Vol. B6 (1975) ACTA PHYSICA POLONICA No 6

THE THEORY AND PHENOMENOLOGY OF COLOURED QUARK MODELS

By F. E. Close*

CERN — Geneva

(Presented at the XV Cracow School of Theoretical Physics, Zakopane, June 6–19, 1975)

A general introduction to coloured quark models is given and their phenomenology is described with particular reference to the new particles. It is shown that there are essentially three types of colour models with colour excitation when the colour group is SU(3) — Han–Nambu, Greenberg and a model which has the same charges as that of Tati and which can be thought of as the Gell-Mann colour scheme with excitation of the colour degrees of freedom. Particular attention is paid to the four problems of colour models for ψ phenomenology — the radiative decays, the G parity conservation, the lack of deep inelastic threshold phenomena and the apparent discovery of dileptons at SPEAR.

FOREWORD

1. These lectures describe the various approaches to the new particles which are classified under the general heading of colour. They complement and supplement those on charm by Ellis [1] and the discussion of other additive quantum number approaches given by Hey, both of which appear in this issue.

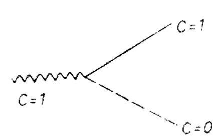
2. Section 6 contains details of work performed in collaboration with J. Weyers. Any reference to that material should include also my earlier work with Weyers [2].

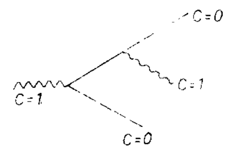
CONTENTS

* Present address: High Energy Physics, Theory Group, Rutherford High Energy Lab. Chilton, Didcot, Berkshire, England.

CHARM AND COLOUR INTRODUCTION

If ψ are hadronic then their metastability at high mass suggests that some new degree of freedom is involved beyond the familiar SU(3). Two particular directions in which one might generalize this group structure are outlined below. This is only intended for orientation; these points are discussed in more detail by Ellis [1]

	SU(3) → SU(N)	SU(3) → SU(3) ⊗ G
Examples	$N = 4$ charm	G = $\overline{\mathrm{SU}(3)}$ colour
New quantum numbers	Additive (like strangeness)	Non-additive (like isospin)
Do ψ and γ have the new degree of freedom?	Hidden [like $\phi(\lambda\bar{\lambda})$]	Manifest, e.g. (i) ω has $I = 0$ and ϱ has $I = 1$ (ii) $\varrho\omega$ have $C = 0$ ψ has $C \neq 0$
Narrow widths	Zweig rule, e.g. $\left(\frac{\phi \to 3\pi}{\phi \to K\bar{K}} \ll 1\right)$	Conservation of C by strong interactions
Theoretical radiative decays	$\psi \to \eta_c\gamma$ $\left.\begin{array}{l}\psi' \to (^3\mathrm{p})\gamma \\ \quad \hookrightarrow \gamma\psi\end{array}\right\} \equiv \psi\gamma\gamma$	ψ has $C = 1$ Ordinary hadrons $C = 0$ γ has $C = 0$ ($\varrho\omega\phi$) and $C = 1$ (ψ) $\psi(C = 1) \to \gamma(C = 1)+$Hadrons ($C = 0$)
Production threshold in e^+e^-	Pair production at $Q^2 \gtrsim 16\,(\mathrm{GeV})^2$	Pair production for exact colour $Q^2 \gtrsim 36\,(\mathrm{GeV})^2$: Broken colour $Q^2 \gtrsim 16$? Can be produced singly 

	SU(3) → SU(*N*)	SU(3) → SU(3) ⊗ G
Examples	$N = 4$ charm	G = $\overline{\text{SU}(3)}$ colour
Increase of neutral energy fraction with Q^2	$D^* \to D\gamma$ if $m_{D^*} - m_D < m_\pi$ where D are charmed mesons	 Photons radiate energy
Deep inelastic thresholds and scaling violation	Thresholds open as $x \to 0$ due to $c\bar{c}$ parton sea	Quarks are colour analysed even as $x \to 1$
$Q^2 = 0$ $\sigma_{tot}(\gamma_p)$	Small rise by VMD	Small rise by VMD
Theory a priori motivation	GIM mechanism for unification of weak and electromagnetic interactions without strangeness changing neutral currents	Baryon statistics. $\pi^0 \to 2\gamma$ rate, etc.,
Electric charge	Not generator of SU(4)	Generator of SU(3)×$\overline{\text{SU}(3)}$ for suitable non-trivial quark charges

The following table repeats the skeleton of the above and gives comments on the various points presented in the first table

	SU(3) → SU(4)	SU(3) → SU(3) ⊗ SU(3)
Narrow widths	A problem? Why is Zweig's rule so good?	Natural explanation of narrow widths
Radiative widths	Problem: None seen among $\psi \to \eta_c\gamma$; $\psi' \to \gamma$+new states.	A priori important. Why is *G* parity concerned in ψ decays? Problem: None seen among $\psi \to \gamma$+(coloured pseudoscalars).
Production threshold in e^+e^-	Natural explanation of σ^{tot} threshold. Magnitude problem? $R \to 3^1/_3$ in model but data $R \gtrsim 5$? No new peaks found in the final states produced. Dileptons "natural".	No new peaks seen in the final states. Exotic states predicted. Why dileptons?

	SU(3) → SU(4)	SU(3) → SU(3) ⊗ SU(3)
Increase of neutral energy	Possible if $m_{D^*} \simeq m_D$	Natural due to radiation of γ in colour decays
Deep inelastic thresholds	Consistent with data	No evidence for big thresholds opening as $x \to 1$. Problem?
$\sigma_{tot}(\gamma_p)Q^2 = 0$	Consistent with data	Consistent with data

1. Why colour?

Originally the motivation for having coloured quarks was in connection with baryons and the spin-statistics theorem. In the quark model one obtains a nice description of the baryon spectrum if one demands that the baryons are built from three quarks and that the total SU(6) ⊗ O(3) three-quark wave function is *symmetric* under the interchange of any pair of quarks. Yet baryons are *antisymmetrized* with respect to one another since they are fermions and obey the Pauli principle.

Consequently, it looks as if quarks are funny. They are symmetrized in sets of three, but antisymmetric otherwise. One can consider "parafermi statistics of rank three" [3] and impose the demand that physical particles are fermions or bosons — and so all physical three-quark systems are totally symmetric.

Alternatively, one can make the three-quark system totally antisymmetric by introducing a new degree of freedom for the quarks. We paint them red, green or blue. This RGB degree of freedom generates an [SU(3)] colour group. Baryons are therefore

$$[\mathrm{SU}(6) \otimes \mathrm{O}(3) \otimes \mathrm{SU}(3)_{\mathrm{colour}}]_{\mathrm{ANTISYMMETRIC}}.$$

The familiar baryons are then

$$[\mathrm{SU}(6) \otimes \mathrm{O}(3)]_{\mathrm{SYMMETRIC}} \otimes [\mathrm{SU}(3)_{\mathrm{colour}}]_{\mathrm{ANTISYMMETRIC}},$$

which demands that they be *colour singlets* (since a totally antisymmetric three-body state in SU(3) is a singlet). Note that the antisymmetric colour state requires the three quarks to be one red, one blue, and one green [4].

Colour is also useful in connection with the $\pi^0 \to 2\gamma$ rate for which Adler [5] and Bell-Jackiw [5] have given an exact formula in a "quark-gluon" model theory. The amplitude is a known constant times

$$(\sum_i e_i^2)_{I_z=+1/2} - (\sum_i e_i^2)_{I_z=-1/2}$$

with e_i the quark charges for $I_z = \pm {}^1/_2$ quarks. To agree with experiment within the errors requires that

$$\sum_i I_z^i e_i^2 = \tfrac{1}{2}\,.$$

Three Fermi-Dirac quarks would yield

$$\tfrac{1}{2}\times\tfrac{4}{9}-\tfrac{1}{2}\times\tfrac{1}{9} = \tfrac{1}{6}$$

a factor three too small in amplitude (nine in rate). With the coloured quarks we get a factor of three multiplying this due to the three freedoms RGB (all this assuming PCAC is alright) [6].

2. *Colour models — general*

There is an SU(3) symmetry structure generated by the three quarks (pnλ) which form a triplet representation of this group and so the nλ have one unit of charge less than the p quark.

Each of these quarks can come in three colours red, blue, or green. These three colours RBG generate a separate $SU(3)_C$ (which we will denote $\widetilde{SU(3)}$). The colours RB generate an $\widetilde{SU(2)}$ subgroup which we will refer to as "colour isospin" or $\tilde{I}$.

The charges of the quarks in the three colour states are

	p	n	λ
R	z'	$z'-1$	$z'-1$
B	z''	$z''-1$	$z''-1$
G	z	$z-1$	$z-1$

(2.1)

subject to the constraint that

$$z+z'+z'' = 2. \tag{2.2}$$

The origin of this constraint will be discussed in the next paragraph. Examples of two *particular* models which are already well known are the Gell–Mann colour [6] ($z = z' = z'' = {}^2/_3$) and Han–Nambu [7] ($z = z'' = 1, z' = 0$) which satisfy the constraint. The reason for this constraint is the demand that baryons be colour singlets. To see this, consider the Δ^{++}. To be a colour singlet the Δ^{++} is ($p_R p_G p_B$) and to have charge +2 then one requires

$$z+z'+z'' = 2.$$

Notice that the "average charge" of pnλ is therefore

$$e_p = \frac{z+z'+z''}{3} = \tfrac{2}{3},$$

$$e_n = e_\lambda = \frac{(z-1)+(z'-1)+(z''-1)}{3} = -\tfrac{1}{3}. \tag{2.3}$$

Consequently all colour singlet baryons will have the same charges as in the familiar uncoloured quark model, since the average (colour singlet) quark charges are the same as for "uncoloured" quarks.

The same constraint on z, z', z'' arises if we want to obtain the correct $\pi^0 \to \gamma\gamma$ rate which required

$$\sum_i I_z^i e_i^2 = \tfrac{1}{2}. \tag{2.4}$$

Since $I_z = \pm^1/_2$ for p, n and zero for λ then

$$(z^2+z'^2+z''^2)-((z-1)^2+(z'-1)^2+(z''-1)^2) = 1 \tag{2.5}$$

and hence

$$z+z'+z'' = 2.$$

At this stage any model, modulo this constraint, will be equally good. The Gell–Mann [6] and Han–Nambu [7] are but two examples of an infinity of models. In Section 5 we will classify these models and in doing so it is useful to notice that the charge is a generator of the group if such a constraint is placed on the charges. This is easily seen as follows. The charge operator is given by

$$Q = \left(I_z + \frac{Y}{2}\right) + \left(\alpha \tilde{I}_z + \beta \frac{\tilde{Y}}{2}\right) \tag{2.6}$$

hence the charges of the p quarks will be

$$z' = \tfrac{2}{3} + \frac{\alpha}{2} + \frac{\beta}{6},$$

$$z'' = \tfrac{2}{3} - \frac{\alpha}{2} + \frac{\beta}{6},$$

$$z = \tfrac{2}{3} \qquad - \frac{\beta}{3} \tag{2.7}$$

and so $z+z'+z'' = 2$. In Section 5 we will classify models by their values of α, β.

3. *Mesons in colour models*

To illustrate our notation and to bring out an important point we will calculate the charge of the π^+ both in uncoloured and coloured models.

3.1. Uncoloured quarks

$$\pi^+ \equiv \mathrm{p\bar{n}}.$$

The charge is given by

$$\langle \mathrm{p\bar{n}} | e_q + e_{\bar{q}} | \mathrm{p\bar{n}} \rangle \Rightarrow e_\mathrm{p} + e_{\bar{\mathrm{n}}} \Rightarrow \tfrac{2}{3} + \tfrac{1}{3} = 1$$

in an obvious notation.

3.2. Coloured quarks

We will for the moment suppose that the π^+ is a colour singlet (which we will denote by $\tilde{\omega}_1$)[1]. Hence its representation in $SU(3) \times \widetilde{SU(3)}$ as

$$(\pi^+, \tilde{\omega}_1)$$

$$= \left(p\bar{n}, \frac{1}{\sqrt{3}}(R\bar{R}+B\bar{B}+G\bar{G})\right) \tag{3.1}$$

$$\equiv \frac{1}{\sqrt{3}}(p_R\bar{n}_R+p_B\bar{n}_B+p_G\bar{n}_G). \tag{3.2}$$

The charge calculation then becomes

$$\langle \pi^+, \tilde{\omega}_1 | \hat{Q} | \pi^+, \tilde{\omega}_1 \rangle = \tfrac{1}{3}\langle p_R\bar{n}_R+p_B\bar{n}_B+p_G\bar{n}_G | e_q + e_{\bar{q}} | p_R\bar{n}_R+p_B\bar{n}_B+p_G\bar{n}_G \rangle$$

$$= \tfrac{1}{3}(z'+z''+z)+\tfrac{1}{3}((1-z')+(1-z'')+(1-z))$$

$$= 1. \tag{3.3}$$

Notice that this result was obtained *independent* of any constraint on z, z', z''. We would have obtained the same result, and absence of constraint, for any colour neutral ($\tilde{\omega}_1, \tilde{\omega}_8, \tilde{\varrho}^0$) assignment. Consequently, the familiar mesons can be in any colour neutral representation of SU(3) and the spectroscopy will not constrain z, z', z'' (in contrast to the baryons).

