ON THE BEHAVIOUR OF THE *N*-PARTICLE SYSTEM IN THE DYNAMICS WITH RETARDATIONS AND IN THE POST-NEWTONIAN APPROXIMATION OF GENERAL RELATIVITY

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The conditions are given which guarantee a complete dispersion of the n-particle system in the dynamics with retardations. The theorems concerning the behaviour of the n-particle system in the Post-Newtonian approximation are proved.

1. Introduction

The equations of motion of N particles, taking into consideration the relativistic effects, have a rather complicated structure, to say nothing of the difficulties of the n-body problem, even in the Newtonian dynamics. These are, for example, the equations of motion of the linear GR approximation, which take into account the retardation effects and the equations of the Post-Newtonian approximation. One can mention also the classical electrodynamics as well as any other theory taking into account the finite speed of propagation of interactions. Because of the difficulties in the solution of the equations, it would be reasonable to present general theorems dealing with the properties and the qualitative behaviour of particle trajectories. In this connection, the work of Driver [1] should be noted, where analogous theorems are given for the one-dimensional two-body problem of electrodynamics. We shall prove the theorems which concern the necessary conditions of the complete dispersion and the behaviour of the N-particle system (in the P-N approximation), depending on the sign of energy integral. These theorems generalize the corresponding results of the Newtonian theory [2, 3]. Considerable interest has been generated in these problems in the Newtonian theory of gravitation, connected with the problems of cosmogony, in particular with the famous Schmidt hypothesis [2, 3] .

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2. The conditions of the complete dispersion in the dynamics with retardations

Having in mind the applications to relativistic dynamics, consider the equations

$$\dot{\vec{u}}_i(t) = \sum_{k=1,k\neq i}^{N} \frac{\vec{F}_k[\ldots, \vec{r}_j(\alpha(\vec{r}_i, \vec{r}_j, t)), \ldots, \dot{\vec{r}}_j(\alpha(\vec{r}_i, \vec{r}_j, t)), \ldots]}{|\vec{r}_k(\alpha(\vec{r}_i, \vec{r}_j, t)) - \vec{r}_i(t)|^2} +$$

$$+ \sum_{k=1,k\neq i}^{N} \frac{\vec{G}_k[\ldots, \dot{\vec{u}}_j(\alpha(\vec{r}_i, \vec{r}_j, t)), \ldots, \dot{\vec{r}}_j(\alpha(\vec{r}_i, \vec{r}_j, t)), \ldots, \vec{r}_j(\alpha(\vec{r}_i, \vec{r}_j, t)), \ldots]}{|\vec{r}_k(\alpha(\vec{r}_i, \vec{r}_k, t)) - \vec{r}_i(t)|}, \qquad (1)$$

where (we put $e = 1$)

$$\vec{u}_i(t) = \dot{\vec{r}}_i(t)\,(1 - \dot{\vec{r}}_i^{\,2}(t))^{-1/2}$$

and $\alpha(\vec{r}_i, \vec{r}_j, t)$ is defined from the equation

$$\alpha(\vec{r}_i, \vec{r}_j, t) = t - \vec{r}_j(\alpha(\vec{r}_i, \vec{r}_j t)) - \vec{r}_i(t), \qquad (2)$$

$\vec{r}_i(t)$ being the three-dimensional trajectory of the i-th particle. The functions $\vec{F}_k[\ldots, \vec{x}_i, \ldots, \vec{y}_i, \ldots]$, $\vec{C}_k[\ldots, \vec{z}_i, \ldots, \vec{x}_i, \ldots, \vec{y}_i, \ldots]$ are supposed to be sufficiently smooth in the domain $\{|\vec{x}_i| < 1,\ \vec{y}_k \neq y_i \text{ for } k \neq i\}$.

