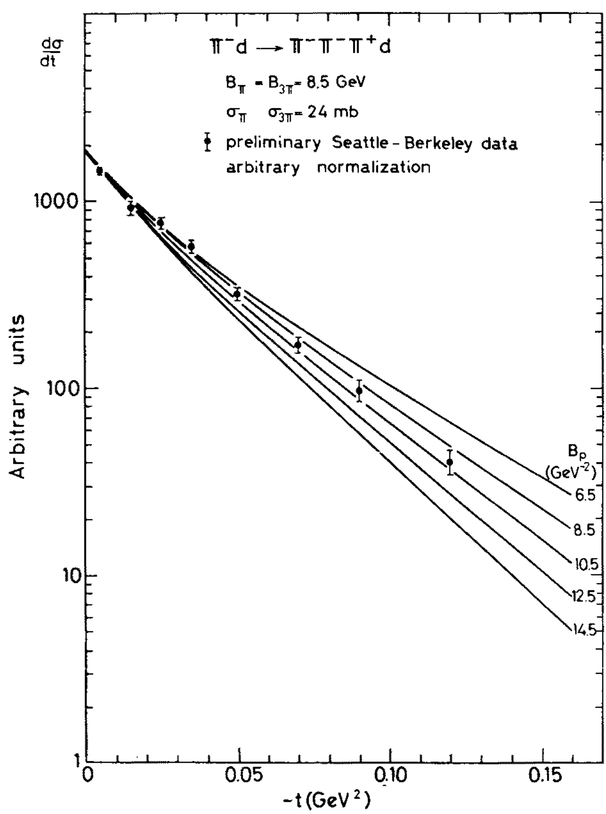

Fig. 2. Dependence on B_p of the calculated cross section for reaction (3.1)

fractive background the authors included in their analysis only the events with all the three pions going (in the centre of mass system) into the forward hemisphere. Even if this selection is not very efficient, the slopes are likely to be reasonable. Averaging them over the mass spectrum reported in Ref. [17] for process (3.1) one finds

$$B_p = (9 \pm 0.5)\ \mathrm{GeV}^{-2}, \tag{3.4}$$

where the error was estimated by comparing the results of various prescriptions for averaging.

Very little is known about elastic (3π) N scattering. Scattering on heavier nuclei yields $\sigma_{3\pi}^{\mathrm{tot}} \approx 24$ mb [1], while the Glauber model calculations, assuming that the (3π) is a $\rho\pi$ system, give 41 mb $\lesssim \sigma_{3\pi} \lesssim 48$ mb [22]. No information about the slope is available, but probably it is about 10 GeV^{-2} as are all the known slopes for scattering on nucleons.

In Fig. 2 the differential cross sections for $d\sigma(\pi^- d \to \pi^-\pi^+\pi^- d)/dt$, calculated assuming for elastic (3π) N scattering

$$\sigma_{3\pi} = 24\ \mathrm{mb}, \qquad B_{3\pi} = 8.5\ \mathrm{GeV}^{-2} \tag{3.5}$$

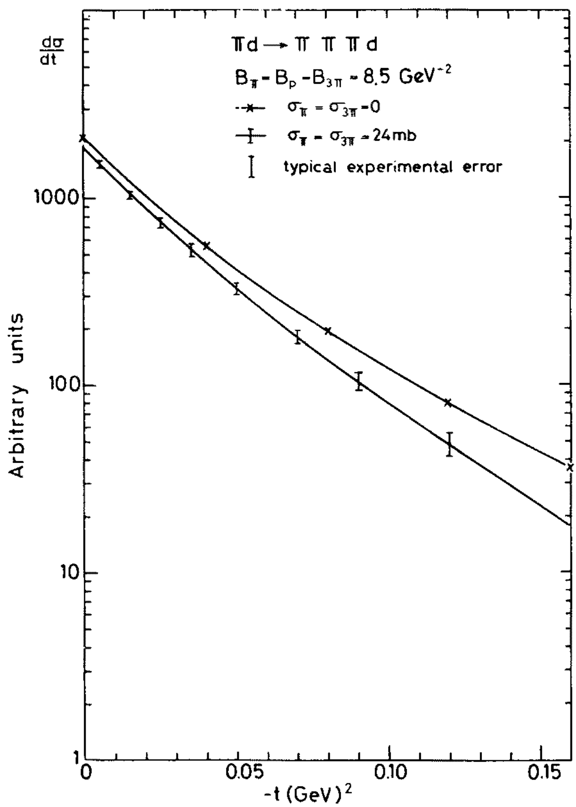

Fig. 3. Effect of the double scattering correction on the calculated cross section of reaction (3.1). The experimental errors are taken from Ref. [17]