It has been conventional to assume [8] that the familiar mesons are colour singlets, like the baryons. However, it is not a priori necessary as evidenced by the above.

4. *The photon in colour models*

One place in the meson world where the colour assignment is important is for the photon. We illustrate this by examples.

4.1. Uncoloured

p	n	λ
$^2/_3$	$-^1/_3$	$-^1/_3$

With the above charges the photon may be written

$$\gamma \sim \tfrac{2}{3}\, p\bar{p} - \tfrac{1}{3}\, n\bar{n} - \tfrac{1}{3}\, \lambda\bar{\lambda} \tag{4.1}$$

and is a $U = 0$ member of an octet (denoted by γ_8).

[1] We will use $\tilde{\varrho}\tilde{\omega}_8\tilde{\omega}_1$ to denote the $\tilde{I} = 1$ and $\tilde{I} = 0$ (octet, singlet) states.

4.2. Gell-Mann [6]

	p	n	λ
R	$^2/_3$	$-^1/_3$	$-^1/_3$
B	$^2/_3$	$-^1/_3$	$-^1/_3$
G	$^2/_3$	$-^1/_3$	$-^1/_3$

(4.2)

Hence the photon is

$$\gamma_{GM} \sim (\tfrac{2}{3}\, p_R\bar{p}_R - \tfrac{1}{3}\, n_R\bar{n}_R - \tfrac{1}{3}\, \lambda_R\bar{\lambda}_R) + (\tfrac{2}{3}\, p_G\bar{p}_G - \tfrac{1}{3}\, n_G\bar{n}_G - \tfrac{1}{3}\, \lambda_G\bar{\lambda}_G)$$
$$+ (\tfrac{2}{3}\, p_B\bar{p}_B - \tfrac{1}{3}\, n_B\bar{n}_B - \tfrac{1}{3}\, \lambda_B\bar{\lambda}_B)$$
$$\Rightarrow (\tfrac{2}{3}\, p\bar{p} - \tfrac{1}{3}\, n\bar{n} - \tfrac{1}{3}\, \lambda\bar{\lambda})\,(R\bar{R} + B\bar{B} + G\bar{G}) \qquad (4.3)$$

and thus is a $(\gamma_8, \tilde{\omega}_1)$ member of $(8,\tilde{1})$ of $SU(3) \otimes \widetilde{SU(3)}$.

Notice that since it is a singlet of the colour SU(3) then the colour degrees of freedom are not excited. The ψ as a new degree of freedom requires here the introduction of new quark(s). For example,

	p	n	λ	c
R	$^2/_3$	$-^1/_3$	$-^1/_3$	$^2/_3$
B	$^2/_3$	$-^1/_3$	$-^1/_3$	$^2/_3$
G	$^2/_3$	$-^1/_3$	$-^1/_3$	$^2/_3$

(4.4)

This is the charm model with hidden colour and is discussed in detail by Ellis in these proceedings [1].

4.3. Han-Nambu [7]

We now examine models where the colour degrees of freedom can be excited. It may be possible that the ψ are associated with excitation of this degree of freedom — this phenomenological question will be discussed in Sections 4.3.1 to 4.4.4.

The quarks form a (3,3*) representation of $SU(3) \otimes \widetilde{SU(3)}$. Hence their charges will be

	p	n	λ
R	$z-1$	$z-2$	$z-2$
B	z	$z-1$	$z-1$
G	z	$z-1$	$z-1$

(4.5)

The constraint $z+z'+z'' = 2$ yields $3z-1 = 2$ and hence $z = 1$. The charges are therefore [7]

	p	n	λ
R	0	−1	−1
B	1	0	0
G	1	0	0

(4.6)

and these are displayed in Fig. 1.

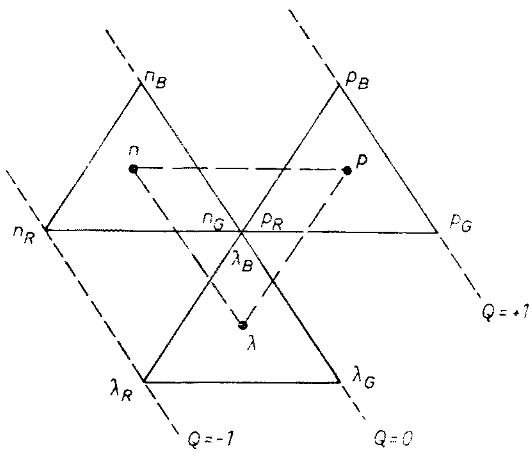


Fig. 1

Note the inverted triangle (triplet) of the $pn\lambda$ and the triangles (antitriplet) that show the spectrum analysis of each quark into RBG of the colour SU(3).

With these charges the photon is written

$$\gamma_{HN} \sim p_G\bar{p}_G + p_B\bar{p}_B - n_R\bar{n}_R - \lambda_R\bar{\lambda}_R$$
$$\Rightarrow p\bar{p}(G\bar{G}+B\bar{B}) - (n\bar{n}+\lambda\bar{\lambda})(R\bar{R}). \tag{4.7}$$

Trivial algebra enables us to rewrite this as

$$\gamma_{HN} = (\tfrac{2}{3}\,p\bar{p} - \tfrac{1}{3}\,n\bar{n} - \tfrac{1}{3}\,\lambda\bar{\lambda})(R\bar{R}+B\bar{B}+G\bar{G}) - (p\bar{p}+n\bar{n}+\lambda\bar{\lambda})(\tfrac{2}{3}\,R\bar{R} - \tfrac{1}{3}\,B\bar{B} - \tfrac{1}{3}\,G\bar{G}) \tag{4.8}$$

and hence

$$\gamma_{HN} = (8,\tilde{1}) - (1,\tilde{8}). \tag{4.9}$$

The $(8,\tilde{1})$ contains the familiar ϱ, ω, ϕ vector mesons which are colour singlets in this model. The $(1,\tilde{8})$ piece of the photon can excite vector mesons which are singlets of SU(3) and octets of $\widetilde{\mathrm{SU}(3)}$. If colour is conserved by the strong interactions, then these $\tilde{8}$ states will not decay by strong interaction to the $\tilde{1}$ hadrons and hence will be narrow. The ψ are assigned to be such $(1,\tilde{8})$ states in this model.

4.3.1. Phenomenology of the new states à la Han-Nambu colour

We have

$$\gamma_{HN} = [(\tfrac{2}{3}\,p\bar{p}-\tfrac{1}{3}\,n\bar{n}-\tfrac{1}{3}\,\lambda\bar{\lambda}),\ \sqrt{3}\,\tilde{\omega}_1]-[\sqrt{3}\,\omega_1,\ \sqrt{\tfrac{2}{3}}\,\tilde{\gamma}]. \tag{4.10}$$

In the [SU(3), $\tilde{1}$) we know that U-spin is not conserved by the strong interactions. The $U = 0$ photon therefore couples not to a single $U = 0$ vector meson but to the isospin eigenstates ϱ and ω_8. Furthermore, the ω_8 is a combination of the physical ω and ϕ and so

$$\gamma_{HN} = \left[\left(\frac{1}{\sqrt{2}}\,\varrho^0+\frac{1}{3\sqrt{2}}\,\omega-\tfrac{1}{3}\,\phi\right),\ \sqrt{3}\,\tilde{\omega}_1\right]-[\sqrt{3}\,\omega_1,\ \sqrt{\tfrac{2}{3}}\,\tilde{\gamma}]. \tag{4.11}$$

For the $\widetilde{SU(3)}$ we do not know a priori whether $\tilde{U}$ or $\tilde{I}$ is conserved, nor whether the $(\omega_1, \tilde{8})$ splits into $(\omega, \tilde{8})$ and $(\phi, \tilde{8})$. When assigning the new vector mesons to the $(1,\tilde{8})$ states there are therefore several possibilities as tabulated below [9].

SU$_3$ \ $\widetilde{SU}_3$	$\tilde{U}$ good	$\tilde{I}$ good	$\tilde{I}$ good Strong breaking
ω_1	I $(\omega_1, \tilde{\gamma})$	II $(\omega_1, \tilde{\varrho})$ $(\omega_1, \tilde{\omega}_8)$	
ω, ϕ	III $(\omega, \tilde{\gamma})$ $(\phi, \tilde{\gamma})$	IV $(\omega, \tilde{\varrho})$ $(\omega, \tilde{\omega}_8)$ $(\phi, \tilde{\varrho})$ $(\phi, \tilde{\omega}_8)$	$(\omega, \tilde{\varrho})$ $(\omega, \tilde{\omega})$ $[\omega, \tilde{\phi}]$ $(\phi, \tilde{\varrho})$ $(\phi, \tilde{\omega})$ $[\phi, \tilde{\phi}]$ $(\varrho, \tilde{\omega})$ $[\varrho, \tilde{\phi}]$

The different categories have been discussed by various authors.
Category II: Alles (1974) [10] and Bailin–Love mark 1 [10].
Category III: Alles (1975) [11] and Bailin–Love mark 2 [11], Bars–Peccei [12], T. C. Yang [13], Krammer et al. [14].
Category IV: Sanda–Terezawa [15], Kenny–Peaslee–Tassie [16], Feldman–Mathews [17], Stech–Marinescu [18].

The strong breaking of Han–Nambu colour has been discussed by Arik et al. [19] and by Marinescu and Stech [18]. The $[\varrho, \omega, \phi; \tilde{\phi}]$ are the familiar vector mesons, the new "coloured" states are $[\varrho, \omega, \phi; \tilde{\omega}]$. Note that these include a $[\varrho; \tilde{\omega}]$ not present in categories I to IV. This is because $[\varrho; \tilde{\omega}]$ as well as $[\varrho; \tilde{\phi}]$ contains a piece in $(8, \tilde{1})$. In categories I to IV $[\varrho, \tilde{\varrho}]$ and $[\varrho, \tilde{\omega}_8]$ states exist but are not excited by the photon which does not contain a piece transforming as $(8,\tilde{8})$. These states may be anticipated to exist around 3 GeV and could be found in the decay

$$\psi'_{3.7}(1,\tilde{8}) \to (\varrho, \tilde{8})+(\pi, \tilde{1}) \tag{4.12}$$

accompanying a single conventional π.

We shall now describe the phenomenology of these five categories and summarize in Table I.

TABLE I

Comparison of models I to IV

Models	I	II	III	IV (*i*)	IV (*ii*)
Assignments	$(\omega_1, \tilde{\gamma})$ 3.1 $(\omega_1, \tilde{\gamma})'$ 3.7 $(\omega_1, \tilde{\gamma})''$ 4.1	$(\omega_1, \tilde{\varrho})$ 3.1 $(\omega_1, \tilde{\omega}_8)$ 3.7 $(\omega_1, \tilde{\varrho})'$ 4.1	$(\omega, \tilde{\gamma})$ 3.1 $(\phi, \tilde{\gamma})$ 3.7 $(\omega, \tilde{\gamma})'$ 4.1	$(\omega, \tilde{\varrho})$ 3.1 $(\omega, \tilde{\omega}_8)$ 3.7 $(\phi, \tilde{\varrho})$ 4.1	$(\omega, \tilde{\varrho})$ 3.1 $(\phi, \tilde{\varrho})$ 3.7 $(\omega, \tilde{\omega}_8)$ 4.1
New states	$(\omega_1, \tilde{\gamma})'''$ 4.8	$(\omega_1, \tilde{\omega}_8)'$ 4.8	$(\phi, \tilde{\gamma})'$ 4.8	$(\phi, \tilde{\omega}_8)$ 4.8	$(\phi, \tilde{\omega}_8)$ 4.8
$\Gamma^{e^+e^-}$	Arbitrary	6 : 2, 6′ : 2′	6 : 3, 6′ : 3′	6 : 2 : 3 : (1)	6 : 3 : 2 : (1)
Strong interaction widths to uncoloured states	Narrow [H_{SB} $\tilde{8}$ small]	Narrow [H_{SB} $\tilde{8}$ small (?)]	Narrow [H_{SB} $\tilde{8}$ small]	Narrow [H_{SB} $\tilde{8}$ small (?)]	Narrow $\tilde{\varrho}$ Broad $\tilde{\omega}_8$ [H_{SB} like usual SU(3). $\tilde{I}$ good, $\tilde{Y}$ broken]
$\psi' \to \psi\pi\pi$	Like charm	Violates $\tilde{I}$ (H_{SB} has 27 plet piece?) ? Dead	H_{SB} 8 of ordinary SU(3) and Zweig rule cuts rate down	Violates $\tilde{I}$ Like model II ? Dead	Like model III
Radiative decays	$\psi(1,\tilde{8}) \to \gamma(1,\tilde{8})+$ hadrons $(1, \tilde{1})$ e.g. η_1, $(\pi\pi)_1$ not $\pi\gamma$, not $\eta_8\gamma$		$\psi(1+8, \tilde{8}) \to \gamma(1, \tilde{8})$ + hadrons $(1+8, \tilde{8})$ $\eta_1\gamma$ and $\eta_8\,\gamma$ not $\pi\gamma$		As model III
$\psi' \to \pi + (\varrho, \tilde{8})$ width	Arbitrary since ψ' is radial excitation		Significant $(\varrho, \tilde{8})$ around 3 GeV		Significant $(\varrho, \tilde{8})$ around 3 GeV