Let $D(v, d)$ ($v < 1$, $d > 0$) be a domain of the values $(\ldots, \vec{x}_i, \ldots, \vec{y}_i, \ldots) \in E_{6N^2}$ (the Euclidean space of $6N^2$ dimensions) in which $|\dot{\vec{x}}_i| < v$, $|\vec{y}_i - \vec{y}_j| > d$ for $i \neq j$, $i, j = 1, \ldots, N$. Suppose that in the case of initial data $\varphi(t)$ for which the following conditions are satisfied: (a) $\vec{\varphi}_i(t) \in C^2_{[t_1, t_0]}$, $\varphi_i(t)$ are Lipschitz continuous on $[t_1, t_0]$, (b) $(\ldots, \dot{\vec{\varphi}}_i(t), \ldots, \vec{\varphi}_i(t), \ldots) \in D(v, d)$, where $v < 1$, $d > 0$, (c) $\alpha(\vec{\varphi}_i, \vec{\varphi}_j, t_0) \geqslant t_1$, (d) the conditions are fulfilled which provide the continuous junction in C^3 of the initial data with the solution at $t = t_0$ (see e. g. [4]), there exists a unique solution $\vec{r}_i(t)$ to Eqs (1) on the segment $[t_0, t_0 + \tau(v, d)]$, which corresponds to the initial data $\vec{\varphi}_i(t)$, and $\vec{r}_i(t) \in C^2$, $\vec{r}_i(t)$ are Lipschitz on $[t_0, t_0 + \tau(v, d)]$, $|\dot{\vec{r}}(t)| < 1$, $\vec{r}_i(t) \neq \vec{r}_i(t)$ for $i \neq j$. Here $\tau(v, d)$ is independent of t_0.

The validity of this statement depends on the properties of the functions $\vec{F}$ and $\vec{G}$ in (1). The restrictions on the $\vec{F}$ and $\vec{G}$ can be found using the extended existence and uniqueness theorem [5]. Our suppositions concerning $\varphi_i(t)$ could be weakened (e. g. the condition (d)), but they are quite relevant for physical applications.

Suppose also

$$|\vec{F}_k[\ldots, x_i, \ldots, y_i, \ldots]| \leqslant m_k(v), \qquad (3)$$

$$|\vec{G}_k[\ldots, z_i, \ldots, x_i, \ldots, y_i, \ldots]| \leqslant g_k(v) \sum_{i=1}^{N} |\vec{z}_i| \qquad (4)$$

for $(\ldots, \vec{x}_i, \ldots, \vec{y}_i, \ldots) \in D(v, d)$.

Our suppositions are satisfied in many practical cases in spite of their seeming complexity. For example, they are fulfilled for the equations of motion of classical electrodynamics (without radiation damping) and in the linear GR approximation. Moreover, the following can be generalized for the equations more general than (1), because in our case only the behaviour of the interparticle interactions at large distances is essential.

Denote $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$, $r_{ij} = |\vec{r}_{ij}|$. Suppose that the initial data satisfy the above conditions with parameters $v < 1$, $d > 0$ and at $t = t_0$

$$\dot{r}_{ij}(t_0) > 0. \tag{5}$$

Evidently, $\bar{D}(v, d) \in D(u_0, d')$, where $d' < d$, $u_0 = 2v(1+3v^2)^{-1/2}$, so the solution to Eqs (1) exists on $[t_0, t_0+\tau(u_0, d')]$, moreover, due to the continuity of the solution there exists $T > t_0$ such that

$$r_{ij}(t) > \tfrac{1}{2}\left[r_{ij}(t_0) + \dot{r}_{ij}(t_0)(t-t_0)\right] \tag{6}$$

and $(\ldots, \dot{\vec{r}}_i(t), \ldots, \vec{r}_i(t), \ldots) \in D(u_0, d')$ for $t \in [t_0, T]$. Let us find the conditions which guarantee the fulfillment of (6) for all $t \in [t_0, \infty.)$

Taking advantage of (1), (3), (4) and the properties of the function $\alpha(\vec{r}_i, \vec{r}_j, t)$ (see e. g. [6]), we have on the segment $[t_0, T]$

$$\sum_{i=1}^{N} |\dot{\vec{u}}_i(t)| \leqslant [f(t)]^{-1} + \sum_{i,k=1, i\neq k}^{N} \frac{2g_k(u_0)(1+u_0)}{r_{ik}(t_0)} \sum_{m,n}^{N} |\dot{\vec{u}}_m(\alpha(r_i, r_n, t))|,$$

where

$$[f(t)]^{-1} = \sum_{i,k=1, i\neq k}^{N} \frac{4m_k(u_0)(1+u_0)^2}{[r_{ik}(t_0) + \dot{r}_{ik}(t_0)(t-t_0)]^2}.$$

