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DIFFICULTIES FOR SU(N) QUARK MODELS OF THE NEW PARTICLES

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If preliminary experimental results on the new particles are confirmed (primarily the $\psi\eta$ decay of the ψ'), and if conventional theoretical prejudices are accepted, it is shown that Harari's SU(6) model is the minimal N-quark model (with hidden color) which can accommodate these constraints.

1. Introduction

The purpose of this note is to emphasize that if recent experimental results on the new particles, $\psi(J)$, ψ', and ψ'', are confirmed, then there already exist extremely tight constraints on theoretical models based on conventional quarks with hidden color. Our conclusion after a systematic search is that Harari's model [1], containing the standard "light" quark triplet (u, d, s) together with a "heavy" antitriplet (t, b, r), appears to be almost the only viable model, and certainly the least unattractive, with fewer than seven quarks.

In order to be as clear as possible, and to emphasize the reasonableness of our starting assumptions, we begin by listing the experimental "mezzo-facts" [2] and the theoretical prejudices we shall accept.

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2. *Experimental "mezzo-facts"*

1) The existence of the ψ and ψ' as very narrow resonances and of the ψ'' as a broad resonance (all assumed to have $J^{PC} = 1^{--}$) [3].

2) The radiative widths of the ψ and ψ' are approximately in the ratio

$$\Gamma_{\psi \to e\bar{e}} : \Gamma_{\psi' \to e\bar{e}} \simeq 4.8 \text{ keV} : 2.2 \text{ keV}$$

(and $\Gamma_{\psi'' \to e\bar{e}}$ is roughly similar) [3].

3) The ψ does not decay to even numbers of pions directly, and the ψ' decays predominately to $\psi\pi\pi$ [3]. This strongly suggests that G parity is a good quantum number for the direct decays and that both ψ and ψ' are $I = 0$ resonances. Up to now, the experimental facts and most of the inferences are fairly well established.

4) From preliminary data, the ψ is not seen decaying into two octet states whose combined C parity [4] is even,

$$\psi \text{ does not decay into } K\bar{K}, K^*\bar{K}^*, K^{**}\bar{K},$$

but $\psi \to K\bar{K}^*$ and this rate is comparable to that of $\psi \to \pi\varrho$ [5]. Although not conclusive we shall accept this as evidence that the ψ is an SU(3) singlet.

5) The equally preliminary reports of the decay $\psi' \to \psi\eta$ with a branching ratio of a few percent [5, 6] would seem to imply a coupling whose strength is not too different from that of $\psi' \to \psi\pi\pi$. The obvious inference is that the ψ' has an SU(3) octet component [7]. For us, the SU(3) nature of the ψ and ψ' will be the *critical assumptions*, greatly restricting available models, but based on very tentative experimental results.

6) Other experimental indications up to now are:

(*i*) $R = \sigma(e^+e^- \to X)/\sigma(e^+e^- \to \mu^+\mu^-) < 10$, at least in the forseeable energy range [3, 5],

(*ii*) a dearth of monoenergetic γ rays from ψ' decay [5, 6].

3. *Theoretical prejudices*

1) The apparent non-observance of large radiative widths and the existence of G parity conserving decays (plus some reluctance to abandon all of the parton model results) causes us to reject models where the new particles are manifestations of color (such as the Han–Nambu model, etc. [8]).

2) "Ordinary" hadrons are based on the conventional Gell-Mann–Zweig "coloured" quark triplet (u, d, s). The standard charge assignment $(^2/_3, -^1/_3, -^1/_3)$, among many other predictions, leads uniquely to the $9:1:2$ prediction for the $\varrho : \omega : \phi$ lepton couplings, in good agreement with experiment.

3) No fractionally charged hadrons exist.

4) Our lack of real theoretical understanding of the extremely narrow widths of the ψ and ψ' is summarized by "Zweig's Rule", for which we have no satisfactory explanation. However, it leads to the notion that the new particles are built entirely out of new "heavy" quarks (with hidden color). Moreover, the dominant decay of ψ' into $\psi\pi\pi$ suggests that ψ' contains some of the same heavy quarks as the ψ [9].