4.3.2. Leptonic widths

We recall that

$$\gamma_{HN} = \left[\left(\frac{1}{\sqrt{2}}\varrho^0 + \frac{1}{3\sqrt{2}}\omega - \tfrac{1}{3}\phi\right), \sqrt{3}\,\tilde{\omega}_1\right] - [\sqrt{3}\,\omega_1, \sqrt{\tfrac{2}{3}}\,\tilde{\gamma}]. \tag{4.13}$$

A calculation of $\Gamma(V \to e^+e^-)$ involves the coupling γ_V of vector meson to photon (Fig. 2). The width will be proportional to γ_V^2. For the colour singlet ϱ, ω, ϕ one will have

$$\langle \varrho, \tilde{\omega}_1 | \gamma \rangle = \sqrt{\tfrac{3}{2}}, \quad \gamma^2_{\varrho,\tilde{\omega}_1} = \tfrac{3}{2},$$

$$\langle \omega, \tilde{\omega}_1 | \gamma \rangle = \frac{\sqrt{3}}{3\sqrt{2}}, \quad \gamma^2_{\omega,\tilde{\omega}_1} = \tfrac{1}{6},$$

$$\langle \phi, \tilde{\omega}_1 | \gamma \rangle = \frac{-\sqrt{3}}{3}, \quad \gamma^2_{\phi,\tilde{\omega}_1} = \tfrac{1}{3} \tag{4.14}$$

and so, ignoring any mass factors, the leptonic widths will be in the ratio $\varrho : \omega : \phi = 9 : 1 : 2 \equiv 6.5 : 0.72 : 1.44$ to be compared with the data (in keV) 6.5, 0.75 and 1.3, respectively. This is very nice, if we had included one power of mass ($\Gamma \sim m\gamma_V^2$) the ϕ

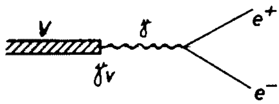


Fig. 2

prediction would have been some 20−30% in error. We shall therefore adopt the "rule" that [20] $\Gamma \sim \gamma_V^2$ in what follows:

Category I:

$$\langle \omega_1, \tilde{\gamma} | \gamma \rangle = -\sqrt{3}\sqrt{\tfrac{2}{3}}; \quad \gamma^2_{\omega_1, \tilde{\gamma}} = 2. \tag{4.14}$$

Hence the ratio $\varrho : \omega : \phi : (\omega_1, \tilde{\gamma})$ for the widths becomes

$$9 : 1 : 2 : 12. \tag{4.15}$$

Category II:

$$(\sqrt{3}\,\omega_1, \sqrt{\tfrac{2}{3}}\,\tilde{\gamma}) \to \left(\sqrt{3}\,\omega_1, \frac{1}{\sqrt{2}}\,\tilde{\varrho} + \frac{1}{\sqrt{6}}\,\tilde{\omega}_8\right) \tag{4.16}$$

and so the 12 of category I splits up 3 : 1 for $(\omega_1, \tilde{\varrho}) : (\omega_1, \tilde{\omega}_8)$. Hence

$$\varrho : \omega : \phi : (\omega_1, \tilde{\varrho}) : (\omega_1, \tilde{\omega}_8) = 9 : 1 : 2 : 9 : 3. \tag{4.17}$$

Category III:

$$(\sqrt{3}\,\omega_1, \sqrt{\tfrac{2}{3}}\,\tilde{\gamma}) \to (\sqrt{2}\,\omega + \phi, \sqrt{\tfrac{2}{3}}\,\tilde{\gamma}) \tag{4.18}$$

and so the 12 of category I splits into 2 : 1 for $(\omega, \tilde{\gamma}) : (\phi, \tilde{\gamma})$. Hence

$$\varrho : \omega : \phi : (\omega, \tilde{\gamma}) : (\phi, \tilde{\gamma}) = 9 : 1 : 2 : 8 : 4. \tag{4.19}$$

Category IV:

$$(\sqrt{3}\,\omega_1, \sqrt{\tfrac{2}{3}}\,\tilde{\gamma}) \to \left(\sqrt{2}\,\omega + \phi, \frac{1}{\sqrt{2}}\,\tilde{\varrho} + \frac{1}{\sqrt{6}}\,\tilde{\omega}_8\right). \tag{4.20}$$

The 2 : 1 splitting of ω_1 into $\omega : \phi$ and 3 : 1 splitting of $\tilde{\gamma}$ into $\tilde{\varrho} : \tilde{\omega}_8$ yields

$$\varrho : \omega : \phi : (\omega, \tilde{\varrho}) : (\phi, \tilde{\varrho}) : (\omega, \tilde{\omega}_8) : (\phi, \tilde{\omega}_8) = 9 : 1 : 2 : 6 : 3 : 2 : 1. \tag{4.21}$$

Strong breaking

Here one can write

$$\gamma_{HN} = \left[\frac{1}{\sqrt{2}}\,\varrho + \frac{1}{3\sqrt{2}}\,\omega - \tfrac{1}{3}\,\phi, \sqrt{2}\,\tilde{\omega} + \tilde{\phi}\right] - \left[\sqrt{2}\,\omega + \phi, \frac{1}{\sqrt{2}}\,\tilde{\varrho} + \frac{1}{3\sqrt{2}}\,\tilde{\omega} - \tfrac{1}{3}\,\tilde{\phi}\right]. \tag{4.22}$$

The phenomenology of $\tilde{\varrho}$ states is as before

$$\Gamma^{e^+e^-}(\omega, \tilde{\varrho}) : (\phi, \tilde{\varrho}) = 2 : 1. \tag{4.23}$$

For the $\tilde{\omega}$ and $\tilde{\phi}$ states one obtains

$$\gamma_{\mathrm{HN}} = \left[\frac{1}{\sqrt{2}}\varrho + \frac{1}{\sqrt{2}}\omega + 0\phi, \tilde{\phi}\right] + \left[1\varrho + 0\omega - \frac{1}{\sqrt{2}}\phi, \tilde{\omega}\right] \tag{4.24}$$

so that for $\tilde{\phi}$, $\varrho : \omega : \phi = 1 : 1 : 0$, while for $\tilde{\omega}$ one has $\varrho : \omega : \phi = 1 : 0 : \frac{1}{2}$. One is completely at sea with regard to the familiar $\varrho : \omega : \phi$ widths of $9 : 1 : 2$ and the model [19] will not be discussed further. If one wishes to make a strong breaking of $\tilde{\omega}_8 \to \tilde{\omega}, \tilde{\phi}$ one must do so in models either

(*i*) where the $\tilde{\phi}$ quarks carry charges $^2/_3$, $-^1/_3$, $-^1/_3$ so that the $\varrho : \omega : \phi$ phenomenology is satisfactory (such a model [2, 21] will be discussed in Section 6) or

(*ii*) the $\tilde{\omega}_{8,1}$ mixing is not of the "ideal" kind $\tilde{\omega}, \tilde{\phi}$ as, for example, Marinescu and Stech [18] (discussed in Section 4.4.4).

4.3.3. Assignments of resonances

1. The $(\omega_1, \tilde{\gamma})$ is the $\psi(3.1)$. The $\psi(3.7)$ will be the first radial recurrence $(\omega_1, \tilde{\gamma})'$ analogous to the charm scheme. The leptonic width of 3.7 relative to 3.1 is arbitrary until further assumptions as to the dynamics are made. However, the $9 : 1 : 2 : 12$ for the ratio of leptonic widths of ψ to ϱ, ω, and ϕ predicts a width

$$\Gamma_{\psi}^{e^+e^-} \simeq 8.7 \text{ keV} \tag{4.25}$$

which is in gross disagreement with data [22] ($\Gamma_{\psi}^{e^+e^-} \simeq 4.8 \pm 0.6$ keV). This leads us to suspect that the $(\omega_1, \tilde{\gamma})$ must fragment either into ω and ϕ states or into $\tilde{\varrho}$, $\tilde{\omega}_8$ or both, as in categories II to IV.

2. The leptonic width ratio of $3 : 1$ for $(\omega_1, \tilde{\varrho}) : (\omega_1, \tilde{\omega}_8)$ and the data

$$\Gamma_{\psi(3.1)}^{e^+e^-} : \Gamma_{\psi(3.7)}^{e^+e^-} = (4.8 \pm 0.6) : (2.2 \pm 0.5) \tag{4.26}$$

suggest that

$$(\omega_1, \tilde{\varrho}) = \psi(3.1),$$
$$(\omega_1, \tilde{\omega}_8) = \psi(3.7) \tag{4.27}$$

and, since $\Gamma_{\varrho\tilde{\omega}_1}^{e^+e^-} : \Gamma_{\omega_1\tilde{\varrho}}^{e^+e^-} = 9 : 9$, one would expect that $\Gamma_{\psi(3.1)}^{e^+e^-} \simeq 6.5$ keV. This is two standard deviations away from the quoted width.

If the 4.2 GeV state is a resonance, then it will be assigned to $(\omega_1, \tilde{\varrho})'$. One necessarily expects that $(\omega_1, \tilde{\omega}_8)'$ also will exist, and on the basis of the observed masses 3.1, 3.7 and 4.2, one expects it to lie around 4.8 GeV in mass. Naive integration over the 4.2 GeV peak [23] suggests that its leptonic width is a few (3 to 5) keV. Hence one predicts a 1 to 2 keV width for the 4.8 GeV state $(\omega_1, \tilde{\omega}_8)'$.

The problem with these assignments is the observation of the cascade $\psi(3.7) \to \psi(3.1)\pi\pi$ with a strong coupling [24]. This decay violates $\tilde{I}$, since a change $\tilde{\varrho}$ to $\tilde{\omega}_8$ must take place.

If the $\tilde{I}$ violation is allowed then why is the $\psi(3.1)$ so narrow? One way out [9] is to allow the colour symmetry breaking to transform as $\underset{\sim}{27}$ under $\overparen{SU}(3)$. This allows $\tilde{\omega}_8$ to $\tilde{\varrho}$, but forbids $\tilde{\varrho}$ to $\tilde{\omega}_1$. Such a solution appears rather ugly since it is quite unlike anything in the familiar SU(3).

3. Since the leptonic widths $(\omega, \tilde{\gamma}) : (\phi, \tilde{\gamma}) = 2 : 1$, then one assigns

$$\psi(3.1) = (\omega, \tilde{\gamma}) \rightarrow \Gamma^{e^+e^-}_{(3.1)} \sim 5.7 \text{ keV}, \tag{4.28}$$

$$\psi(3.7) = (\phi, \tilde{\gamma}) \rightarrow \Gamma^{e^+e^-}_{(3.7)} \sim 2.8 \text{ keV}, \tag{4.29}$$

the leptonic widths following from the 9 : 1 : 2 : 8 : 4 ratios discussed previously for this category. These numbers are quite satisfactory in comparison with the data (4.8 ± 0.6 and 2.2 ± 0.5 keV). Similar to category II the radial recurrence remarks apply to the 4.2 GeV $(\omega, \tilde{\gamma})'$ and predicted 4.8 GeV $(\phi, \tilde{\gamma})'$ states.

The cascade $\psi(3.7) \rightarrow \psi(3.1)\pi\pi$ is allowed due to a piece in the symmetry-breaking Hamiltonian transforming as *8* under familiar SU(3) (not $\overparen{SU}(3)$ as in the category II).

4. There are two schools of thought on the assignment here. One school (Sanda–Terezawa [15]) notes that the leptonic ratios

$$(\varrho, \tilde{\omega}_1) : (\omega, \tilde{\varrho}) : (\omega, \tilde{\omega}_8) : (\phi, \tilde{\varrho}) : (\phi, \tilde{\omega}_8) = 9 : 6 : 2 : 3 : 1 \tag{4.30}$$

are in nice agreement with

$$(\varrho, \tilde{\omega}_1) : \psi(3.1) : \psi(3.7) : \psi(4.1) : [\psi(4.8)] = 6.5 : 4.8\pm0.6 : 2.2\pm0.5 : 3 \text{ to } 5 : [1] \tag{4.31}$$

and hence their assignments. This model has the same problem with the $\psi(3.7)$ cascade as that of category II. Feldman and Mathews assign [17]

$$\begin{array}{lll} \psi(3.1), & (\omega, \tilde{\varrho}), & \Gamma^{e^+e^-} \sim 4.4 \text{ keV}, \\ \psi(3.7), & (\phi, \tilde{\varrho}), & \Gamma^{e^+e^-} \sim 2.2 \text{ keV}, \\ \psi(4.1), & (\omega, \tilde{\omega}_8), & \Gamma^{e^+e^-} \sim 1.5 \text{ keV}, \\ \psi(4.8), & (\phi, \tilde{\omega}_8), & \Gamma^{e^+e^-} \lesssim 1 \text{ keV}. \end{array}$$

The leptonic widths for $\psi(3.1)$ and $\psi(3.7)$ are excellent. For the $\psi(4.1)$ we must wait and see. These author's works are typical of the recent ideas [17–19, 21] that one should break $\overparen{SU}(3)$ but conserve $\overparen{SU}(2)$, hence the narrow $\psi(3.1)$ and $\psi(3.7)$ yet broad $\psi(4.1)$ and $\psi(4.8)$.

Note that all models agree that there is a state to be found around 4.8 GeV.

