Vol. B6 (1975) ACTA PHYSICA POLONICA No 1

PARAMETRIZATION OF THE MISSING MASS DISTRIBUTION IN AN INCLUSIVE PROCESS

BY A. ESKREYS

Institute of Nuclear Physics, Cracow*

AND B. MURYN

Institute of Nuclear Physics and Techniques, Academy of Mining and Metallurgy, Cracow**

(*Received May 22, 1974; Revised version received June 27, 1974*)

A parametrization of the missing mass distribution in an inclusive process $a+b \to c+X$ is proposed in terms of the variable $\xi = \frac{M}{\sqrt{s}-m_c}$ (m_c being the mass of the inclusive particle c). On the basis of $K^+ + p \to K^0 + X$, $K^+ + p \to \pi^+ + (K^0 + X)$, $K^+ + p \to \pi^- + (K^0 + X)$ data at 5 and 8.2 GeV/c, it is found that the distribution $\frac{1}{\langle n_c \rangle} \frac{d\sigma}{d\xi}$ scales better than the Feynman invariant cross-section and has a simple exponential form.

1. *Introduction*

It is known that for high energies scaling of the Feynman invariant cross-section

$$\frac{E_c}{\pi\sqrt{s}} \frac{d^2\sigma}{dx dp_\perp}$$

leads to the scaling behaviour of $\frac{d^2\sigma}{d\left(\frac{M}{\sqrt{s}}\right)dt}$ because of the relation

$$|x| = \left(1 - \frac{M^2}{s}\right). \tag{1}$$

* Address: Instytut Fizyki Jądrowej, Kawiory 26a, 30-055 Kraków, Poland.

** Address: Instytut Fizyki i Techniki Jądrowej, Akademia Górniczo-Hutnicza, Kawiory 26a, 30-055 Kraków, Poland.

However, this relation is true only in the asymptotic region. Fig. 1 shows the experimental plot of $x = 2p_L/\sqrt{s}$ vs. $M/\sqrt{s}$ for the reaction $K^+ + p \to K^0 + X$ at 8.2 GeV/c. One can see that relation (1) does not hold at all at energy of a few GeV/c. In Figs. 2a, b, c we show