5) Other possible constraints on the new quarks are:

(*i*) $R = 3\sum_i Q_i^2$ (the 3 is for hidden color). The GMZ quarks contribute 2.

(*ii*) In building a unified theory for weak and electromagnetic interactions, the strangeness changing neutral currents should be suppressed at the right level and the triangle anomalies should cancel [10].

With this experimental and theoretical "package", we now discuss the various possibilities. We shall ignore constraints from the weak interactions except to guarantee that there exists sufficient freedom to suppress strangeness changing neutral currents.

4. *SU(4) charm models* [11]

Under the usual SU(3), the basic quark representation must reduce as

$$\underset{\sim}{4} \rightarrow \underset{\sim}{3} + \underset{\sim}{1}$$

and the quark-antiquark sector as

$$\underset{\sim}{4} \times \underset{\sim}{\bar{4}} = \underset{\sim}{15} + \underset{\sim}{1} \rightarrow \underset{\sim}{8} + \underset{\sim}{3} + \underset{\sim}{\bar{3}} + \underset{\sim}{1} + \underset{\sim}{1}.$$

Thus, in $q\bar{q}$ combinations, there is no possibility of producing an $\underset{\sim}{8}$ of SU(3) made of "charmed" quarks alone. An octet ψ' cannot therefore be accommodated by the simplest and original charm scheme.

5. *SU(5) models*

Such models, with two charmed quark singlets, fail for the same reason as SU(4) models.

6. *SU(6) models*

There are four possible SU(3) reductions of the basic $\underset{\sim}{6}$ representation of SU(6):

(a) $\underset{\sim}{6} \rightarrow \underset{\sim}{6}$,

(b) $\underset{\sim}{6} \rightarrow \underset{\sim}{3} + \underset{\sim}{1} + \underset{\sim}{1} + \underset{\sim}{1}$,

(c) $\underset{\sim}{6} \rightarrow \underset{\sim}{3} + \underset{\sim}{3}$,

(d) $\underset{\sim}{6} \rightarrow \underset{\sim}{3} + \underset{\sim}{\bar{3}}$.

Resonance phenomenology rules out the first possibility (a) since we require a basic quark triplet to construct the usual baryon and meson states.

Possibility (b) is eliminated for the same reason as SU(4) and SU(5): there is no room for an octet ψ'. We therefore turn to the two remaining branches.

Case (c): $\underset{\sim}{6} \rightarrow \underset{\sim}{3}+\underset{\sim}{3}$

Here there is a basic light quark triplet (u, d, s) plus a "charmed" or "heavy" quark triplet (u', d', s'). The standard SU(3) is generated by the sum of the light and heavy SU(3) generators. Now consider the charge assignment for these new quarks.

For the light quarks we have the usual Gell–Mann–Nishijima relation,

$$Q^{(L)} = I_3 + \frac{Y}{2},$$

but for the heavy quarks we allow more freedom,

$$Q^{(H)} = \alpha I_3 + \beta\frac{Y}{2} + \gamma\frac{H}{3},$$

where $H = +1$ for the heavy quarks. What can be said about α, β, and γ?

(*i*) If ψ is to be the SU(3) singlet state, the requirement that it couples to the photon is

$$\sum_i Q_i^{(H)} \neq 0$$

which requires

$$\gamma \neq 0.$$

The ψ' and ψ'' must then be the $I = 0$ and $I = 1$ octet states, respectively [12]. As is conventionally assumed, the ψ'' is wide because it lies above the threshold for producing mixed quark hadrons (mesons made from one light and one heavy quark).

(*ii*) In the absence of extra mass factors, we expect the lepton couplings of the new particles to be in the ratio

$$\Gamma_{\psi\rightarrow e\bar{e}} : \Gamma_{\psi'\rightarrow e\bar{e}} : \Gamma_{\psi''\rightarrow e\bar{e}} = 6\gamma^2 : 3\beta^2 : 9\alpha^2. \tag{13}$$

Experiment roughly tells us that $|\beta| \simeq |\gamma|$, but we shall use the considerably weaker constraint,

$$\tfrac{1}{2}|\gamma| \leqslant |\beta| \leqslant 2|\gamma|,$$

which allows for symmetry breaking effects up to a factor of four in leptonic widths.