4.3.4. Radiative decays

Colour conservation by strong interactions prevented the decays $\psi(\tilde{8}) \rightarrow$ hadrons $(\tilde{1})$ and hence gave a natural explanation of the narrow widths of these states. Nothing prevents the radiative decays $\psi(\tilde{8}) \rightarrow$ hadrons $(\tilde{1})+\gamma(\tilde{8})$ and naively one has expected [25] that these will be the dominant decay modes of the ψ state. This in turn has led to much criticism of colour models based upon the apparent success of G parity in the ψ decays and the calculations that these radiative widths should be of order 15 MeV or so, in contrast to the observed *total* width of some 50 keV. We shall discuss these questions in detail in the next section. Here we discuss the quantum numbers that are allowed for the colour singlet hadrons produced in the radiative decays.

If the ψ is ω_1 of SU(3) then $\psi(1,\tilde{8}) \to \gamma(1,\tilde{8})$+hadrons $(1,\tilde{1})$. Consequently $\eta\gamma$ and $\eta'\gamma$ are allowed by their η_1 piece, but $\pi\gamma$ is forbidden.

If the ψ is ω of SU(3) (as in categories III and IV) then $\psi(1+8,\tilde{8}) \to \gamma(1,\tilde{8})$ + hadrons $(1+8,\tilde{8})$. The decays to $\eta\gamma$ and $\eta'\gamma$ are allowed by both η_1 and η_8 pieces. However $\pi\gamma$ is still forbidden by isospin. The $\pi\gamma$ mode will be, in general, allowed at some, suppressed, rate by symmetry-breaking $(\psi(1,\tilde{8}) \to V(1,\tilde{1}) \to \pi(8,\tilde{1})+\gamma(8,\tilde{1}))$ or by $\psi \to \gamma \to \pi^0\gamma$.

4.4. The four problems of Han–Nambu colour phenomenology

4.4.1. Radiative decays of ψ

4.4.1.1. Naive estimate

The width for a vector meson (V) to decay into a pseudoscalar (P) and photon is written

$$\Gamma(V \to P\gamma) = \alpha g^2 \frac{|\vec{p}_{cm}|^3}{3}, \tag{4.32}$$

where g is the intrinsic coupling constant and the momentum of the emitted photon in the rest frame of V is

$$|\vec{p}_{cm}| = \frac{m_V^2 - m_P^2}{2m_V} \tag{4.33}$$

with $m_{V,P}$ the respective meson masses.

A naive estimate of the $\psi \to \eta\gamma$ width can be made by taking the known width for $\omega \to \pi\gamma$ (0.9 MeV), for which $|\vec{p}_{cm}| \simeq 380$ MeV and scaling up by $|\vec{p}_{cm}|^3$ to compute $\Gamma(\psi \to \eta\gamma)$ (for which $|\vec{p}_{cm}| \simeq 1500$ MeV). This yields around 60 MeV; however, the Clebsch–Gordan coefficient g^2 for $\psi \to \eta\gamma$ is only $^2/_7$ of that for $\omega \to \pi\gamma$, so an estimate of 15 to 20 MeV is obtained. This is clearly inconsistent with the data on the ψ width and has been regarded as a serious defect of the Han–Nambu colour interpretation of ψ.

Several arguments have been made in the literature, pointing out that the above calculation is too naive. I outline here the argument which I personally find the most convincing.

4.4.1.2. Quark model estimate [26]

The process $V \to P\gamma$ is a magnetic dipole transition and in a quark model the rate is proportional to the square of the quark's magnetic moment. The quark magnetic moment operator may be written

$$\mu \frac{eQ_i}{2m} \vec{\sigma}, \tag{4.34}$$

where $\vec{\sigma}$ is a Pauli spin matrix, $\mu \sim 2.7$, and $m = m_p = 940$ MeV. This yields $m_0 \equiv m/\mu \sim 330$ MeV, which is the parameter which determines the over-all rate and which may be thought of as the "effective mass" of a quark which has the "normal" moment (for colourless ground-state mesons and baryons).

Traditionally one can imagine the quark and antiquark which form the meson to be in some SU(6) state χ (or more generally SU(6) $\otimes$ $\widetilde{\text{SU}}(3)$ state) and to be bound by some potential. In this potential they are in some eigenstate described by the wave function $\phi(\vec{r})$, where $\vec{r}$ is the spatial separation of the two quarks. The matrix element for V → Pγ may be formally written

$$M_{\mathrm{VP}}(|\vec{p}|) = \frac{e}{2m_0} [\int \phi_{\mathrm{P}}^*(\vec{r}) e^{i\vec{p}\cdot\vec{r}} \phi_{\mathrm{V}}(\vec{r}) d^3\vec{r}] [\langle \chi_{\mathrm{P}} | Q_i \vec{\sigma} | \chi_{\mathrm{V}} \rangle]. \tag{4.35}$$

The naive estimate of the previous section only considered $\langle(\chi_{\mathrm{P}}|Q_i\vec{\sigma}|\chi_{\mathrm{V}})\rangle$, all the remaining contributions to $M_{\mathrm{VP}}(|\vec{p}|)$ being implicitly assumed to be the same for $\omega \to \pi\gamma$ and $\psi \to \eta\gamma$. Specifically it was assumed that:

(*i*) $m_0 \equiv m/\mu$ is the same throughout the multiplet,

(*ii*) $\phi(\vec{r})$ is the same throughout the multiplet,

(*iii*) M_{VP} is $|\vec{p}|$ independent.

Let us discuss the validity of these three assumptions.

(*i*) m_0 *the same throughout the multiplet.* We know that there is mass splitting in the SU(3) world and that there is some suggestion that the m_0, which scales the magnetic moment of the quarks, reflects this. For example, the ratio of the Λ to nucleon magnetic moment is improved over the SU(3) prediction if $m_\lambda \simeq 1.5\, m_{\mathrm{p,n}}$ for the colour singlet quarks [27]. The effective quark mass in the ψ is of order 1.5 GeV which is some 4 to 5 times the uncoloured quark with 330 MeV. If one takes

$$m_0^{\text{coloured}} \sim 4m_{\mathrm{p,n}}^{\text{uncoloured}}$$

then this alone will suppress the naive estimate of the $\psi \to \eta\gamma$ width by a factor of 16.

(*ii*) $\phi(\vec{r})$ *is the same throughout the multiplet.* For a simple harmonic oscillator potential $\phi(\vec{r}) \sim \exp(-r^2/a^2)$ for the ground state, where a^2 is a measure of the size of the quark-antiquark system (meson). Intuitively one may feel that the heavier masses of the ψ relative to colour singlet ω suggest that the ψ be "smaller" than ω and hence $a^2_{\text{coloured}} < a^2_{\text{uncoloured}}$.

(*iii*) $M_{\mathrm{VP}}(|\vec{p}|)$ *is* $|\vec{p}|$ *independent.* In any dynamical model where the hadron has finite extent it may be anticipated that $M(|\vec{p}|)$ will be $|\vec{p}|$ *dependent.* For example, for a harmonic oscillator potential where $\phi(\vec{r}) \sim \exp(-\vec{r}^2/a^2)$, then $M_{\mathrm{VP}}(\vec{p}) \sim \exp(-\vec{p}^2/b^2)$ and hence is very strongly damped as $|\vec{p}|$ increases.

Phenomenological support for such a dramatic damping with $|\vec{p}|$ comes from the comparison of quark model and resonance decays data of the familiar hadrons — see papers by Feynman et al., Faiman and Hendry, and Burkhardt and Hey [28] in this respect.

More generally in any composite picture of hadrons, one expects that the electromagnetic form factors $|F(q^2)|$ as a function of the photon mass squared (q^2) will tend to zero as $q^2 \to \infty$. Empirically $F(q^2) \sim (1+q^2/A^2)^{-n}$ with $n \sim 1$ or 2 for the pion and nucleon. In a non-relativistic composite model $F(q^2) \to F(\vec{q}^2)$ and the same damping factor emerges when the three momentum of the photon varies even though $q^2 = 0$ is fixed.

It is not a priori clear what relevance a non-relativistic calculation has to the highly relativistic decay $\psi \to \eta\gamma$. However, the physical, and phenomenological, arguments suggest that it may not be unreasonable at zeroth order to take the $F^{em}(q^2)$ as a guide and parametrize the photo-emission "form factor" as [26]

$$V \to P\gamma \quad \left[1+\left(\frac{p}{2\alpha}\right)^2\right]^{-2} \tag{4.36}$$

$$V_c \to P_c\gamma \quad \left[1+\left(\frac{p}{2\beta}\right)^2\right]^{-2} \tag{4.37}$$

$$V_c \to P\gamma \sim \left[1+\left(\frac{p}{\alpha+\beta}\right)^2\right]^{-2} \tag{4.38}$$

where $V(V_c)$ stand for uncoloured (coloured) states, respectively, and $(\alpha, \beta)^{-1}$ are the "sizes" of colourless (coloured) mesons. The effects on the width calculations are shown in Table II (after Ref. [26]).

TABLE II

Vector meson radiative widths

	(Clebsch)²	$\vec{p}$	Γ_{naive}	$\alpha/\beta = 2$	$\alpha/\beta = 3$	$e^{-6\lvert p\rvert}$
$\omega \to \pi\gamma$	$^1/_4$	380 MeV	1.1 MeV	0.3 MeV	0.3 MeV	0.6 MeV
$\omega \to \eta\gamma$	$^1/_{108}$	200 MeV	6 keV			
$\psi \to \eta\gamma$	$^2/_{27}$	360 MeV	0.28 MeV			
$\tilde{\omega} \to \eta\gamma$	$^2/_{27}$	1500 MeV	20 MeV	1.4 keV	0.4 keV	0.15 keV
$\tilde{\phi} \to \eta\gamma$	$^4/_{27}$	1810 MeV	70 MeV	1.4 keV	0.4 keV	0.08 keV

In the table the naive width calculations are seen and the effect of including the $F^{em}(q^2)$ in Eqs (4.36)–(4.38) for two choices of α/β is also exhibited. A phenomenologically motivated $e^{-6|p|}$ is also shown for comparison. The dramatic consequence is that not only are the widths reduced from the naive estimates and hence compatible with the total width of some 50 keV, but even suggest that the radiative widths could be significantly less than 50% of the total width!

If this is really true, then the naive beliefs that in colour models radiative decays dominate and hence G-parity will be irrelevant could be quite wrong. The dominant decays are non-radiative and so the colour must be broken. This brings us to the G-parity question.

4.4.2. G-parity and hadronic decays

If SU(3) is conserved by strong interactions, then $\psi(\tilde{8}) \to$ hadrons $(\tilde{1})$ can only proceed by $\psi \to \gamma \to$ hadrons, for which G-parity even and odd final states will be a priori equiprobable. The existence of direct decays $\psi(\tilde{8}) \to$ hadrons $(\tilde{1})$ can only be understood in this colour model by allowing there to exist in the Hamiltonian a symmetry-breaking piece H_{SB}, transforming as $\tilde{8}$ for example. The quantum numbers of the produced hadrons will be determined by the specific transformation property of H_{SB} folded in with the ψ representation in $SU(3) \otimes \widetilde{SU(3)}$.

A priori we do not know what are the transformation properties of H_{SB}; one suggestion, discussed in some detail elsewhere [29], has been to take the known behaviour of H_{SB} in familiar SU(3) and make the simplest generalization to $SU(3) \otimes \widetilde{SU(3)}$. For example, it is known that familiar SU(3) breaking can be described by a term in the effective Hamiltonian which transforms like the hypercharge operator Y. The generalization to $SU(3) \otimes \widetilde{SU(3)}$ is taken to be [29]

$$H_{SB} \sim (Y, \tilde{1}) + (1, \tilde{Y}). \tag{4.39}$$

At this stage I and $\tilde{I}$ are separately conserved and the ψ would be stable against decay to $\tilde{I} = 0$ hadrons[2] by conservation of $\tilde{I}$ in the strong interaction.

In familiar SU(3), electromagnetic tadpole interactions generate pieces transforming as I_3 which break the symmetry but are of much weaker strength than the Y piece discussed above. The generalization to $SU(3) \otimes \widetilde{SU(3)}$ becomes

$$H_{SB} \sim (I_3, \tilde{1}) + (1, \tilde{I}_3). \tag{4.40}$$

The $(I_3, \tilde{1})$ piece is well-known and is responsible for

a) $$(\varrho, \tilde{1}) \leftrightarrow (\omega, \tilde{1}), \tag{4.41}$$

b) $$(\eta, \tilde{1}) \to (3\pi, \tilde{1}), \tag{4.42}$$

c) $$(\Delta I = 1, \Delta \tilde{I} = 0) \text{ part of mass differences,} \tag{4.43}$$

and its strength phenomenologically appears to be of order 5–10 MeV [30].

The $(1, \tilde{I}_3)$ piece will cause the analogous transitions

a) $$(1, \tilde{\varrho}) \to (1, \tilde{\omega}) \tag{4.44}$$

and be responsible for

b) $$(\Delta I = 0, \Delta \tilde{I} = 1) \text{ part of mass differences.} \tag{4.45}$$

The transition $(1\tilde{\varrho}) \to (1\tilde{\omega})$ contains $\psi(1\tilde{\varrho}) \to$ hadrons $(1\tilde{\omega}_1)$, hence the colour isospin changes by one unit while familiar isospin is conserved, i. e. *G-parity is conserved.* Figuratively

$$(I_3, \tilde{1}) \begin{cases} \text{violates} & G \text{ parity,} \\ \text{conserves} & \tilde{G} \text{ parity} \end{cases} \tag{4.46}$$

$$(1, \tilde{I}_3) \begin{cases} \text{conserves} & G \text{ parity} \\ \text{violates} & \tilde{G} \text{ parity.} \end{cases} \tag{4.47}$$

[2] Other than $\psi \to \gamma \to$ hadrons of course.