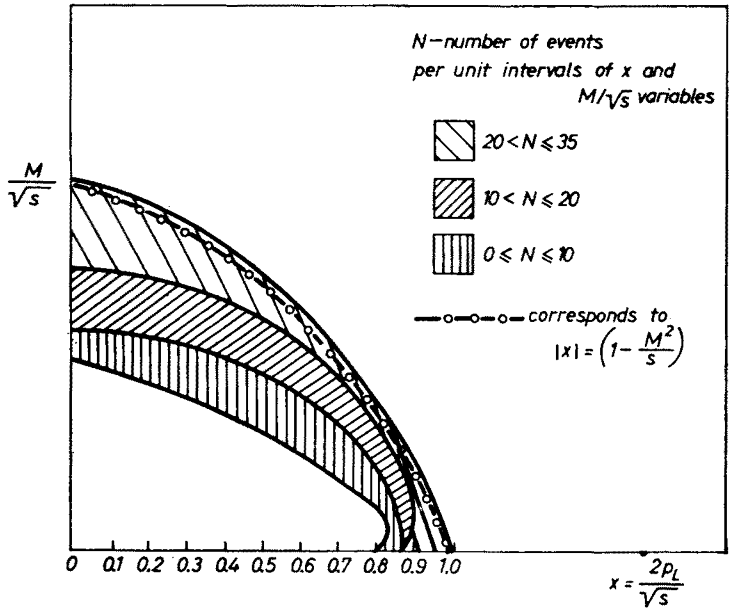

Fig. 1. Distribution of $M/\sqrt{s}$ versus the x variable at 8.2 GeV/c

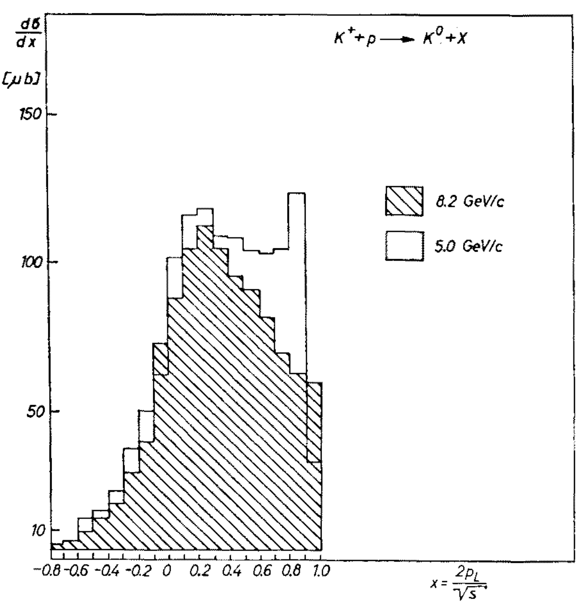

Fig. 2a. The inclusive cross-sections $d\sigma/dx$ for $K^+ + p \to K^0 + X$ at 5 and 8.2 GeV/c

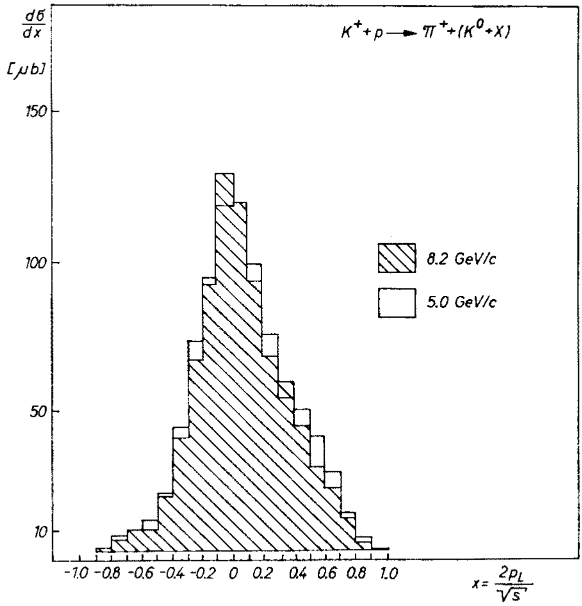

Fig. 2b. The inclusive cross-section $d\sigma/dx$ for $K^+ + p \to \pi^+ + (K^0 + X)$ at 5 and 8.2 GeV/c

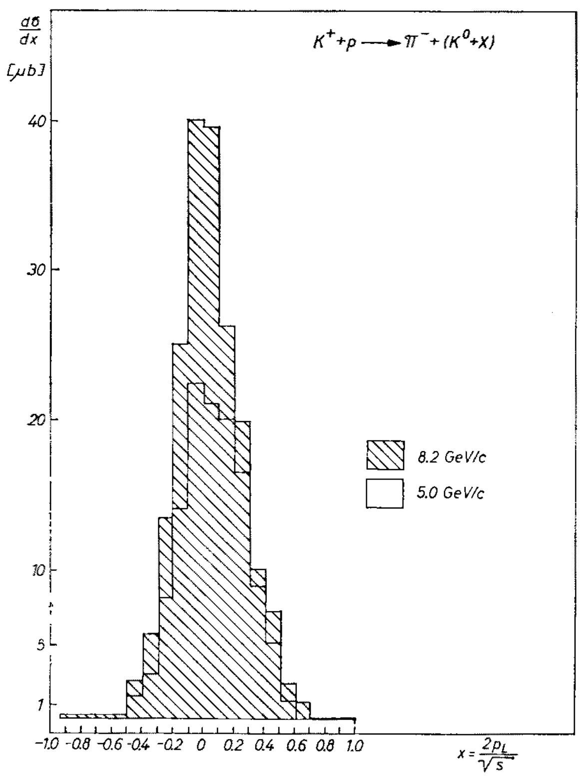

Fig. 2c. The inclusive cross-section $d\sigma/dx$ for $K^+ + p \to \pi^- + (K^0 + X)$ at 5 and 8.2 GeV/c

the Feynman invariant cross-section for processes

$$K^+ + p \to K^0 + X, \tag{2}$$

$$K^+ + p \to \pi^+ + (K^0 + X), \tag{3}$$

$$K^+ + p \to \pi^- + (K^0 + X), \tag{4}$$

at 5 and 8.2 GeV/c. Except for the distribution for π^+'s, clear differences between 5 and 8.2 GeV/c are observed. Additionally, recent results on the inclusive Regge model suggest the use of the $M/\sqrt{s}$ variable. Therefore it is interesting to look for scaling behaviour in terms of $M/\sqrt{s}$ variable. In order to obtain the same upper limit of the chosen variable for different inclusive reactions, instead of variable $M/\sqrt{s}$ we use the variable

$$\xi = \frac{M}{\sqrt{s} - m_c}.$$

2. *Experimental data*

We used the data on inclusive reactions [2–4] at 5 and 8.2 GeV/c.