(*iii*) A strong constraint is provided by demanding integer charges for the mixed quark hadrons. Moreover, the obvious requirement that isospin eigenstates have a definite charge implies $Q(u\bar{u}') = Q(d\bar{d}')$ which determines

$$\alpha = +1.$$

Similarly demanding $Q(s\bar{s}') = Q(u\bar{u}')$ would require $\beta = +1$. However, we shall not impose this constraint since the $I = 0$ states made from strange quarks versus non-strange quarks could have different charges, an ugly alternative but possible with magic mixing.

With $\alpha = +1$, integer charges imply:

$$\gamma - \beta = 2\text{mod}3 \text{ and } \beta = \pm\text{odd integer}.$$

(*iv*) Assuming that R measures the sum of the quark charges squared and is less than 10 implies:

$$2\gamma^2+\beta^2 < 13.$$

Satisfying all three constraints on β and γ yields only two solutions:

a) $\alpha = +1$, $\beta = -1$, $\gamma = +1$ corresponding to the triplet $(^2/_3, -^1/_3, ^2/_3)$,

b) $\alpha = +1$, $\beta = -1$, $\gamma = -2$ corresponding to the triplet $(-^1/_3, -^4/_3, -^1/_3)$.

For case (a), $R \to 5$ and $\Gamma_\psi : \Gamma_{\psi'} : \Gamma_{\psi''} = 6 : 3 : 9$ which is not too bad. For case (b), $R \to 8$ and $\Gamma_\psi : \Gamma_{\psi'} : \Gamma_{\psi''} = 24 : 3 : 9$ which is not particularly good. In both cases, the mixed quark states fall into octets and singlets with the unpleasant feature of different charges for the $I = 0$ states made from strange quarks versus non-strange quarks. Thus, neither alternative with the heavy quarks in a $\underset{\sim}{3}$ is very attractive. We must therefore turn to the last choice.

Case (d) $\underset{\sim}{6} \to \underset{\sim}{3}+\underset{\sim}{\bar{3}}$

This model comprises a basic quark triplet (u, d, s) with a heavy anti-triplet (t, b, r) in the six dimensional representation of SU(6). It is easily verified that the above constraints still apply except that $\beta \to -\beta$. Consequently again there are only two solutions:

a) $\alpha = +1$, $\beta = +1$, $\gamma = +1$ corresponding to the anti-triplet $(^2/_3, -^1/_3, ^2/_3)$,

b) $\alpha = +1$, $\beta = +1$, $\gamma = -2$ corresponding to the anti-triplet $(-^1/_3, -^4/_3, -^1/_3)$.

Both solutions require only an additive quantum number modification to the Gell–Mann–Nishijima relation. An interesting prediction is that the mixed quark states fall into $\underset{\sim}{\bar{6}}+\underset{\sim}{3}$ for $H = +1$ and $\underset{\sim}{6}+\underset{\sim}{\bar{3}}$ for $H = -1$. The first solution is Harari's model, [1] which has $R \to 5$ and $\Gamma_\psi : \Gamma_{\psi'} : \Gamma_{\psi''} = 6 : 3 : 9$ in good agreement with the data. The second one has $R \to 8$ and $\Gamma_\psi : \Gamma_{\psi'} : \Gamma_{\psi''} = 24 : 3 : 9$, which is nearly excluded by the data although compatible with our conservative bounds [14].

Thus, Harari's model is the most acceptable solution with the fewest number of quarks. Moreover, using slightly more stringent limits on R or on leptonic widths, and requiring the $I = 0$ SU(3) eigenstates (mesons made from light plus heavy quarks) to have a definite charge, allows *only* his solution. As Harari has shown [1], it has the freedom to suppress strangeness changing neutral currents [15] (although one new heavy lepton would be required to cancel the anomalies in a unified theory). While the mass splitting chain from SU(6) is ad hoc [16] and the more exact "Zweig's Rule" must be invoked for the narrow widths, Harari's minimal solution is the least unattractive model assuming the listed experimental and theoretical prejudices.