This same tadpole yields the $\Delta I = 0$, $\Delta \tilde{I} = 1$ mass splitting which is of order 2.3 GeV ($m_\psi - m_{\omega\tilde{1}}$). Stech [29] writes down the following relation between this Δm, the strength of the tadpole, and the width of ψ:

$$\Gamma_\psi = \frac{(\text{strength})^2}{\Delta m}. \tag{4.48}$$

Since $\Delta m = 2.3$ GeV, if one assumes the strength to be some 8 MeV (as is the case for the ($I_3\ \tilde{1}$) tadpole), then one obtains

$$\Gamma_\psi = \frac{(8\ \text{MeV})^2}{2.3\ \text{GeV}} \simeq 30\ \text{keV} \tag{4.49}$$

which is excellent phenomenologically.

One might be concerned that this Γ_ψ of 30 keV came from a tadpole which is $O(e^2)$ in the amplitude, while the $O(e)$ radiative decays are expected to be only of order 10 to 15 keV. In this respect, we should remind ourselves of the η which in the colour singlet world is a meson that is stable against hadronic decays and whose decays are $O(e)$ radiative or by the (I_3, $\tilde{1}$) tadpole at $O(e^2)$. In this case we know that

$$O(e)[\Gamma(\eta \to \pi\pi\gamma)] \ll O(e^2)[\Gamma(\eta \to \gamma\gamma;\ \pi\pi\pi)]. \tag{4.50}$$

If the same ratio of $O(e)$ to $O(e^2)$ was found for the ψ decays, then one would have[3]

$$\Gamma_{\psi \to \text{hadrons}+\gamma} \simeq 2\ \text{to}\ 10\ \text{keV}. \tag{4.51}$$

From the purely phenomenological point of view it would be amusing to see if such a result held for the ψ.

4.4.3. Deep inelastic threshold phenomena

4.4.3.1. Threshold phenomena at large Q^2

If we work within the framework of the quark-parton model, then the electromagnetic structure functions take the following form (given the quark charges of the Han-Nambu model).

Below colour threshold

$$F_2^{\text{P}}(x) \sim 3[\tfrac{4}{9}\,\text{p}(x) + \tfrac{1}{9}\,\text{n}(x) + \tfrac{1}{9}\,\lambda(x) + (\bar{\text{p}}, \bar{\text{n}}, \bar{\lambda})]$$

$$F_2^{\text{N}}(x) \sim 3[\tfrac{1}{9}\,\text{p}(x) + \tfrac{4}{9}\,\text{n}(x) + \tfrac{1}{9}\,\lambda(x) + (\bar{\text{p}}, \bar{\text{n}}, \bar{\lambda})]. \tag{4.52}$$

---

[3] The absence of $\gamma\gamma$ decays for ψ generates uncertainty into this guesstimate.

Above colour threshold

$$F_2^{\mathrm{P}}(x) \sim \mathrm{p_B}(x)+\mathrm{p_G}(x)+n_{\mathrm{R}}(x)+\lambda_{\mathrm{R}}(x)+(q \leftrightarrow \bar{q})$$

$$F_2^{N}(x) \sim \mathrm{n_B}(x)+n_{\mathrm{G}}(x)+\mathrm{p_R}(x)+\lambda_{\mathrm{R}}(x)+(q \leftrightarrow \bar{q}). \tag{4.53}$$

Comparing the expressions below and above colour threshold one can compute the magnitude of the threshold rise in $F_2^{\mathrm{N,P}}(x)$. This is summarized in Eq. (4.54) for proton (P), neutron (N) and $I = 0$ targets.

	ΔF_2^{P} (%)	ΔF_2^{N} (%)	$\Delta F_2^{I=0}$ (%)
$x \to 1$ $\mathrm{p}(x)$ dominates	50	200	80
$x \to \frac{1}{3}$ $\mathrm{p}(x) \simeq 2\mathrm{n}(x)\begin{pmatrix}\lambda(x) = 0\\ \bar{q}(x) = 0\end{pmatrix}$	66	100	80
$x \to 0$ $\mathrm{p}(x) = \mathrm{n}(x) = \lambda(x) = \bar{\mathrm{p}}(x) = \bar{\mathrm{n}}(x) = \bar{\lambda}(x)$	100	100	100

(4.54)

The criticism [1] levelled at the Han-Nambu model is that there is no evidence in the FNAL data [31] for such dramatic behaviour in the deep inelastic experiments.

4.4.3.2. Photoproduction threshold

Using the vector meson dominance model and the data on the new vector mesons one estimates that the rise at colour production threshold will be only a few per cent. This will be discussed in more detail in Section 4.4.3.4.

4.4.3.3. Parton model and VMD

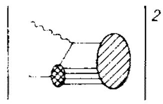
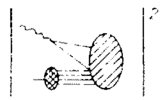
The VMD model predicts a threshold rise in σ_{tot} (γp) for $Q^2 = 0$ of a few per cent, whereas the parton model gave a rise of order 100% in $\sigma_{\mathrm{tot}}(\gamma\mathrm{p})$ for large Q^2.

Since the VDM and parton models are dynamically quite unrelated, there may be no unease in one's mind at their significantly differing predictions for the magnitudes of the threshold rise in $\sigma(\gamma\mathrm{p})$. However, when one is only discussing quantum number effects (as in the present case) the two models are almost in 1 : 1 correspondence. This we shall illustrate by means of a few examples which will also give some indication of the implicit assumptions that entered into the parton model results of Section 3.1. The comparison is most readily facilitated by reference to Table III.

Therefore the vector meson dominance approach appears to give the same results for the threshold rises as the quark-parton model. Why then was it stated in Section 3.2 that the VDM predicted a threshold rise of only a few per cent? This will now be discussed.

TABLE III

Comparison of VDM and parton models

	Parton	Vector meson	Result (either way)
Diagrammatic representation of γN collision total cross-section			
Quark quantum numbers and vector meson analogues	$\lambda(x)$, $n(x)$, $p(x)$	$\lambda\bar{\lambda} \sim \phi$, $n\bar{n}$, $p\bar{p} \sim \varrho, \omega$	
Ratio of non-strange to strange components in $\sigma(\gamma N)$ for $I = 0$ target	$\frac{4}{9}p+\frac{1}{9}n+\frac{1}{9}\lambda+(q\to\bar{q})$ $\Rightarrow \frac{5}{9}(pn)+\frac{1}{9}\lambda$ $+(q\leftrightarrow\bar{q})$	$9\varrho+1\omega+2\phi$ $\Rightarrow 10(\varrho\omega)+2\phi$	5 : 1 Non-strange: strange
Colour threshold (*i*) Diffraction $x \to 0$	P N Below $2+2 = 4$ Above $4+4 = 8$	Below $9+1+2 = 12$ Colour component $8+4 = \underline{12}$ Total above 24	100%
(*ii*) $x \to {}^1/_3$ Non-diffractive	No $\lambda(x)$ P N Below $3+2 = 5$ Above $5+4 = 9$	No ϕ meson Below $9+1 = 10$ Colour component $8 = \underline{8}$ Total above 18	80%
(*iii*) $x \to 1$	p quark dominance P N Below ${}^4/_3+{}^1/_3 = {}^5/_3$ Above $2+1 = 3$	As above	80%

4.4.3.4. VMD in a world where $m_\varrho = m_\omega = m_\psi$ and the real world

We shall see that the quark parton model results correspond to the VMD results in a world where all dynamical effects (other than those which are a function of the quark charges) can be ignored — in particular $m_\varrho = m_\omega = m_\psi$, etc.

$Q^2 = 0$ and VMD

In the vector dominance model, the total photo-absorption cross-section for on-shell photons is given by

$$\sigma(\gamma p) = \sum_V \frac{4\pi\alpha}{f_V^2(0)}\, \sigma(Vp) \tag{4.55}$$

where $em^2/f(0)$ is the photon-vector meson V coupling for on-shell photons.

The leptonic width of a vector meson is given by

$$\Gamma(\mathrm{V} \to e^+e^-) = \frac{\alpha^2}{3}\, m_\mathrm{V} \frac{4\pi}{f_\mathrm{V}^2(m_\mathrm{V}^2)} \tag{4.56}$$

we can therefore write

$$\sigma(\gamma\mathrm{p}) = \sum_\mathrm{V} \frac{3f^2(m_\mathrm{V}^2)}{\alpha f^2(0)}\, \sigma(\mathrm{Vp})\, \frac{\Gamma(\mathrm{V} \to e^+e^-)}{m_\mathrm{V}}\,. \tag{4.57}$$

The VDM assumption is that $f^2(m_\mathrm{V}^2) = f^2(0)$ and so

$$\sigma(\gamma\mathrm{p}) \sim \sum_\mathrm{V} \sigma(\mathrm{Vp})\, \frac{\Gamma(\mathrm{V} \to e^+e^-)}{m_\mathrm{V}}\,. \tag{4.58}$$

(*i*) *Equal mass world.* We are interested in the relative sizes of the contributions to $\sigma(\gamma\mathrm{N})$ coming from $\varrho\omega\phi$ and the coloured vector mesons. We will present the discussion in terms of the $(\omega, \tilde{\gamma})$ and $(\phi, \tilde{\gamma})$ model (category II). The results are essentially unchanged if we use any one of the other models; the use of this particular one enables us most easily to take these calculations over to the charm model, since in both models the 3.1 coupling to the photon happens to be the same [$^8/_9$ of the $\varrho(770)$].

We may formally write

$$\sigma(\gamma) \sim 9\sigma(\varrho)+1\sigma(\omega)+2\sigma(\phi)+[8\sigma(\tilde{\omega})+4\sigma(\tilde{\phi})] \tag{4.59}$$

in an obvious notation. In this equal mass example there is no ambiguity as to whether the $9:1:2$, etc., apply to Γ, Γ/m, etc. If $\sigma(\mathrm{V}) \sim m_\mathrm{V}^{-2}$, then for equal masses $\sigma(\varrho) = \sigma(\omega) = \ldots = \sigma(\tilde{\omega}) = \sigma(\tilde{\phi})$. Hence

$$\sigma(\gamma) \sim 12\sigma\,(\text{uncoloured}) + [12\,\sigma\,(\text{coloured})] \tag{4.60}$$

and the 100% rise of $\sigma(\gamma)$ in the diffractive region is again found. Notice that for the non-diffractive piece, removal of $\sigma(\phi)$ and $\sigma(\tilde{\phi})$ (which are pure diffractive) yields

$$\sigma^{\mathrm{ND}}(\gamma) \sim 10\sigma(\text{uncoloured}) + [8\sigma(\text{coloured})] \tag{4.61}$$

and the 80% rise in the non-diffractive part is seen.

(*ii*) *Real world.* The results of the equal mass model would carry over to the real world immediately if:

— $9:1:2:8:4$ applied to the $\Gamma(\mathrm{V} \to e^+e^-)/m_\mathrm{V}$,

— $\sigma(\mathrm{VN})$ is the same for all V.

Empirically, however, we know that:

— $9:1:2:8:4$ apply to $\Gamma(\mathrm{V} \to e^+e^-)$ *not* Γ/m_V, and also that

— $\sigma(\mathrm{VN}) \sim m_\mathrm{V}^{-2}$,

since $\sigma(3.1) \simeq (1/10)\,\sigma(\phi) \simeq (1/20)\,\sigma(\omega, \varrho) \simeq 1$ mb. Hence, in the real world there is an over-all factor m_V^{-3} relative to the equal mass calculation (m_V^{-1} from leptonic widths and

m_V^{-2} from $\sigma(VN)$). Therefore we have now

$$\sigma(\gamma) \sim \frac{9+1}{(0.78)^3} + \frac{2}{(1)^3} + \left[\frac{8}{(3.1)^3} + \frac{4}{(3.7)^3}\right] \tag{4.62}$$

and the colour threshold rise is less than 2% as against 100% previously.

This is the origin of the remark in Section 4.4.3.2 that VMD predicts a rise of only a few per cent at $Q^2 = 0$; the naive 100% rise has been suppressed by the fact that

$$\sigma(\psi p) \ll \sigma(\varrho p) \tag{4.63}$$

and that

$$\frac{\Gamma(\psi \to e^+e^-)}{m_\psi} \ll \frac{\Gamma(\varrho \to e^+e^-)}{m_\varrho}. \tag{4.64}$$

$Q^2 \neq 0$ and generalized vector meson dominance (GVMD)

Instead of $\varrho\omega\phi$ ... used in VMD the idea in GVMD [32] is to use $\varrho\varrho'\varrho''$ ..., $\omega\omega'$ ..., etc., a whole sequence of vector mesons as suggested for example by dual models. With some assumptions on the mass spectrum of these mesons and their couplings to the photon one can obtain $\nu W_2(\nu, q^2)$ which scales at large q^2. For the details I refer you to the literature [32], here I just quote the answer. Essentially one has

$$\sigma(\gamma N)_{Q^2} \sim \sum_{V=\varrho\omega\phi} \sigma(\gamma N)^V_{Q^2=0} \frac{m_V^2}{m_V^2+Q^2}. \tag{4.65}$$

The tower of mesons yields the scaling behaviour ($1/Q^2$ as $Q^2 \to \infty$), the mass parameter appearing $[m^2/(m^2+Q^2)]$ is the mass of the lowest meson in each tower ($\varrho, \omega, \phi, \tilde{\omega}$ ...).