$$\frac{1}{\langle n_c \rangle} \frac{d\sigma}{d\xi}$$

distributions were studied, where the factor $1/\langle n_c \rangle$ takes into account the energy dependence of average multiplicity of the inclusive particle c. These distributions are shown in Figs. 3a, b, c. As can be seen, the distributions at 5 and 8.2 GeV/c coincide within the experimental errors. Additionally they follow the exponential behaviour[1]

$$\frac{1}{\langle n_c \rangle} \frac{d\sigma}{d\xi} = a \cdot e^{b\xi}. \tag{5}$$

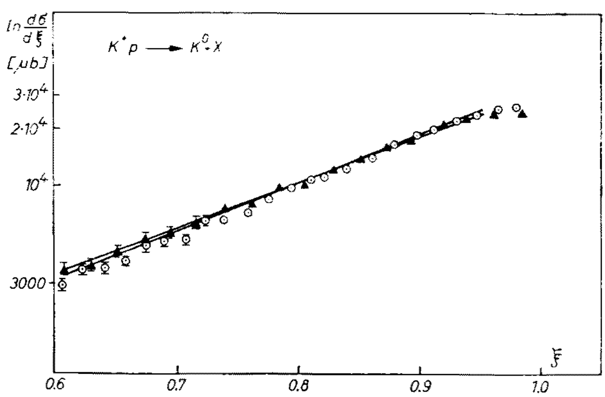

Fig. 3a. Distribution $d\sigma/d\xi$ for $K^+ + p \to K^0 + X$ at 5 (▲) and 8.2 (⊙) GeV/c

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[1] It has been found [1] that distribution $d\sigma/dM$ is proportional to e^{kM} what is in agreement with our parametrization.

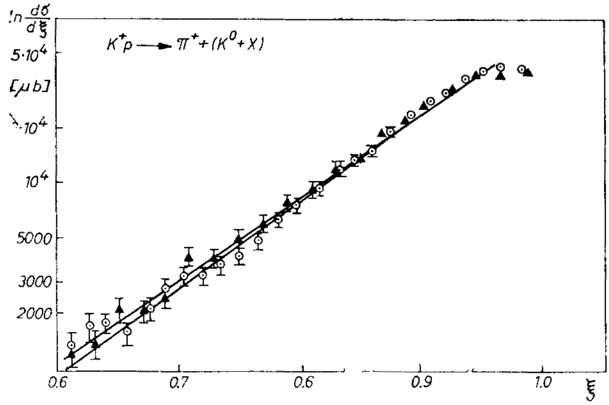

Fig. 3b. Distribution $d\sigma/d\xi$ for $K^+ + p \to \pi^+ + (K^0 + X)$ at 5 (▲) and 8.2 (⊙) GeV/c

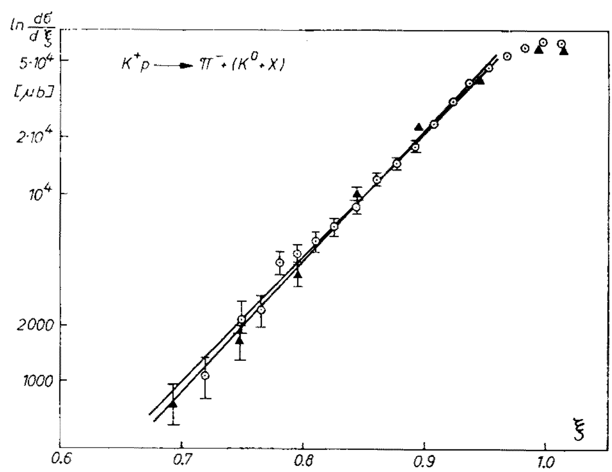

Fig. 3c. Distribution $d\sigma/d\xi$ for $K^+ + p \to \pi^- + (K^0 + X)$ at 5 (▲) and 8.2 (⊙) GeV/c