Denote $t' = \max t_{ij}$, where t_{ij} is such that $\alpha(r_i, r_j, t_{ij}) = t_0$. Using the properties of the function inverse to $\alpha(r_i, r_j, t)$ (see [6]), we have

$$t' - t_0 \leqslant \max_{i,j} \{r_{ij}(t_0)(1-u_0)^{-1}\}.$$

For $t \in [t_0, t']$

$$\sum_{i=1}^{N} |\dot{\vec{u}}_i(t)| \leqslant M + g \sum_{i=1}^{N} \|\dot{\vec{u}}_i(s)\|_{t_1}^{t}, \tag{7}$$

where

$$\|\vec{x}(s)\|_{t_1}^{t} = \sup_{s\in[t_1, t]} |\vec{x}(s)|,$$

$$M = \sup_{t\in[t_0, t']} f(t) = \sum_{i,k=1, i\neq k}^{N} \frac{4m_k(u_0)(1+u_0)^2}{r_{ij}^2(t_0)},$$

$$g = N \cdot \sum_{i,k=1, i\neq k}^{N} \frac{2g_k(u_0)(1+u_0)}{r_{ik}(t_0)}.$$

Note that in most practical cases $g \ll 1$. For $g < 1$ we have from (7) ($t \in [t_0, t']$)

$$\sum_{i=1}^{N} f(t)|\dot{\vec{u}}_i(t)| \leqslant L^{-1} \max\left\{\frac{M}{1-g},\ M+g\sum_{i=1}^{N}\left\| \frac{\ddot{\vec{\varphi}}_i}{(1-\dot{\vec{\varphi}}_i^2)^{1/2}} + \right.\right.$$

$$\left.\left. + \frac{\dot{\vec{\varphi}}_i(\ddot{\vec{\varphi}}_i\dot{\vec{\varphi}}_i)}{(1-\dot{\vec{\varphi}}_i^2)^{3/2}} \right\|_{t_1}^{t}\right\}, \tag{8}$$

where

$$L = \inf_{t\in[t_0,t']}[f(t)]^{-1} \geqslant \sum_{i,k=1, i\neq k}^{N} \frac{4m_k(u_0)(1+u_0)^2}{[r_{ij}(t_0)+r_{ij}(t_0)\max r_{k,n}(t_0)(1-u_0)^{-1}]^2}.$$

Using the properties of $\alpha(r_i, r_j, t)$ we have for $t \leqslant t'$

$$[f(\alpha(r_i, r_j, t))]^{-1}f(t) \leqslant \max_{i,j,k,n}\left\{2+u_0+\frac{\dot{r}_{ij}r_{kn}}{r_{ij}(1-u_0)}\right\}^2.$$

Then

$$\sum_{i=1}^{N} f(t)\,|\dot{\vec{u}}_i(t)| \leqslant 1+g'\sum_{i=1}^{N}\|f(s)\dot{\vec{u}}_i(s)\|_{t_0}^{t}, \tag{9}$$

where

$$g' = g\max_{i,j,k,n}\left\{2+u_0+\frac{\dot{r}_{ij}r_{kn}}{r_{ij}(1-u_0)}\right\}^2.$$

From (9) we find for $g' < 1$

$$\sum_{i=1}^{N} f(t)\,|\dot{\vec{u}}_i(t)| \leqslant \max\{(1-g')^{-1}, 1+g'\sum_{i=1}^{N}\|f(s)\dot{\vec{u}}_i(s)\|_{t_0}^{t'}\}, \qquad t \geqslant t', \tag{10}$$

where $\sum_{i=1}^{N}\|f(s)\dot{\vec{u}}_i(s)\|_{t_0}^{t'}$ is defined from (8). Thus we have $\forall\, t \in [t_0, T]$

$$\sum_{i=1}^{N}\|f(s)\dot{\vec{u}}_i(s)\| \leqslant K, \tag{11}$$

where the constant K is defined from the inequalities (8), (10). It should be noted that in the case of small velocities $K \simeq 1$.