Approximately for $\varrho\omega\phi$ one has $1/(1+Q^2)$ for this term, while for ψ one has $10/(10+Q^2)$. Hence roughly

$$\nu W_2(\omega, Q^2) \sim \text{constant} + 2\% \times \frac{10}{10+Q^2} \times \frac{1+Q^2}{1} \tag{4.66}$$

and so the magnitude of the colour threshold becomes

$Q^2 = 0$	2%
$Q^2 = 5\ \text{GeV}^2$	8%
$Q^2 = 10$	11%
$Q^2 = \infty$	20%

(4.67)

These magnitudes are consistent with the FNAL data. Notice that the heavy mass of ψ relative to the $\varrho\omega\phi$ leads to a violation of scale invariance.

Conclusion

It is not clear whether GVMD or the parton model is applicable to nature. At the very least I think that this discussion shows that one should be cautious in applying the parton model ignoring all dynamical differences between uncoloured and coloured sectors. Depending upon your taste this question of deep inelastic thresholds is (partons) or is not (GVMD) a serious problem for the Han-Nambu colour scheme.

4.4.4. Dileptons

The recent observation of dileptons

$$e^+e^- \to e^{\pm}\mu^{\mp} + \text{neutrals} \tag{4.68}$$

at 4.8 GeV in the c.m. at SPEAR suggests that new states are being *pair* produced and then decaying weakly into leptons and neutrinos [33]. If this interpretation is correct, and if these new particles are new hadrons associated with the ψ (as against heavy leptons, for example), then one must break the colour strongly. This is because the unbroken colour would describe the threshold rise for $Q^2 > (4\text{ GeV})^2$ as being due to production of a single coloured state ($m \sim 3$ GeV) in conjunction with a conventional meson. This would not be consistent with the dilepton interpretation. Within the colour framework one must break the colour strongly ((WC in Section 6) or in Han-Nambu as in Marinescu--Stech [18]).

In Ref. [18] there is mixing allowed between $\tilde{\omega}_8$ and $\tilde{\omega}_1$. The 4.1 state is dominantly $\tilde{\omega}_8$ with a mixing angle $\tan\theta \simeq 0.2$ which retains the good phenomenology of the familiar mesons and mixes in some (ϱ, $\tilde{\omega}_1$) to the 4.1 peak in addition to the (ω, $\tilde{\omega}_8$) already there. The phenomenology of the decays of the 4.1 into pions ($G = +$ and $G = -$) will be similar to the WC model of Section 6. The 4.1 can decay readily into conventional hadrons by means of this mixing. Alternatively one requires a model like that of Greenberg [9], discussed in the next section.

4.5. Greenberg's model [9]

Unlike Han-Nambu, for which the quarks formed a (3,3*) representation of SU(3) ⊗ SU(3), the Greenberg model takes them to be (3,3). Hence their charges will be

	p	n	λ
R	$z+1$	z	z
B	z	$z-1$	$z-1$
G	z	$z-1$	$z-1$

(4.69)

The constraint $z+z'+z'' = 2$ yields $3z+1 = 2$ and hence $z = {}^1/_3$. The charges are therefore [9]

	p	n	λ
R	$^4/_3$	$^1/_3$	$^1/_3$
B	$^1/_3$	$-^2/_3$	$-^2/_3$
G	$^1/_3$	$-^2/_3$	$-^2/_3$

(4.70)

and these are displayed in Fig. 3. Note that all triangles are inverted (3,3), since the spectrum analysis is a triplet as against antitriplet in the Han-Nambu model (Fig. 1).

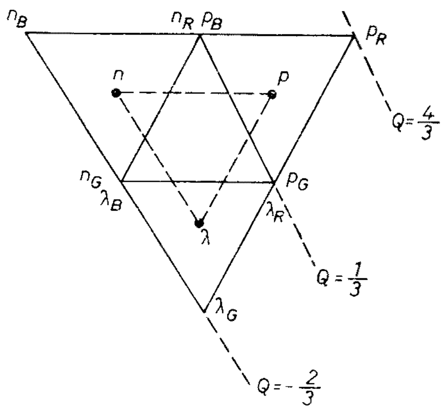


Fig. 3

The (3,3) transformation property under $SU(3) \otimes \widetilde{SU}(3)$ enables one to reclassify the quarks under the diagonal SU(3) subgroup

$$SU(3) \otimes \widetilde{SU}(3) \longrightarrow (SU(3))_{\text{diagonal}} \tag{4.71}$$

$$(3 \otimes 3) \longrightarrow 6 + 3^*$$

One can imagine these as a 6 and 3* of new quarks as in Fig. 4 where we have used Greenberg's notation. These new quarks are linear combinations of the previously defined

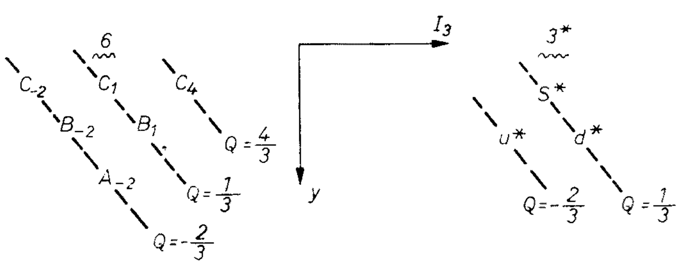


Fig. 4

quarks. These combinations are shown in Fig. 5 (normalization factors should also be included, but have been omitted for ease of visualization — further details can be found in Greenberg's paper [9].

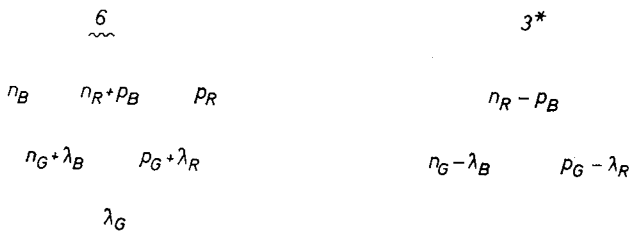


Fig. 5

Since $m(u^*, d^*) < m(s^*)$, then one expects $m(A) < m(B) < m(C)$. Furthermore, since $\underset{\sim}{6}$ is triangular

$$m^2(B)-m^2(A) = m^2(C)-m^2(B) \tag{4.72}$$

and so equal spacing of the meson masses squared will obtain if they are $A\bar{A}$, $B\bar{B}$, $C\bar{C}$ states.

Quark-antiquark states are as follows:

(*i*) $\bar{3}\times 3$ — familiar mesons — $\underset{\sim}{1}+\underset{\sim}{8}$, (4.73)

(*ii*) $\underset{\sim}{6}\times\underset{\sim}{\bar{6}}$ — new mesons like in "hidden charm" — $\underset{\sim}{1}+\underset{\sim}{8}+\underset{\sim}{27}$, (4.74)

(*iii*) $\begin{matrix}\underset{\approx}{6}\times\underset{\approx}{3} \\ \underset{\sim}{\bar{6}}\times\underset{\sim}{\bar{3}}\end{matrix}$ — states with "manifest charm" — $\begin{matrix}\underset{\sim}{8}+\underset{\sim}{10}. \\ \underset{\sim}{8}+\underset{\sim}{\overline{10}}\end{matrix}$ (4.75)

One can visualize this as a model with six charmed quarks. The new mesons contain hidden "sixness" (hidden charm) and the usual Zweig arguments of the charm model apply. This model started out as a colour model but has transmogrified into a "charm" like one.

The $6\times\bar{6}$ new states are taken by Greenberg to be as in the table. The predicted leptonic widths may be a problem however

		$\Gamma^{e^+e^-}$
3.1	$\psi(A\bar{A})$	8
3.7	$\psi'(B_1\bar{B}_1+B_{-2}\bar{B}_{-2})\dfrac{1}{\sqrt{2}}$	9
4.1	$\psi''(C_4\bar{C}_4+C_1\bar{C}_1+C_{-2}\bar{C}_{-2})\dfrac{1}{\sqrt{3}}$	6

(4.76)

There is a natural feeling for the equal spacing in squared masses, as noted above due to the $\underset{\sim}{6}$ triangular representation for the new quarks. One finds

$$m^2(\psi')-m^2(\psi) = m^2(\psi'')-m^2(\psi'). \tag{4.77}$$

In common with the Weyers-Close model of the next section, this model expects baryon pair production to be important since the baryons contain both 3* and 6 quarks. However, the A quark in $\psi(A\bar{A})$ is λ_G which does not occur in N or Δ so $N\bar{N}$ and $\Delta\bar{\Delta}$ are not important decay modes.

If the 6 new quarks were given a new quantum number called "heaviness", while the 3* had zero heaviness, then this model would contain the model of Harari [34] (see also Dr Hey's talk at this school). The main difference between the two models lies in the new spectroscopy as shown in Table IV. In particular charge 2 states are present in the Greenberg as against Harari model[4].

TABLE IV

Spectroscopy in Harari's and Greenberg's models

	Harari	Greenberg
Usual	$3\times\bar{3} = 1+8$	$3\times\bar{3} = 1+8$
New states ψ	$\bar{3}_H\times 3_H = (1+8)$	$\bar{6}\times 6 = 1+8+27$
"Charm-like" states	$3\times 3_H = 6+\bar{3}$ $\bar{3}_H\times\bar{3} = \bar{6}+3$	$\bar{6}\times\bar{3} = 8+\overline{10}$ $3\times 6 = 8+10$

5. *The classification of colour models*

In the first lecture we noted that the constraint $z+z'+z'' = 2$ could also be written (by exploiting the fact that the charge is a generator of the group)

$$Q = I_3 + \frac{Y}{2} + \left(\alpha\tilde{I}_3 + \beta\frac{\tilde{Y}}{2}\right). \tag{5.1}$$

We can classify models by their α, β values. If we demand that no fractionally charged mesons occur in the theory then α, β and $(\alpha+\beta)/2$ are integers.

The value of

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)_{\text{QED}}} = 2 + \frac{3\alpha^2+\beta^2}{2}. \tag{5.2}$$

The first model that we discussed was that of Gell-Mann, for which $\alpha = 0$, $\beta = 0$. In order to excite colour one requires $|\alpha|+|\beta| \geqslant 2$. At the simplest level one has $\alpha = \pm 1$, $\beta = \pm 1$; $\alpha = \pm 1$, $\beta = \mp 1$; $\alpha = \pm 2$, $\beta = 0$; $\alpha = 0$, $\beta = \pm 2$. Of these, several are identical modulo a trivial interchange of red, blue, green labels. One has three independent sets of charges:

[4] The prediction of charge > 1 states is a consequence of any sensible model with colour excitation.

(*i*) $\alpha = 1$, $\beta = 1$. This gives the Greenberg model [9].
(*ii*) $\alpha = 1$, $\beta = -1$. This gives the Han-Nambu model [7].
(*iii*) $\alpha = 2$, $\beta = 0$. This leads to a model not yet discussed. Since $R = 2+(3\alpha^2+\beta^2)/2$ this model will have $R = 8$ as against 4 for the models (*i*) and (*ii*). The charges of this model are

	p	n	λ
R	$^5/_3$	$^2/_3$	$^2/_3$
B	$-^1/_3$	$-^4/_3$	$-^4/_3$
G	$^2/_3$	$-^1/_3$	$-^1/_3$

(5.3)

These charges first arose in the work of Tati [35], who used $SU(3) \otimes \widetilde{SU(2)}$ and took the triplet representation of $\widetilde{SU(2)}$ (like $\widetilde{\pi^+}$, $\widetilde{\pi^0}$, $\widetilde{\pi^-}$). These charges also arise in the model of Weyers and Close [2] (referred to here as the WC model). One can think of it as the Gell–Mann model with colour excitation (split the identical charges). The reasons that led us into these charges were to some extent inductive and so it is interesting that the model also arises naturally as the "missing minimal-excitation colour-model"[5].