Fitted values of parameters a and b for different inclusive particles are listed in Table I. It is worth noting that for the inclusive process

$$K^+ + p \to K^0 + X$$

because of $\langle n_{K^0} \rangle = 1$ at the studied energies, the parametrization of the form (5) is consistent with constant total cross-section for reaction (2). Indeed, integrating

$$\int_{M_{\min}}^{M_{\max}} \frac{d\sigma}{dM} dM = \int_{M_{\min}/M_{\max}}^{1} \frac{d\sigma}{d\xi} d\xi = \int_{M_{\min}/M_{\max}}^{1} a e^{b\xi} d\xi = \frac{a}{b}\left[e^{b} - e^{b\frac{M_{\min}}{M_{\max}}}\right], \tag{6}$$

TABLE I

Reaction	Momentum GeV/c	b	a [μb]
$K^+ + p \to K^0 + X$	5.0	5.55 ± 0.20	94 ± 8
	8.2	5.76 ± 0.25	111 ± 9
$K^+ + p \to \pi^+ + (K^0 + X)$	5.0	11.5 ± 0.50	0.43 ± 0.04
	8.2	12.1 ± 0.50	0.30 ± 0.04
$K^+ + p \to \pi^- + (K^0 + X)$	5.0	15.4 ± 0.60	0.0115 ± 0.0012
	8.2	16.0 ± 0.60	0.0084 ± 0.0008

and neglecting the term $e^{b\, M_{\min}/M_{\max}}$, which is very small compared with term e^b, we obtain

$$\int_{M_{\min}/M_{\max}}^{1} \frac{d\sigma}{d\xi}\, d\xi = \frac{a}{b}\, e^b,$$

which is constant on the basis of $d\sigma/d\xi$ scaling properties. On the other hand, the integrated cross-section (6) corresponds to the total cross-section $K^+ + p \to K^0$ which is found constant and equal $1/3 \cdot \sigma_{\text{inel}}(K^+ + p)$ (see Ref. [1]).

3. *Conclusions*

A parametrization of the missing mass distribution in an inclusive process $a+b \to c+X$ is proposed in terms of the variable

$$\xi = \frac{M}{\sqrt{s} - m_c}$$

On the basis of $K^+ + p$ inclusive data at 5 and 8.2 GeV/c it is shown that distribution

$$\frac{1}{\langle n_c \rangle} \frac{d\sigma}{d\xi}$$

scales and has an exponential form $ae^{b\xi}$, where a and b are the energy independent parameters. It is worth stressing that at the same energies the invariant Feynman cross-section does not scale. The proposed parametrization of the inclusive missing mass distribution in terms of the ξ variable leads to very interesting consequences regarding the approach to scaling of the $S\, d\sigma/dM^2$ distribution for $M/\sqrt{s} = \text{const} < 1$. One can see that, assuming our parametrization, one gets the following expression

$$s \frac{d\sigma}{dM^2} = \frac{a \cdot s}{M(\sqrt{s} - m_c)}\, e^{b \frac{M}{(\sqrt{s} - m_c)}} = f_1\left(\frac{M}{\sqrt{s}}\right) + \frac{1}{\sqrt{s}} f_2\left(\frac{M}{\sqrt{s}}\right), \tag{7}$$

in which the non-scaling part vanishes as $1/\sqrt{s}$. Qualitatively, the same result can be obtained from Regge theory [2], where the second term in the relation (7) is interpreted to be the meson exchange contribution and the first one the Pomeron exchange scaling part.

The authors are indebted to CERN-Brussels K^+ Collaboration for kind permission to use their data. Special thanks are due to Professors A. Białas, F. Grard, Y. Goldschmidt-Clermont, and K. Zalewski for their interest and many valuable remarks. One of us (B. Muryn) would like to express his gratitude to Professor F. Grard for the hospitality, extended to him during his stay at Université de l'État in Mons.

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[2] Chan Hong-Mo, *Proceedings of the IV International Conference on High Energy Collisions*, Oxford, 1972.