7. *SU*(7) *to SU*(9)

Models involving SU(N) for $N = 7, 8, 9, \ldots$, are clearly possible, yet even these are fairly well constrained by $R < 10$ (assuming that R measures the sum of the squares of the quark charges times three for hidden color). Also, as above, the coefficient of I_3 in the Gell–Mann–Nishijima relation must be $+1$ (i.e. all the α's $= +1$). The only possible SU(3) decompositions for $N = 7$ and 8 are $\underset{\sim}{3}+\underset{\sim}{3}$ or $\underset{\sim}{3}+\underset{\sim}{\bar{3}}$ plus $\underset{\sim}{1}$'s. Eliminating the trivial cases (the previous SU(6) models plus disjoint singlets) leaves the ones with mixing between the new $I = 0$ vector mesons. As one example, an $SU(3)\times SU(4)$ model is possible with heavy quarks (u', d', s', c') having charges $(^2/_3, -^1/_3, -^1/_3, ^2/_3)$, as in the original charm scheme except that the SU(4) is for heavy quarks alone. The assignments for the new particles would be as follows:

$$\psi = \frac{u'\bar{u}'+d'\bar{d}'+s'\bar{s}'-3c'\bar{c}'}{\sqrt{12}}, \quad \psi' = \frac{u'\bar{u}'+d'\bar{d}'-2s'\bar{s}'}{\sqrt{6}}, \quad \psi'' = \frac{u'\bar{u}'-d'\bar{d}'}{\sqrt{2}}$$

and a new state which couples to the photon,

$$\psi'' = \frac{u'\bar{u}'+d'\bar{d}'+s'\bar{s}'+c'\bar{c}'}{2} \quad [17].$$

This new vector meson could be made heavier than ψ'' since it is the SU(4) singlet. Of course, models with $N > 6$ will usually have such new states coupling to the photon. There are obviously other SU(N) models for $N = 7$ and 8 (having different mixing and quark charges) which are consistent with our assumed experimental constraints. For $N = 9$, the SU(3) decompositions are: $\underset{\sim}{3}+\underset{\sim}{3}+\underset{\sim}{1}+\underset{\sim}{1}+\underset{\sim}{1}$, $\underset{\sim}{3}+\underset{\sim}{3}+\underset{\sim}{3}$, $\underset{\sim}{3}+\underset{\sim}{6}$ and the obvious conjugate possibilities. The first case is similar to $N = 7$ and 8; the last one leads inexorably to $R > 10$. For the case $\underset{\sim}{3}+\underset{\sim}{3}+\underset{\sim}{3}$ (and conjugates), there are several possibilities. However, $R < 10$ allows only heavy quarks with charges $^2/_3$ and $-^1/_3$, and the SU(3) $\underset{\sim}{1}$ nature of ψ excludes the most symmetric case of three identical triplets all with $(^2/_3, -^1/_3, -^1/_3)$.

None of the possibilities for $N > 6$ seems sufficiently attractive to warrant the increase in new meson and baryon states. Thus, one is being forced into either accepting Harari's model (unless one wants more than six quarks), or rejecting some of our current theoretical prejudices, or hoping that the data will change. Experiment must confirm or deny Harari's model, but one cannot escape the suspicion that one is on the wrong track. If the experimental package is confirmed, theorists are in a tight corner.

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[2] This terminology is due to L. Lederman, Rutherford Conference, January 1975.

[3] J. J. Aubert et al., *Phys. Rev. Lett.* **33**, 1404 (1974); J. E. Augustin et al., *Phys. Rev. Lett.* **33**, 1406 (1974); G. S. Abrams et al., *Phys. Rev. Lett.* **33**, 1453 (1974); *Phys. Rev. Lett.* **34**, 1181 (1975); J. E. Augustin et al., *Phys. Rev. Lett.* **34**, 764 (1975); J. A. Kadyk, *Proceedings of the Tenth Rencontre de Moriond*, Méribel-lès-Allues (France), ed. J. Tran Thanh Van; A. M. Bayarski et al., *Phys. Rev. Lett.* **34**, 1357 (1975).