6. *The WC-Tati model* [*21, 35*]

Under $\widetilde{SU(3)}$ we consider the states

$$V(G\bar{G}) \quad \tilde{I} = 0 \quad (\tilde{\phi}), \tag{6.1}$$

$$\chi\left(\frac{R\bar{R}+B\bar{B}}{\sqrt{2}}\right)\tilde{I} = 0 \quad (\tilde{\omega}), \tag{6.2}$$

$$\psi\left(\frac{R\bar{R}-B\bar{B}}{\sqrt{2}}\right)\tilde{I} = 1 \quad (\tilde{\varrho}). \tag{6.3}$$

Each type V, χ, ψ (i.e. $\tilde{\phi}$, $\tilde{\omega}$, $\tilde{\varrho}$) will have three vector mesons which we write as follows (neglecting the normalization factors for clarity)

$$p_G\bar{p}_G+n_G\bar{n}_G \quad [\omega, \tilde{\phi}], \tag{6.4}$$

$$p_G\bar{p}_G-n_G\bar{n}_G \quad [\varrho, \tilde{\phi}], \tag{6.5}$$

$$\lambda_G\bar{\lambda}_G \quad [\phi, \tilde{\phi}], \tag{6.6}$$

$$(p_R\bar{p}_R+n_R\bar{n}_R)\pm(p_B\bar{p}_B+n_B\bar{n}_B) \quad \begin{bmatrix}\omega, \tilde{\omega}\\ \omega, \tilde{\varrho}\end{bmatrix}, \tag{6.7}$$

[5] $\alpha = 0$, $\beta = 2$ leads again to Han-Nambu.

$$(p_R\bar{p}_R - n_R\bar{n}_R) \pm (p_B\bar{p}_B - n_B\bar{n}_B) \quad \begin{bmatrix} \varrho, \tilde{\omega} \\ \varrho, \tilde{\varrho} \end{bmatrix}, \tag{6.8}$$

$$\lambda_R\bar{\lambda}_R \pm \lambda_B\bar{\lambda}_B \quad \begin{bmatrix} \phi \;\; \tilde{\omega} \\ \phi, \tilde{\varrho} \end{bmatrix}. \tag{6.9}$$

Unlike the previous lectures, where the $\tilde{I} = 0$ physical states have been $\widetilde{SU(3)}$ singlet ($\tilde{\omega}_1$) or octet ($\tilde{\omega}_8$), we shall now consider the possibility that they are the ideally mixed colour states $\tilde{\omega}$ and $\tilde{\phi}$ and hence that the colour is strongly broken at $\widetilde{SU(3)}$ level but a good symmetry at $\widetilde{SU(2)}$ level.

In order to help orient the reader we list here the assignments of the states

	ϱ	ω	ϕ
$\tilde{\phi}$	770	780	1020
$\tilde{\varrho}$	?	3100	3700
$\tilde{\omega}$	? ← 4.1 →	?	?

(6.10)

We shall show that we are naturally led into this pattern of classification. Indeed, we would like the 3100 and 3700 narrow states to be $\tilde{\varrho}$, since they are forbidden to decay into ordinary $\tilde{\phi}$ states due to:

(*i*) $\tilde{I} = 1 \nleftrightarrow \tilde{I} = 0$,

(*ii*) Zweig rule in colour.

But why are there only *two* narrow states, and which one is ω, ϕ, (ϱ?)?

6.1. Leptonic widths of mesons

We return to our original general definition for the quark charges

	p	n	λ
R	z'	$z'-1$	$z'-1$
B	z''	$z''-1$	$z''-1$
G	z	$z-1$	$z-1$

(6.11)

A simple calculation gives the photon-vector meson couplings (defined as in Section 4.3.2). For the $\tilde{\phi}(G\bar{G})$ states one finds

$$\gamma_\varrho : \gamma_\omega : \gamma_\phi = \tfrac{1}{2} : z - \tfrac{1}{2} : \frac{1}{\sqrt{2}}(z-1). \tag{6.12}$$

Empirically these appear to be consistent with $^1/_2 : {^1/_6} : -1/3\sqrt{2}$, and so we will take

$$z = {^2/_3} \tag{6.13}$$

i.e. a triplet with the familiar charges $^2/_3$, $-^1/_3$, $-^1/_3$ for p_G, n_G, λ_G.

Similarly it is easy to see that for the $\tilde{\varrho}$ and $\tilde{\omega}$ states one will have

$$\gamma_\varrho : \tfrac{1}{2} \pm \tfrac{1}{2},$$

$$\gamma_\omega : (z' - \tfrac{1}{2}) \pm (z'' - \tfrac{1}{2}),$$

$$\gamma_\phi : \frac{1}{\sqrt{2}}(z'-1) \pm \frac{1}{\sqrt{2}}(z''-1). \tag{6.14}$$

The $I_c = 0(\tilde{\omega})$ states have the relative + sign and since

$$z' + z'' = 2 - z = \tfrac{4}{3} = 2z \tag{6.15}$$

we have the $3 : 1 : -\sqrt{2}$ couplings just as for the $\tilde{I} = 0$ [$\tilde{\phi}$(G$\overline{\text{G}}$)] states. This already prevents one assigning the 3100 and 3700 states to $I_c = 0$, since one would have the embarrassing failure to see a ϱ-like state with $\Gamma^{e^+e^-} \sim 20$ keV!

For the $\tilde{I} = 1(\tilde{\varrho})$ states, with the relative − sign, one immediately has that

$$\gamma_\varrho : \gamma_\omega : \gamma_\phi = 0 : z' - z'' : \frac{1}{\sqrt{2}}(z' - z''). \tag{6.16}$$

Hence we finally have for the widths (apart from mass factors)

$\tilde{I} = 0$	$\tilde{I} = 1$
$\phi = 2\omega$; $\varrho = 9\omega$	$[\varrho]\omega = 2\phi$

(6.17)

The $\tilde{I} = 1$ phenomenology is now perfect for the 3100 and 3700 identification. Empirically

$$\Gamma^{e^+e^-}(3.7) \sim \tfrac{1}{2}\Gamma^{e^+e^-}(3.1) \tag{6.18}$$

and so we identify

$$[\omega, \tilde{\varrho}] \equiv 3.1, \tag{6.19}$$

$$[\phi, \tilde{\varrho}] \equiv 3.7. \tag{6.20}$$

Furthermore, both are $I = 0$ as empirically seems the case. The $[\varrho, \tilde{\varrho}]$ does not couple to e^+e^- *and hence only two narrow states arise.*

Note also that $\omega = 2\phi$ for $\tilde{I} = 1$ in contrast to $\phi = 2\omega$ for $\tilde{I} = 0$. It is this fact that caused the ϕ to be identified wih the 3.7 (due to its smaller leptonic width) rather than the 3.1. This is rather nice because in the masses ϕ is heavier than ω both in the familiar $\tilde{\phi}$(G$\overline{\text{G}}$) world and now also here for the $\tilde{\varrho}$. In fact

$$\left(\frac{\phi - \omega}{\phi + \omega}\right)_{\tilde{\varrho}} \simeq \left(\frac{\phi - \omega}{\phi + \omega}\right)_{\tilde{\phi}} \tag{6.21}$$

to within about 3%. This fact suggests that we might scale up all masses for the vector mesons from the $\tilde{\phi}$ to the $\tilde{\varrho}$ world. Doing so we expect

$$[\varrho, \tilde{\varrho}]\ (3.05), \tag{6.22}$$

$$[\omega, \tilde{\varrho}]\ (3.1), \tag{6.23}$$

$$[\phi, \tilde{\varrho}]\ (3.7) \tag{6.24}$$

and also

$$[K^*, \tilde{\varrho}]\ (3.4). \tag{6.25}$$

6.2. Exotic states

Since we have empirically that $[\omega, \tilde{\varrho}]$ and $[\phi, \tilde{\varrho}]$ do couple to e^+e^- [22] then we conclude

$$z' \neq z''. \tag{6.26}$$

This is the point of departure from the Gell–Mann colour model, where $z' = z''$ and no excitation occurs. The simplest choice for the charges then becomes

	p	n	λ	Lepton
R	$^5/_3$	$^2/_3$	$^2/_3$	$+1$
G	$^2/_3$	$-^1/_3$	$-^1/_3$	0
B	$-^1/_3$	$-^4/_3$	$-^4/_3$	-1

(6.27)

Note that the average charges of the R, G, B states are $+1, 0, -1$ like the leptons μ^+, ν, e^- (and hence the fourth column if one wishes to incorporate leptons with the quarks).

An immediate consequence of the above is the existence of exotic states.

In the vicinity of 3 to 4 GeV should exist the $\tilde{I} = 1$, $\tilde{I}_z = \pm 1$ states $(R\bar{B}, B\bar{R})$, partners of the $\tilde{I} = 1$, $\tilde{I}_z = 0$, $\tilde{\varrho}$ mesons at 3.1 and 3.7. Examples of some of the exotic states that will occur in these representations are, with their charges

$$p_R\bar{n}_B,\ p_R\bar{\lambda}_B \ -\ \text{charge 3}, \tag{6.28}$$

$$p_R\bar{p}_B,\ n_R\bar{n}_B,\ \lambda_R\bar{\lambda}_B \ -\ \text{charge 2} \tag{6.29}$$

and exotic states with strangeness -1.

$$\lambda_R\bar{p}_B,\ \lambda_R\bar{n}_B \tag{6.30}$$

with charges $+1$, $+2$, respectively.

The most naive mass formulae would then lead us to predict that an isoscalar doublet ($\tilde{I} = {^1/_2}$, $R\bar{G}$) family exists around 2 GeV. The (ordinary) SU(3) nonets which correspond to it will contain doubly charged states

$$p_R\bar{n}_G,\ p_R\bar{\lambda}_G. \tag{6.31}$$

These $\tilde{I} = {}^1/_2$ states (referred to as $\tilde{K}^*$) will in many ways be similar to the charm model states (think of G as non-charmed and R as charmed quarks). Their decays will presumably be weak and so they will show up as narrow peaks in their decay topologies. Also, by their weak decays they could give rise to dilepton events of the type

$$e^+e^- \to (\tilde{I} = \tfrac{1}{2})^+ + (\tilde{I} = \tfrac{1}{2})^- \qquad (6.32)$$
$$\quad\;\; \hookrightarrow l^+\nu\ldots \qquad \hookrightarrow l^-\bar{\nu}\ldots$$

which manifest themselves, for example, as

$$e^+e^- \to e^\pm\mu^\mp + \ldots \qquad (6.33)$$

The apparent observation of such events at SPEAR [33] is consistent with such a strongly broken colour model as it is also with charm. The crucial test to distinguish from charm is the presence of doubly charged states around 2 GeV.

6.3. The $\tilde{\omega}$ states

The three states $[\varrho\tilde{\omega}]$, $[\omega\tilde{\omega}]$, $[\phi\tilde{\omega}]$ all couple to the photon and so can be produced directly in e^+e^- annihilation. They all have $\tilde{I} = 0$, so we can make general remarks on their properties which will be applicable to the state at 4.1 GeV, irrespective of whether or not it is $[\varrho\tilde{\omega}]$, $[\omega\tilde{\omega}]$, or $[\phi\tilde{\omega}]$.

The direct decays into ordinary $\tilde{\phi}$ mesons are allowed by $\tilde{I}$ conservation, but suppressed by a (colour) Zweig rule. Baryon-antibaryon production is also allowed by $\tilde{I}$ but now, since baryons are colour singlets and contain all colours of quarks as a consequence, there is no Zweig (colour) rule suppression Therefore we may anticipate that an important decay mode should be into baryon-antibaryon and related channels. The final-state interaction will readily allow ordinary mesons to be produced (the direct production of these states may be possible if Zweig's rule is not exact for colour).

In particular, the isocolour conservation will yield the consequence that the $\tilde{\omega}$(4.1 GeV) state should *not* significantly cascade into the $\tilde{\varrho}$(3.1, 3.7) states. More precisely cascading into the $\tilde{\varrho}$ states is allowed in first order in electromagnetism and hence should account for a few MeV *at most* of the total width of the 4.1.

If the isocolour doublet states predicted in our model have indeed a mass of ~2 GeV, a significant fraction of the 4.1 width could be due to decay into a pair of these states. If this is the case, narrow structures could be seen in *doubly charged* channels. It should be pointed out that our model does predict the existence of such states, but depending on the complications one is willing to accept in the mass formulae their masses could be almost anything (unfortunately!).

If we scale up masses from the $\tilde{\phi}$ world to the $\tilde{\omega}$ world, then it may be that the $\tilde{\omega}$(4.1) is a mixture of $[\omega\tilde{\omega}]$ and $[\varrho\tilde{\omega}]$. If so then one might look for apparent G-parity violations as one moves through the peak and interesting $\varrho-\omega$ interference effects could exist. One would then also anticipate a $[\phi\tilde{\omega}]$ with mass around 5 GeV with leptonic width $\leqslant 2$ keV. This suggests that the $[\phi\tilde{\omega}]$ could be a broad hump barely visible in the total cross-section. However, it may be visible in topological cross-sections, for example those involving strange hadronic final states.

6.4. Properties of ψ(3.1 and 3.7) in this scheme

As in the Han–Nambu model the strong decay of the ψ states is forbidden into ordinary mesons by $\tilde{I}$ conservation, while first order electromagnetic processes are, in principle, allowed. In this present model with the ideal colour mixing $\tilde{\omega} \to \tilde{\phi}\gamma$ is further suppressed by the (colour) Zweig rule. One might expect that

$$\psi(3.1) \to \mathrm{B\bar{B}}\gamma$$

could be an important member of the class of radiative decays.

The presence of $\tilde{I} = 0$, $\tilde{\omega}$ states around 4 GeV naturally suggests a mechanism whereby the 3.1 and 3.7 states can decay into $\tilde{\phi}$ hadrons [36]. As in Section 4.4.2 we will suppose that there exists a tadpole transforming as $(I_3, \tilde{1})+(1, \tilde{I}_3)$. In the familiar meson world it is known, from the magnitude of $(\omega\tilde{\phi}) \to (2\pi\tilde{\phi})$, that the tadpole mechanism, Fig. 6a,

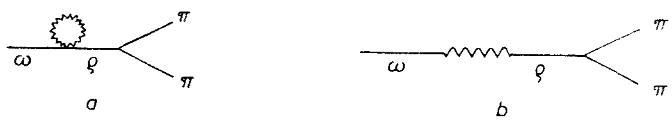


Fig. 6a, b

is dominant by far over Fig. 6b. If we suppose that this is true in general, then a tadpole of type (Fig. 6a) will mix the $\tilde{\varrho}$ with $\tilde{\omega}$ states. Hence the 3.1 mixes to a 4.1 $\tilde{\omega}$ state and then decays into conventional hadrons conserving $\tilde{I}$. We can even make quantitative estimates of this rate.