and choosing various values of B_p, are compared with the experimental data from Ref. [17]. Slopes B_p from the interval 7–11 GeV^{-2} are clearly favoured, but with the present experimental accuracy deuterium data seem to give less information about the slope B_p than data from scattering on nucleons. Therefore, we keep estimate (3.4) and proceed to the discussion of A_p and the parameters (3.5).

In Fig. 3 results from the full calculation of $d\sigma/dt$ are compared with the results obtained by neglecting the double scattering corrections D_{mn}. It is seen that the discrepancy between the two curves exceeds the experimental errors from Ref. [17]. Thus, the double

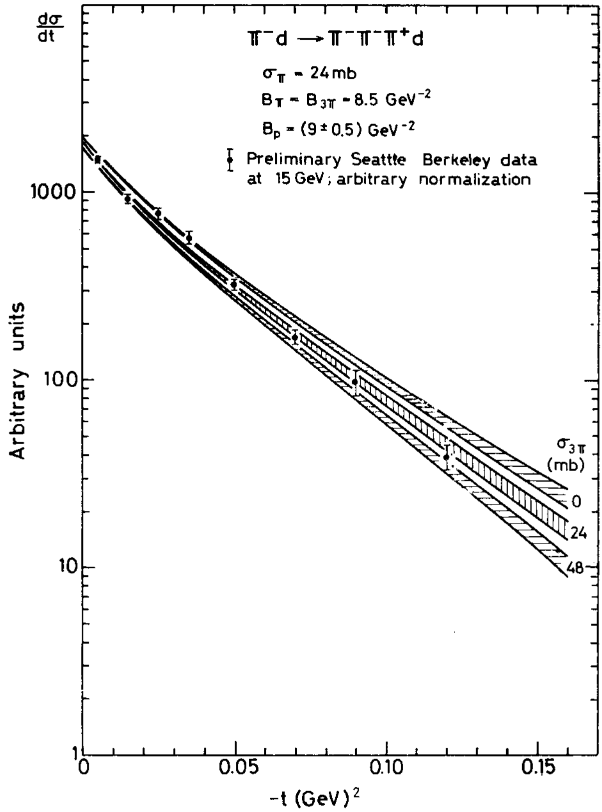

Fig. 4. Dependence of the calculated cross section for process (3.1) and the $(3\pi)-N$ total cross section: Comparison with experiment (Ref. [17])

scattering terms should be included in the analysis of the data for reaction (3.1). Moreover, one can hope that the bubble chamber data on this reaction can give information about (3π) N elastic scattering.

In Fig. 4 the dependence of $d\sigma/dt$ on the (3π) N total cross section is shown. The curves, calculated using the parameters (3.2), $B_{3\pi} = 8.5$ GeV^{-2} and $\sigma_{3\pi} = 0$, 24 and 48 mb, are compared with the arbitrarily normalized data from Ref. [17]. The parameter A_p (irrelevant for this comparison) was taken equal to $10^4/16\pi$ mb^2. The corridors of errors, shown in the figure, correspond to the uncertainty (3.4) in B_p. Analysing both the shape and the normalization of the curves shown in Fig. 4 one can obtain information about elastic (3π) N scattering. The experimental data used in the present paper are unsufficient to reduce the uncertainty on $\sigma_{3\pi}$. The estimates of $B_{3\pi}$ and $\sigma_{3\pi}$ are correlated. We see from Eq. (2.6) that the relevant parameter is approximately given by

$$\lambda \approx \frac{\sigma_{3\pi}}{B_d + \frac{1}{2}(B_p + B_{3\pi})}, \tag{3.6}$$

where B_d is essentially the slope of the deuteron form factor $S_0(t)$ ($B_d \approx 33$ GeV^{-2} for the Gaussian wave function). This shows that there is little chance for getting soon useful information about $B_{3\pi}$.

