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THE MELOSH TRANSFORMATION: THEORY AND EXPERIMENT

By A. J. G. Hey

Southampton University*

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This article surveys the theoretical and experimental developments in the search for a transformation between current and constituent quarks.

1. *Introduction*

In the past year or so there has been a revival of interest in SU(6) schemes for resonance classification and decays. Melosh initiated this movement by providing a useful theoretical framework in which to discuss relativistic $SU(6)_w$ symmetries. The purpose of this review is to survey the theoretical and experimental developments in the search for a transformation between current and constituent quarks. We therefore begin [1] with some general remarks on the "constituent quark" SU(6) algebra, followed by a brief outline of the approach to $SU(6)_w$ via current algebra and "current quarks". In the fourth section we then discuss the theoretical necessity for distinguishing between current and constituent quarks; in other words, the necessity for a non-trivial Melosh transformation. After commenting on attempts to derive a theoretical Melosh transformation, we turn in Section 5 to phenomenology. We discuss the present state of $SU(6)_w$ models for π, γ and ϱ decays of baryon resonances, and contrast the difficulties of more explicit quark models with the success of an algebraic approach based on the Melosh transformation. The concluding section contains a list of the urgent experimental questions and some of the many difficult problems remaining for theory to answer.

2. *Constituent quarks*

The symmetry SU(3) assigns hadron resonances of the same spin and parity into multiplets which are "approximately degenerate" in mass. Baryons are found to occur only in $\underset{\sim}{10}$'s $\underset{\sim}{8}$'s and $\underset{\sim}{1}$'s and Mesons only in $\underset{\sim}{8}$'s and $\underset{\sim}{1}$'s. The concept of "constituent quark"

* Address: Department of Physics, Southampton University, Southampton S09 5NH, England.

triplets of SU(3) provides a mnemonic for explaining the non-observation of other SU(3) representations. Baryons are made up of three quarks and this leads immediately to the desired result

$$B \sim qqq$$

$$\underset{\sim}{3} \otimes \underset{\sim}{3} \otimes \underset{\sim}{3} = \underset{\sim}{10} \oplus \underset{\sim}{8} \oplus \underset{\sim}{8} \oplus \underset{\sim}{1}.$$

Thus, $\underset{\sim}{27}$ representations, for example, cannot be constructed from three quarks.

Similarly, mesons are thought of as a quark-antiquark pair

$$M \sim q\bar{q}$$

$$\underset{\sim}{3} \otimes \underset{\sim}{\bar{3}} = \underset{\sim}{8} \oplus \underset{\sim}{1},$$

and $\underset{\sim}{8}$'s and $\underset{\sim}{1}$'s are the only "allowed" representations. The natural next step is to look for further regularities. In both the baryon and meson spectra one observes rough "mass bands of resonances" alternating in parity with increasing mass. (See Fig. 1). This is strongly

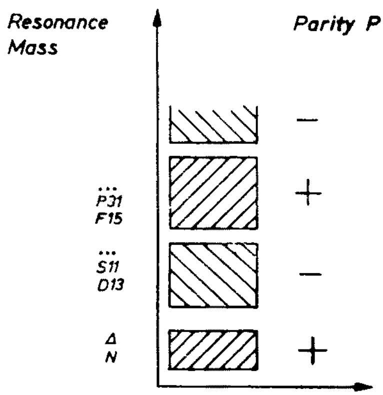

Fig. 1. Idealized view of the Baryon spectrum

reminiscent of a rotational excitation spectrum. Furthermore, in the lowest positive parity band for the baryons there is the nucleon octet with $J^P = \frac{1}{2}^+$, and the delta decuplet with $J^P = \frac{3}{2}^+$. Why do these states lie lowest?