Before mixing, let $\psi(\omega, \tilde{\varrho})$ have mass M_1 and width to hadrons zero, and $(\omega, \tilde{\omega})$ have mass M_0 width Γ_0.

Turn on a tadpole of strength ε and the mass matrix becomes

$$\begin{pmatrix} M_1 & \varepsilon \\ \varepsilon & M_0 - i\Gamma_0 \end{pmatrix}. \tag{6.34}$$

In the approximation that $M_0 - M_1 \gg \Gamma_0 \gg \varepsilon$, one finds for the eigenvalues and eigenvectors

Eigenvalues	Eigenvectors
$m_{4.1} \approx M_0 - i\Gamma_0$	$\begin{pmatrix} \varepsilon/(m_0 - m_1) \\ 1 \end{pmatrix}$
$m_{3.1} \approx M_1 - i\dfrac{\varepsilon^2 \Gamma_0}{(M_1 - M_0)^2}$	$\begin{pmatrix} 1 \\ -\varepsilon/(M_0 - M_1) \end{pmatrix}$

(6.35)

and so the 3.1 has acquired a width $\varepsilon^2\Gamma_0/(M_1-M_0)^2$. If $\Gamma_0 \simeq 100$ MeV (remembering that $(\varrho\tilde{\omega})$ as well as $(\omega\tilde{\omega})$ is present in the 4.1 GeV bump) and $m_1-m_0 \simeq 1$ GeV, then

$$\Gamma_{3.1}^{\text{hadrons}} \simeq 40 \text{ keV} \tag{6.36}$$

if $\varepsilon \simeq 20$ MeV. This is very encouraging, since $\varepsilon \simeq 5$ to 8 MeV in calculations of the nucleon mass differences and hence is some $^1/_2$ to 1% of the mass. Scaling up for a ψ mass of 3 GeV leads to $\varepsilon \simeq 20$ MeV.

If one played the same game with the 3.7 then, by virtue of the fact that its mass is closer to 4.1, so would its width be larger.

However, a problem is that the 3.7 final states are rather different in content from the 3.1. In the specific model discussed here one anticipates that the $(\phi\tilde{\varrho})3.7$ should be mixed with the $(\phi\tilde{\omega})4.9$ state and so its final state should reflect those of the 4.9 as against the 4.1 that generates the 3.1 decay modes. One then predicts

$$\frac{\Gamma_{3.7}}{\Gamma_{3.1}} = \left(\frac{\varepsilon_{3.7}}{\varepsilon_{3.1}}\right)^2 \left(\frac{4.1-3.1}{4.9-3.7}\right)^2 \frac{\Gamma_{\text{tot}}(4.9)}{\Gamma_{\text{tot}}(4.1)} \simeq \frac{3}{2} \frac{\Gamma_{\text{tot}}(4.9)}{\Gamma_{\text{tot}}(4.1)}. \tag{6.37}$$

The non-cascade width of 3.7 is of order 100 keV [37] and hence some $^5/_2$ times that of the 3.1. Bearing in mind that the 4.1 peak contains ϱ and ω in our model, then a $\Gamma_{\text{tot}}(4.9) \simeq 200$ MeV is required.

This mixing mechanism is certainly more generally applicable than to the specific colour model discussed here, and can be applied to general models with "zero width" ψ that can be mixed to wide 4.1 states. The ψ need not even be hadronic perhaps [38]. One could even imagine that the 3.7 decays always into 3.1 either $\psi'(3.7) \to \psi X$ or $\psi' \to X\psi \to X\gamma$. This latter "vector dominance in reverse" can easily generate widths of several keV. Some fraction of $\psi \to$ (all) should come from this mechanism which could be part of the explanation for the missing neutrals in the 3.7 final state.

This same mechanism that mixes 3.1 and 4.1 will cause the cascade $4.1 \to 3.1$ to exist (a priori it would have been zero by $\tilde{I}$ conservation). This should be of the same order as the 3.1 total hadronic width generated by this mechanism and so possibly less than the cascade $3.7 \to 3.1$. If this could be shown to be the case, namely

$$\Gamma(4.1 \to 3.1) \lesssim \Gamma(3.7 \to 3.1) \tag{6.38}$$

then this "anti-phase-space" result would be very significant for our ideas.

6.5. Discussion

We have performed "ideal mixing" in the colour SU(3). This was possible due to the quark charges in the model but was not forced upon us. Indeed, with ideal mixing one might already be in trouble with $\psi \to p\bar{p}\gamma$ which has no colour space Zweig suppression and so naively looks as if it should be dominant among the final states that contain a photon. Relaxing the ideal mixing could alleviate the problem; the $\bar{p}/\pi$ enhancements would still be expected though at a less dramatic level than for ideal mixing.

From a theoretical point of view, if one is committed to the religion that for E_{cm}

$\lesssim 3.5$ GeV one has R = constant $\simeq 2$ in the data and that this is immediately related to the squared charges of the quarks in this domain then the ideal mixing is too much [39]. In the past we have so much wanted to believe that $R = 2$ for $E_{cm} \lesssim 3.5$ GeV that we have tended to see it in the data. However, I think that an unbiased look at the data does not support the belief that there is scaling at low E_{cm}. In this model $R = {}^2/_3$ *plus* contributions arising from intermediate baryon pairs for which all colours of quarks contribute. Furthermore, a *small* deviation from ideal mixing gives a dramatic increase in the predicted R in the model at low E_{cm}.

Notice also that one does not necessarily lose the phenomenology of $\pi^0 \to \gamma\gamma$ which seemed to require RBG quarks in the π^0 to be consistent with the observed π^0 lifetime. With the π^0 only containing green quarks it looks just like a traditional π^0 with uncoloured quarks. It was noted by Drell [40] that such a situation is not in discord with the data in the framework of weak PCAC. This necessitated the introduction of a further π^0 state (Drell called it $\pi^{0\prime}$). In the WC model such a state arises naturally.

Alternatively one could allow the axial current to mix the π^0 with red and blue quarks which would then contribute canonically to the anomaly. In this case, one would anticipate that ψ states can be readily produced singly in π initiated reactions.

6.6. How to confirm the model?

The most clear-cut prediction is charge 2 and 3 meson states around 3 GeV in mass.

Charge 2 states are also predicted to exist, but their precise masses are somewhat model dependent. Naively one may anticipate that they are around 2 GeV in mass (containing green (light) quarks as well as red blue (heavy) quarks) and are responsible for the dilepton production at $E_{cm} = 4.8$ GeV [33]. These states are in many ways analogous to the charmed mesons of the SU(4) charm models. Their exotic charges should distinguish them from "conventional" charmed mesons.

If one could show experimentally that

$$\Gamma(4.1 \to 3.1\ X) \lesssim \Gamma(3.7 \to 3.1\ X)$$

then this would probably be unnatural in charm models, since the extra phase space would suggest a faster rate. In the class of colour models where 3.1 and 3.7 are $\tilde{\varrho}$ and 4.1 is $\tilde{I} = 0$, then colour isospin conservation leads one to expect that, in order of magnitude

$$\Gamma(4.1 \to 3.1\ X) \simeq \Gamma(3.1 \to \text{hadrons}).$$

Present data suggest that the 4.1 cascade width is small, but it not yet at the level of testing whether or not it is this small.

Finally, one anticipates that the decays of the 4.1 to ordinary hadrons may be related to baryon-antibaryon production for which there is no (colour) Zweig rule violation. These may be virtual states which produce the final-state mesons through final-state interaction. This still apparently violates the colour Zweig rule, but duality diagrams arguments applied to $B\bar{B}$ channels are known to be unreliable as well as uncalculable. If one was lucky one might find a larger $\bar{B}/\pi$ production around 4.1 GeV than at lower Q^2 values.

I wish to thank J. Ellis, A. J. G. Hey, and the organizers and the students of the Cracow School for their questions, comments and suggestions. The lectures are the result of many hours spent talking with J. Weyers, whom I also thank for commenting on this manuscript.

REFERENCES

[1] J. Ellis, Preprint CERN TH. 1996 (1975).
[2] F. E. Close, J. Weyers, *Proc.* 10*e Recontre de Moriond*, Méribel-les-Allues, France, June 1975, ed. J. Tran Thanh Van (to be published).
[3] O. W. Greenberg, *Phys. Rev. Lett.* **13**, 598 (1964).
[4] For a detailed description of the quark model states in SU(6) ⊗ O(3), see F. E. Close, *How high are higher symmetries and ψ chology for Jwalkers*, Rutherford Lab. report RL-75-091 (1975). The current empirical situation concerning the baryon spectrum is described by P. Litchfield, *Proc. 17th Internat. Conf. on High-Energy Physics*, London 1974, Rutherford Lab., Chilton, Didcot, 1974, p. II–65; and A. Hey in these proceedings.
[5] S. Adler, *Phys. Rev.* **177**, 2426 (1969). J. S. Bell, R. Jackiw, *Nuovo Cimento* **60A**, 47 (1969).
[6] See, for example, M. Gell–Mann, *Acta Phys. Austriaca Suppl.* **9**, 733 (1972); O. Greenberg, D. Zwanziger, *Phys. Rev.* **150**, 1177 (1966).
[7] M. Han, Y. Nambu, *Phys. Rev.* **139**, B 1006 (1965).
[8] See, for example, the various colour models in Refs [9–17].
[9] After O. W. Greenberg, *Proc. Second Orbis Scientiae*, Univ. of Miami, January 1975, Maryland report 75-064 (1975).
[10] W. Alles, Bologna preprint (1974). D. Bailin, A. Love, Univ. of Sussex preprint (1974).
[11] As Ref. [10]. Supplements produced in 1975.
[12] I. Bars, R. Peccei, *Phys. Rev. Lett.* **34**, 985 (1975).
[13] T. C. Yang, Univ. of Maryland report 75-052.
[14] M. Krammer, D. Schildknecht, F. Steiner, Report DESY 74/64 (1974).
[15] A. Sanda, H. Terezawa, *Phys. Rev. Lett.* **34**, 1403 (1975).
[16] B. G. Kenny, D. C. Peaslee, L. J. Tassie, *Phys. Rev. Lett.* **34**, 429 (1975); **34**, 1482 (1975).
[17] G. Feldman, P. Mathews, Imperial College London report (1975); P. Mathews, Invited talk at *EPS Internat. Conf. on High-Energy Physics*, Palermo, June 1975 (to be published).
[18] N. Marinescu, B. Stech, Heidelberg report HD-THEP-75-4 (1975).
[19] M. Arik, D. Coon, S. Yu, Brookhaven report BNL-19827 (1975).
[20] D. Yennie, *Phys. Rev. Lett.* **34**, 239 (1975).
[21] See also F. E. Close in Ref. [4].
[22] R. F. Schwitters, Invited talk in the Anaheim meeting of the APS, January 1975; H. L. Lynch, SLAC-PUB-1536; A. M. Boyarski et al., SLAC-PUB-1572.
[23] J. E. Augustin et al., *Phys. Rev. Lett.* **34**, 764 (1975).
[24] G. Abrams et al., *Phys. Rev. Lett.* **34**, 1181 (1975).
[25] See, for example, H. Harari, *ψ chology*, SLAC-PUB-1514; The CERN Theory Boson Workshop, *The narrow peaks in pp → e^+e^-X and e^+e^- annihilation* (1975).
[26] See, for example, M. Kuroda, Y. Yamaguchi, Report UT-248 (1975) and also O. W. Greenberg, Ref. [9].
[27] See, for example, K. Johnson in these proceedings; also Ref. [26].
[28] R. P. Feynman, M. Kislinger, F. Ravndal, *Phys. Rev.* **D3**, 2706 (1971); D. Faiman, A. W. Hendry, *Phys. Rev.* **173**, 1720 (1968); **180**, 1609 (1969); H. Burkhardt, A. J. G. Hey (to be published).
[29] B. Stech, *Proc. of 12th Internat. Winter School*, Karpacz, Poland, February 1975; Heidelberg preprint (1975).
[30] See, for example, H. Osborn, D. Wallace, *Nucl. Phys.* **B20**, 23 (1971) and *Erratum* **B22**, 658 (1970).

[31] D. J. Fox et al., *Phys. Rev. Lett.* **33**, 1504 (1974).
[32] See, for example, M. Greco, *Nucl. Phys.* **B63**, 398 (1973).
[33] Private communication and M. Perl et al, SLAC-PUB (in preparation).
[34] H. Harari, SLAC-PUB-1568, to be published in *Phys. Lett.* and SLAC-PUB-1589.
[35] T. Tati, *Prog. Theor. Phys.* **35**, 126 (1966).
[36] H. Osborn, private communication. We are indebted to Dr. Osborn for suggesting this mechanism.
[37] Private communication from T. Walsh.
[38] For example one might apply this mechanism to the qualitative remarks of T. Neff et al., SLAC-PUB-1540.
[39] Such a possibility has also been considered by M. Böhm and M. Krammer (private communication, unpublished).
[40] S. D. Drell, *Phys. Rev.* **D7**, 2190 (1973), especially Eq. (20) et seq.