[4] The C parity of an octet is the charge conjugation of the $I = 0$ member, and the combined C parity of two symmetrically coupled octets is the product of their individual C parities. This quantum number is conserved if SU(3) is exact.

[5] Rumour concerning SPEAR data at SLAC. See also G. Feldman, M. L. Perl, *Electron-Positron Annihilation above 2 GeV and the New Particles, Physics Reports*, to be published.
[6] Preliminary data from DESY reported by K. Berkelman at a Rutherford seminar.
[7] This conclusion assumes that SU(3) breaking (which the selection rule in ψ decays seems to indicate is small) or a singlet admixture in the η (which is mainly octet) is insufficient to account for a P-wave decay that has such little phase space. (A rough calculation indicates that $\psi' \to \psi\eta$ and $\psi' \to \psi\pi\pi$ are both suppressed by approximately two orders of magnitude from strong decays). It is obviously important to look for $K\bar{K}$ and other decay modes of the ψ' which are allowed by an octet assignment.
[8] M. Y. Han, Y. Nambu, *Phys. Rev.* **139B**, 1006 (1974). See also the review by O. W. Greenberg, *Proceedings of the* 1975 *Coral Gables Conference*, and the paper by F. E. Close, J. Weyers, Rutherford Lab. preprint.
[9] The conventional explanation for the enhancement of the $\psi\pi\pi$ decay is that the ψ' is a radial excitation of the ψ, which for us is ruled out by the assumed SU(3) assignments. As Harari has emphasized, [1] this reassignment of the ψ' allows the $L = 1$ $q\bar{q}$ excitations to lie higher in mass and, thus, not appear in ψ' radiative cascade decays. Of course, the pseudoscalar $q\bar{q}$ states are still expected in the 3–4 GeV mass range and, if light enough, should be seen in ψ or ψ' radiative decays.
[10] See, for example, the discussion by C. H. Llewellyn-Smith *Phenomenology of Particles at High Energies* (Proceedings of the Fourteenth Scottish Universities Summer School in Physics), Academic Press, 1974.
[11] For references and review, see M. A. Beg, A. Sirlin, *Gauge Theories of Weak Interactions, Annual Reviews of Nuclear Science* **24**, 379 (1974); and M. K. Gaillard, B. W. Lee, J. L. Rosner, *Search for Charm, Rev. Mod. Phys.* **47**, 277 (1975).
[12] Although incompatible with our assumed "mezzo-facts", an interesting two triplet model (same charges for both 3's) was proposed by R. M. Barnett, *Phys. Rev. Lett.* **34**, 41 (1975) and Harvard preprint, Feb. 1975. The heavy "ϱ" and "ω" are degenerate at 3.1 GeV; the first is the ψ produced in e^+e^- and the second is the J produced in pp. The ψ decays via electromagnetic mixing with the wider J; and the ψ' is the radial excitation of the ψ.
[13] With exact SU(6) between light and heavy quarks (which is probably grossly broken), the overall scale is the same as for $\varrho : \omega : \phi = 9 : 1 : 2$.
[14] A better leptonic ratio of 18 : 9 : 9 would be achieved with singlet-octet mixing by an angle θ equal to either arctan $(-\sqrt{2}/5)$ or arctan $(\sqrt{2})$. Only the former appears not to be in contradiction with $\psi \nrightarrow K\bar{K}, K^*\bar{K}^*, K^{**}\bar{K}$ (giving a suppression factor of 10). We thank Frank Close for emphasizing this point.
[15] Strangeness changing neutral currents can also be suppressed in the model with a quark of charge $-4/3$ by generalizing the work of T. Goto, V. S. Mathur, Rochester preprint COO-3065-108 (1975).
[16] As shown by Harari [1], the mass splitting does have the nice feature that mass decreases as hypercharge increases for both the light and heavy quarks (making the isosinglet heavy quark lighter than the isodoublet).
[17] The relative lepton couplings predicted by this assignment are $\Gamma_\psi : \Gamma_{\psi'} : \Gamma_{\psi''} : \Gamma_{\psi'''} = 6 : 3 : 9 : 2$. However, unlike the previous models, not all the off-diagonal neutral currents can be cancelled.