To obtain predictions for spins and parities of resonances clearly requires more than SU(3) symmetry. The obvious first guess is to endow the constituent quark with spin $\frac{1}{2}$, and use this as a building block. We are therefore led to consider SU(6) as a possible symmetry

$$q \sim (u, d, s) \times (\uparrow, \downarrow) \sim \underset{\sim}{6}.$$

The allowed representations are now:

For the Baryons $B \sim qqq$,

$$\underset{\sim}{6} \otimes \underset{\sim}{6} \otimes \underset{\sim}{6} = \underset{\sim}{56} \oplus \underset{\sim}{70} \oplus \underset{\sim}{70} \oplus \underset{\sim}{20},$$

and under SU(3)×SU(2), the representations decompose as follows

$$\underset{\sim}{56} = {}^4\underset{\sim}{10} + {}^2\underset{\sim}{8},$$

$$\underset{\sim}{70} = {}^4\underset{\sim}{8} + {}^2\underset{\sim}{10} + {}^2\underset{\sim}{8} + {}^2\underset{\sim}{1},$$

$$\underset{\sim}{20} = {}^4\underset{\sim}{1} + {}^2\underset{\sim}{8},$$

where the superscript is $2S+1$, with S the total quark spin.

For the Mesons $M \sim q\bar{q}$

$$\underset{\sim}{6} \otimes \underset{\sim}{\bar{6}} = \underset{\sim}{35} \oplus \underset{\sim}{1}$$

and

$$\underset{\sim}{35} = {}^3\underset{\sim}{8} \oplus {}^3\underset{\sim}{1} \oplus {}^1\underset{\sim}{8},$$

$$\underset{\sim}{1} = {}^1\underset{\sim}{1}.$$

How does this constituent quark SU(6) scheme correspond to the experimental spectrum? We illustrate this for the lowest lying baryon multiplets.

Baryon spectrum

The angular momentum $\boldsymbol{J}$ of the resonance is decomposed into quark spin $\boldsymbol{S}$ and orbital angular momentum $\boldsymbol{L}$

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}.$$

For the parity, we take, as a first guess, the prescription

$$P = (-1)^L$$

(although for three body system this need not necessarily be the case [2]). The lowest state we therefore expect to have $L = 0$ and positive parity. Counting the spin and unitary spin states of the nucleon and delta multiplets leads to an astonishing result!

$$\left.\begin{array}{ll} \Delta\ {}^4 10 & = 40 \\ \mathrm{N}\ {}^2 8 & = 16 \end{array}\right\} \text{ 56 spin and unitary spin states.}$$

Thus we assign these to a $\{\underset{\sim}{56} : L^P = 0^+\}$ representation of SU(6) ⊗ O(3). Fresh from this success, if we now look in detail at the negative parity states, we find the remarkable result that *all* negative parity resonances below about 2 GeV can be assigned to a $\{\underset{\sim}{70} : 1^-\}$ representation of SU(6) ⊗ O(3). This is *not* a trivial prediction: the existence of a well established "*extra*" state would have caused the model grave difficulties. Fig. 2 shows the present state of the $Y = 0$ and $Y = 1$ states of the $\{\underset{\sim}{70}, 1^-\}$. Proceeding through the Data Tables one can identify a positive parity $\{\underset{\sim}{56}, 2^+\}$ multiplet higher in mass than the $\{\underset{\sim}{70}, 1^-\}$, and indications of a $\{\underset{\sim}{56}, 4^+\}$. Another $\{\underset{\sim}{56}, 0^+\}$ multiplet is needed to accommodate the Roper resonance, the $P11(1430)$. (In a harmonic oscillator model this is visualized as a radial excitation, ($n = 2$ $L = 0$), of the ground state, ($n = 0$ $L = 0$).) Thus from the baryon spectrum we see that it does indeed make sense to classify resonances into SU(6) supermultiplets. What of the mesons?

		$^{4}8$	
$5/2^-$	N^* (1670)	Σ (1765)	Λ (1830)
$3/2^-$	N^* (1710)	Σ (1940)	Λ (?)
$1/2^-$	N^* (1660)	Σ (?)	Λ (1670)
		$^{2}8$	
$3/2^-$	N^* (1520)	Σ (1660)	Λ (1690)
$1/2^-$	N^* (1510)	Σ (?)	Λ (?)
		$^{2}10$	
$3/2^-$	Δ (1700)	Σ (1580)	
$1/2^-$	Δ (1610)	Σ (1740)	
		$^{2}1$	
$3/2^-$			Λ (1520)
$1/2^-$			Λ (1405)