It is easy to see that

$$\ddot{r}_{ij}(t) \geqslant -|\ddot{\vec{r}}_{ij}(t)| \geqslant -2|\dot{\vec{u}}_i(t)|-2|\dot{\vec{u}}_j(t)|.$$

Using (11) we find from Eqs (1)

$$r_{ij}(t) \geqslant \dot r_{ij}(t_0)\,(t-t_0)-2K\int_{t_0}^{t} ds\int_{t_0}^{s} ds'[f(s')]^{-1}+r_{ij}(t_0) \geqslant$$

$$\geqslant r_{ij}(t_0)+\left\{\dot r_{ij}(t_0)-2K\sum_{\substack{n,k=1\\ n\neq k}}^{N}\frac{4m_k(u_0)\,(1+u_0)^2}{r_{nk}(t_0)\dot r_{nk}(t_0)}\right\}(t-t_0),$$

$$|\vec u_i(t)| \leqslant \frac{v}{(1-v^2)^{1/2}}+K\int_0^t ds[f(s)]^{-1} \leqslant$$

$$\leqslant \frac{v}{(1-v^2)^{1/2}}+K\sum_{\substack{n,k=1\\ n\neq k}}^{N}\frac{4m_k(u_0)\,(1+u_0)^2}{r_{nk}(t_0)\dot r_{nk}(t_0)}\,.$$

By analogy with [2], theorem (3.3), it is easy to see[1] that if

$$K\sum_{\substack{i,k=1\\ i\neq k}}^{N}\frac{8m_k(u_0)\,(1+u_0)^2}{r_{ik}(t_0)r_{ik}(t_0)} \leqslant \min_{i,k}\frac{r_{ik}(t_0)}{2}\,, \tag{12}$$

then the inequality (6) is fulfilled in $[t_0, t_0+\tau(u_0, d')]$ and $(\dots, \dot{\vec r}_i(t), \dots, \vec r_i(t), \dots) \in D(u_0, d')$. Thus the solution can be extended to $[t_0, t_0+2\tau(u_0, d')]$: Continuing this process one can prove (6) for any $t \geqslant t_0$.

We are now in a position to formulate the following theorem.

Theorem 1. Let the initial data $\bar\varphi(t)$ given in $[t_1, t_0]$ satisfy the conditions (a)–(d). Let (6), (12) and $g' < 1$ be fulfilled. Then

$$r_{ij}(t) \to \infty \text{ for } t \to \infty,\ i,j = 1, \dots, N,\ i \neq j.$$

The theorem has a simple physical meaning. The conditions $\dot r_{ij}(t_0) > 0$ imply that at $t = t_0$ the distances between the particles are increasing. If the distances are sufficiently large (therefore (12) and $g' < 1$ should be satisfied), the interaction between the particles is small and slightly affects their motion, i.e. the system continues to disperse.

Let us find the conditions of the complete dispersion in another case.

Denote $\vec v_{ij} = \dot{\vec r}_{ij}(t_0)$, $v_{ij} = |\vec v_{ij}|$. It is easy to see that

$$\vec r_{ij}(t_0)+\vec v_{ij}(t-t_0) = \vec\varrho_{ij}+\vec v_{ij}(t-t_{ij}),$$

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[1] Suppose (for contradiction) that these statements are not true beginning with some $t = T$. Then the estimates proved will lead to contradiction because of (12).

where

$$t_{ij} = -e_{ij}\frac{r_{ij}(t_0)}{v_{ij}}, \qquad e_{ij} = \frac{(\vec{r}_{ij}(t_0)\vec{v}_{ij})}{v_{ij}r_{ij}(t_0)},$$

$$\vec{\varrho}_{ij} = \vec{r}_{ij}(t_0)+\vec{v}_{ij}(t_{ij}-t_0), \qquad \varrho_{ij} \overset{\text{df}}{=} |\vec{\varrho}_{ij}| = \sqrt{1-e_{ij}^2}\cdot r_{ij}(t_0).$$

Evidently,

$$|\vec{r}_{ij}(t_0)+\vec{v}_{ij}(t-t_0)| = [v_{ij}^2(t-t_{ij})^2+\varrho_{ij}^2]^{1/2}.$$

We shall take $\varrho_{ij} > 0$.

Consider a segment $[t_0, T]$ such that

$$r_{ij}(t) > \tfrac{1}{2}[v_{ij}^2(t-t_{ij})^2+\varrho_{ij}^2]^{1/2}.$$

By analogy to the above considerations we obtain

$$\sum_{i=1}^{N} h(t)\,|\dot{\vec{u}}_i(t)| \leqslant \tilde{K},$$

where

$$[h(t)]^{-1} = \sum_{\substack{i,k=1\\ i\neq k}}^{N} \frac{4m_k(u_0)\,(1+u_0)^2}{(\varrho_{ij}^2+v_{ij}^2(t-t_{ij})^2)}, \tag{13}$$

the constant $\tilde{K}$ is defined by means of the function $h(t)$ just as K is defined by means of $f(t)$.