In Fig. 5 the theoretical relation between the integrated cross section for $\pi^- \to \pi^-\pi^+\pi^-$ coherently on deuterium and diffractively on protons is plotted. The calculation is done for the same set of parameters as in Fig. 4. From Fig. 5 we can read (for different $\sigma_{3\pi}$) the

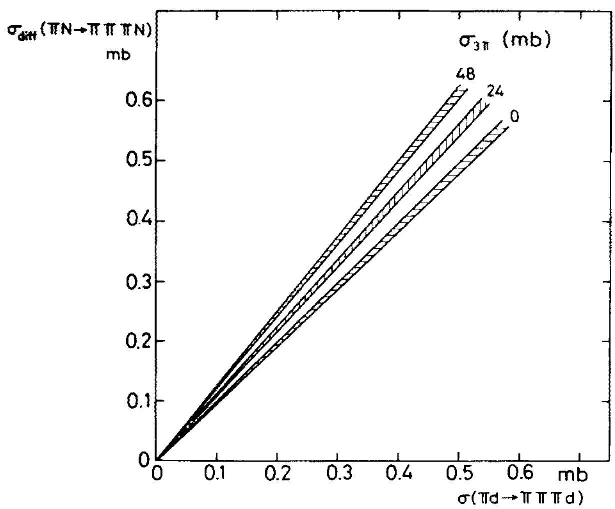

Fig. 5. Predicted relation between the integrated cross sections for the coherent 3π production process on deuterium and for the diffractive process $\pi^-N \to (\pi^-\pi^+\pi^-)N$

values of the deuterium cross sections corresponding to the estimate (3.3). In this way we obtain 0.27, 0.30, and 0.34 mb for $\sigma_{3\pi} = 48, 24$, and 0 mb, respectively. Taking into account the ± 55 mb error of (3.3) we obtain extreme limits 0.22 and 0.40 mb for the integrated deuterium cross section at 205 GeV/c. These numbers are in general agreement with measured cross sections on deuterium in the 5–12 GeV/c range [23–25]. The existing data, however, are not sufficiently accurate to study in detail the energy dependence of the 3π coherent production cross section and the diffractive cross section for $\pi \to 3\pi$ on nucleons. The theoretical correlation presented in Fig. 5 may help us to reduce errors of these cross sections. From the numbers quoted above we infer that an experimental accuracy of the data must be much better than 10% in order to get a valuable information about the (3π)–N total cross section.

4. Discussion

It is found that the bubble chamber data on the coherent process

$$\pi^- d \to (\pi^-\pi^+\pi^-)d \tag{4.1}$$

can give valuable information of the diffractive dissociation process

$$\pi^- \mathrm{N} \to (\pi^-\pi^+\pi^-)\,\mathrm{N} \tag{4.2}$$

and on the elastic $(3\pi)-\mathrm{N}$ scattering. Using data for the process (4.2) we calculated the cross section for the reaction (4.1) at 205 GeV/c finding that it is consistent with the existing deuterium data in the 5–12 GeV/c range. The correlation between the cross sections for both processes: (4.1) and (4.2), may help in the determination of the energy dependence of the diffractive process on nucleons (4.2). To this aim, however, new and precise measurements on deuterium in a wide energy range are required. Now the main source of uncertainty is the error in the integrated cross section for each of the processes (4.1) and (4.2). Other sources of uncertainty in order of decreasing importance are point to point errors in the measured cross section of reaction (4.1) and error in the slope parameter for process (4.2). There are also some theoretical problems discussed briefly in Section 2.

For elastic (3π)–N scattering the data shown in Fig. 4 tend to favour $\sigma_{3\pi} = 24$ mb over 48 mb. Because of the normalization uncertainty, however, this is not significant. We see in Fig. 4 that the splitting of the curves corresponding to the different values of $\sigma_{3\pi}$ rises very quickly with growing momentum transfer. For example, comparing the $\sigma_{3\pi} = 24$ mb and 48 mb cases we get only 6% difference at $t = 0$ but *more than factor 2 in the region of* $-t \approx 0.5$ GeV^2 *when double scattering dominates*. Therefore, an experiment on deuterium in this t region should give conclusive results for the (3π)–N scattering. Let us remember here about similar case, when the ϱ-photoproduction experiment on deuterium at large momentum transfers gave a very significant improvement over the data for the ϱ–N total cross section obtained from scattering on heavy nuclei [26].

Information about the slope of the elastic (3π)–N scattering can, in principle, be obtained from deuterium chamber data; this would require, however, much bigger statistics than presently available. Once again the region of large $|t|$ values is especially suitable for getting the value of the $B_{3\pi}$ parameter.

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