Fig. 2 $Y = 0$ and $Y = 1$ states of the $\{70, 1^-\}$

Meson spectrum

The quark-antiquark system has the same quantum number rules as the $N\bar{N}$ system

$$J = L+S,$$

$$P = (-1)^{L+1},$$

$$C = (-1)^{L+S}.$$

The ground state $\underset{\sim}{35}+\underset{\sim}{1}$ has therefore $L = 0$ and negative parity: the pion and rho meson nonets fall into the $\{\underset{\sim}{35}+\underset{\sim}{1} : L^P = 0^-\}$ multiplet of SU(6).

$$\left.\begin{array}{l} J^{PC} = 0^{-+}: \text{“}\pi\text{”}\ {}^{1}8+{}^{1}1 = 9 \\ J^{PC} = 1^{--}: \text{“}\varrho\text{”}\ {}^{3}8+{}^{3}1 = 27 \end{array}\right\} \text{ 35+1 Spin and unitary spin states.}$$

At the $L = 1$ level the picture is much less clear. States with the predicted spin and parity,

$$L^P = 1^+: J^{PC} = 1^{+-}, 2^{++}, 1^{++}, 0^{++}$$

have been observed but there are many missing states. Two views of this $L = 1$ multiplet are shown in Fig. 3. For higher L, some of the leading trajectory states (such as the $g\ 3^{--}$) have been observed, but the situation is much less satisfactory than for the baryons. This probably reflects the fact that meson spectroscopy is much more difficult than the standard “formation” baryon resonance phase shift analysis, and requires greater statistics than we have at present [3].

An optimistic summary of the Hadron spectrum is shown in Fig. 4, adapted from Rosner's [1] *Michelin Guide to the Hadrons.*

From the experimental spectrum, we nevertheless conclude that single particle (resonance) states do appear to fall into approximate SU(6) multiplets. What does this mean? It means there must exist a set of generators $\hat{W}_\alpha$ such that when acting on a resonant state in a given SU(6) multiplet, they transform it, to a good approximation, to another a)

J^{PC}	$I = 1$	$I = 1/2$	$I = 0$	$I = 0$
2^{++}	A_2	K^{**}	f	f'
1^{++}	A_1	Q_A	D	E
0^{++}	δ	κ	ε	S^*
1^{+-}	B	Q_B		

b)

J^{PC}	$I = 1$	$I = 1/2$	$I = 0$	$I = 0$
2^{++}	A_2	K^{**}	f	f'
1^{++}				
0^{++}				
1^{+-}	B			

Fig. 3. Status of $L = 1$ meson multiplet: (a) optimistic, (b) pessimistic

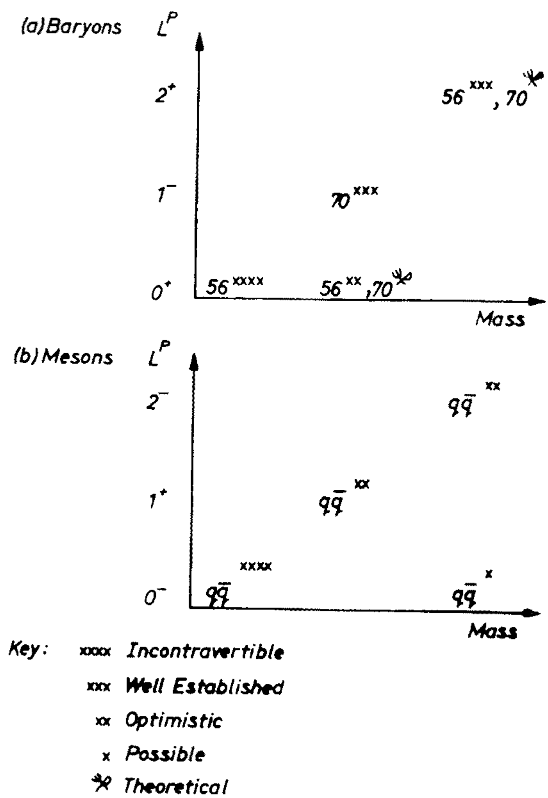

Fig. 4. SU(6)×O(3) multiplets in the Baryon and Meson spectra

state *within the same SU(6) multiplet.* This is the meaning of an approximate SU(6) symmetry

$$\hat{W}_\alpha \; |1 \text{ particle state} \rangle \approx |1 \text{ particle state in same SU(6) multiplet} \rangle$$

where the $\hat{W}_\alpha$'s satisfy an SU(6) algebra under commutation.