Making use of this estimate, one can prove the following theorem.

Theorem 2. Let the initial data given in $[t_1, t_0]$ satisfy the conditions (a)–(d), $k' < 1$, where k' is defined by means of the function $h(t)$ just as g' is constructed by means of $f(t)$, and

$$\tilde{K}\cdot\sum_{\substack{i,k=1\\ i\neq k}}^{N} \frac{16\pi m_k(u_0)\,(1+u_0)^2}{\varrho_{ik}(t_0)v_{ik}} \leqslant \min_{i,k}\frac{v_{ik}}{2}.$$

Then $r_{ij}(t)\to\infty$ for $t\to\infty$, $i, j = 1, \ldots, N$, $i\neq j$.

3. *The Post-Newtonian approximation*[2]

The theorems proved above give us some information on the behaviour of dispersing system of particles moving asymptotically with uniform velocity. In the case of finite

[2] When we were writing this paper we were not familiar with the paper by B. R. Hoffman and P. Havas (*Phys. Rev.* **B140**, 1162 (1965)) where theorems are proved which deal with the same subject as our Theorems 3 and 4. We are grateful to Professor Havas for sendirg us this paper. However, there is a technical difference in our proof: starting from the Lagrangian (14) we treat the problem from a rigorous viewpoint without neglecting higher order terms. This involves some complications in the proof of Theorem 4.

motions the behaviour of the system depends strongly on the type of equations, so the derivation of the common criteria which are not connected with the concretization proves to be difficult. Moreover, the equations with retardations do not admit the energy integral, while it would be interesting to obtain the estimates concerning the sign of energy[3]. Because of this we shall consider the behaviour of the N-particle system described by the Post-Newtonian Lagrangian (see e. g. [7])

$$\mathscr{L} = T_0(\dot{r})+T_1(\dot{r})+U(\dot{r}, r)+V_0(r)+V_1(r), \tag{14}$$

where

$$T_0(\dot{\vec{r}}) = \frac{1}{2}\sum_{i=1}^{N} m_i\dot{\vec{r}}_i^2, \quad T_1(\dot{\vec{r}}) = \frac{1}{8c^2}\sum_{i=1}^{N} m_i(\dot{\vec{r}}_i^2)^2,$$

$$U(\dot{\vec{r}}, \vec{r}) = \frac{\gamma}{4c^2}\sum_{\substack{i,j=1\\ i\neq j}}^{N} m_im_jr_{ij}^{-1}[3\dot{\vec{r}}_i^2+3\dot{\vec{r}}_j^2-7(\dot{\vec{r}}_i\dot{\vec{r}}_j)]-$$

$$-\frac{\gamma}{4c^2}\sum_{\substack{i,j=1\\ i\neq j}}^{N} m_im_jr_{ij}^{-3}(\dot{\vec{r}}_i\vec{r}_{ij})(\dot{\vec{r}}_j\vec{r}_{ij}),$$

$$V_0(r) = \frac{\gamma}{2}\sum_{\substack{i,j=1\\ i\neq j}}^{N} m_im_jr_{ij}^{-1},$$

$$V_1(r) = -\frac{\gamma^2}{4c^2}\sum_{\substack{i,j=1\\ i\neq j}}^{N} m_im_j(m_i+m_j)r_{ij}^{-3}-$$

$$-\frac{\gamma^2}{6c^2}\sum_{\substack{i,j,k=1\\ i\neq j\neq k\neq i}}^{N} m_im_jm_k[r_{ij}^{-1}r_{jk}^{-1}+r_{jk}^{-1}r_{ki}^{-1}+r_{ki}^{-1}r_{ij}^{-1}].$$

Here m_i are the masses of the particles, γ is the gravitational constant, c is the speed of light.

Note that this case one can also prove the theorems analogous to Theorems 1 and 2, because the interaction falls off sufficiently quickly at large distances between the particles.

From (14) we have the energy integral

$$H = T_0(r)+3T_1(r)+U(r, r)-V_0(r)-V_1(r). \tag{15}$$

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[3] The term integral is meant in the sense of ordinary differential equations, i. e. as a function of positions and velocities.

So far as the Post-Newtonian approximation is valid in the case of a weak gravitational field we put

$$\frac{\gamma M}{c^2 \varrho(t)} \leqslant a, \tag{16}$$

where a is sufficiently small, $\varrho(t) = \min\limits_{i,j(i \neq j)} \{r_{ij}(t)\}$, $M = \sum\limits_{i=1}^{N} m_i$.

The condition (16) can be treated as the condition of the absence of collisions, because it shows that the particles do not approach each other too closely. This requirement is quite natural in the case of extended particles of finite sizes. It should be noted that when the Post-Newtonian approximation is valid one can choose $a \ll 1$.

Let us consider the behaviour of the system of particles for $H < 0$. From (15) it follows, after elementary estimates are made, that

$$\frac{M^2\gamma}{2\varrho(t)} > |H| + \sum_{i=1}^{N} \frac{m_i r_i^2}{2}\left(1 - \frac{\gamma}{c^2} \sum_{j=1, j\neq i}^{N} \frac{m_i}{r_{ij}}\right). \tag{17}$$

This proves the following theorem.

Theorem 3. Suppose that in the system of particles described by the Lagrangian (14) collisions are absent for $a < 1$.

Then for $H < 0$

$$\varrho(t) \leqslant \frac{\gamma M^2}{2|H|}. \tag{18}$$

Indeed, for $a < 1$

$$\frac{\gamma}{c^2} \sum_{j=1, j\neq k}^{N} \frac{m_j}{r_{kj}} < \frac{M\gamma}{c^2 \varrho(t)} < 1$$

and (18) follows from (17).

Consider now the case $H > 0$.

Introduce the quantity

$$K = \sum_{i=1}^{N} (\vec{r}_i p_i),$$

where $\vec{p}_i$ are the generalized momenta corresponding to (14)

$$p_i = \frac{\partial \mathscr{L}}{\partial \dot{r}_i}.$$

Taking into account the Lagrange equations and the Euler theorem we easily obtàin

$$\frac{dK}{dt} = H + T_0(\dot{r}) + T_1(\dot{r}) - V_1(r). \tag{19}$$

Denote

$$L = \tfrac{1}{2} \sum_{i,j=1}^{N} m_i m_j (\vec{r}_{ij} \vec{v}_{ij}),$$

where

$$\vec{v}_{ij} = \vec{v}_i - \vec{v}_j, \qquad \vec{v}_i = \frac{\vec{p}_i}{m_i}.$$

The Lagrangian (14) is invariant with respect to space translations, whence

$$\sum_{i=1}^{N} \vec{p}_i = \text{const}.$$

Consider the coordinate system where

$$\sum_{i=1}^{N} \vec{p}_i = 0. \tag{20}$$

In order that (20) be satisfied, it is sufficient to choose the initial conditions in an appropriate manner. In this coordinate system we have

$$L = M \sum_{i=1}^{N} (\vec{p}_i \vec{r}_i) = MK. \tag{21}$$

Theorem 4. Suppose that in the coordinate system in which (20) is fulfilled, the energy constant $H > 0$ and the condition of absence of collisions is satisfied for $a < 1$.

Then

$$\sum_{i,j=1}^{N} r_{ij}(t) \to \infty \quad \text{for} \quad t \to \infty.$$

Proof.
Using (19), (21), we have

$$L(t) = L(t_0) + \int_{t_0}^{t} [H + T_0(r) + T_1(r) - V_1(r)]dt \geqslant L(t_0) + MH(t - t_0).$$

Thus $L(t) \to \infty$ for $t \to \infty$.
By analogy with (17), we obtain for $a < 1$ that

$$H \geqslant \frac{1}{2} \sum_{i=1}^{N} m_i \dot{\vec{r}}_i^2 (1-a) - \frac{Mc^2 a}{2},$$

whence it follows that $|\dot{\vec{r}}_i|$ are bounded for all t. Then using (16) and the expression for $\vec{p}_i$ we have

$$|\vec{p}_i| \leqslant B,$$

where B is a constant.

Thus $|\vec{v}_{ij}|$ are bounded for all t. From the inequality

$$|L(t)| < \frac{1}{2} \sum_{i,j=1}^{N} m_i m_j |\vec{v}_{ij}(t)| \sum_{k,n=1}^{N} r_{kn}(t),$$

we obtain the statement of the theorem.

From this theorem it follows that for $H > 0$ at least one body goes to infinity with respect to other bodies.

It should be noted that the results obtained in this paper are quite similar in form to those of the Newtonian dynamics [2, 3].

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