So far, all the discussion has been based on experimental data of resonant states of rest. Yet if there is a larger symmetry than SU(3) for the states, is it possible to have a larger symmetry than SU(3) for decay processes?
e. g. $A \to B+C$

$$\underset{C}{\longleftarrow}\!\!\underset{A}{\circ}\!\!\underset{B}{\longrightarrow} \quad \text{collinear decay.}$$

There are two snags.

(*i*) Decays necessarily involve moving particles, and in the decay of, say, a $\Delta(1700)$ into a nucleon and pion, the system is highly relativistic. Thus it is not surprising that a naive application of a basically nonrelativistic spin symmetry leads to absurd results (e. g. no $\Delta \to N\pi$, no $\varrho \to \pi\pi$). In order to have a chance, one must use a relativistic version of quark spin — W-spin [4]. This is analogous to using helicity $\lambda = \boldsymbol{J} \cdot \boldsymbol{P}/P$ for a moving particle instead of M_z, the component of spin in the rest frame. Apart from the W–S flip subtlety in the classification of the mesons, this is rather a minor modification. It means that if we want to discuss moving states we should refer to the $SU(6)_w$ classification (rather than SU(6)) of the states. For the baryons the two are the same [5].

(*ii*) More serious difficulties arise, however, in attempts to treat any spin type symmetry like an "ordinary" internal symmetry such as SU(3). Demanding both a sort of "spin invariance" and Lorentz invariance can give trouble — the infamous "No go" theorems of SU(6). In some attempts, for example, one is forced to the result that the S matrix is unity

$$S = 1,$$

i. e. the symmetry requirements are too strong to allow any interactions! Rather than delve deeper into this mire [6], we shall draw the moral that $SU(6)_w$ should be treated with care. Therefore, even though a one-nucleon state may be classified in a $\underset{\sim}{56}$ of $SU(6)_w$ we shall *not* assume that *two* particle states — which involve interactions via the S-matrix — are simply classified.

We now have an approximate $SU(6)_w$ symmetry of one-particle states — forced on us from experiment — and consequently corresponding generators $\hat{W}_\alpha$, which close on the algebra of $SU(6)_{w,\,\text{constituents}}$ i. e.

$$[\hat{W}_\alpha, \hat{W}_\beta] = C_{\alpha\beta\gamma} \hat{W}_\gamma,$$

where the $C_{\alpha\beta\gamma}$ are the structure constants of $SU(6)_w$. We may ask two questions

1) Are the generators $\hat{W}_\alpha$ physically measurable operators?

2) How can we obtain a higher symmetry scheme for two-body decays?

To answer these questions we turn to current algebra.

3. *Current quarks*

Current matrix elements are measurable quantities. For example, photo-excitation of nucleon resonances measures the square of the amplitude

$$M \sim \varepsilon^{\mu} \langle \mathrm{N}^* | J_{\mu} | \mathrm{N} \rangle$$

representing the process

$$\gamma + \mathrm{N} \rightarrow \mathrm{N}^*.$$

Similarly, neutrino excitation measures matrix elements of the weak current. The SU(3) properties of the electromagnetic current are summarized by the relation

$$J_{\mu}^{\mathrm{e.m}} = F_{\mu}^{3} + \frac{1}{\sqrt{3}} F_{\mu}^{8},$$

where F_{μ}^{i} $(i = 1 \ldots 8)$ are SU(3) currents, which integrates to give the Gell-Mann Nishijima relation

$$Q = I_3 + Y/2.$$

The weak currents in the conventional Cabibbo theory measure the 1, 2, 4 and 5 components of the vector SU(3) currents and also the same components of an octet of axial vector currents. Current algebra postulates that the equal time charges of the physical current have the same algebraic properties under commutation as the analogous charges in a quark field theory. Here, the general current is written in a very simple form

$$F_{\Gamma}^{i} \sim \bar{q} \Gamma \frac{\lambda^{i}}{2} q$$

where q and $\bar{q}$ are spin $\frac{1}{2}$ quark fields obeying the usual equal-time anticommutation relations, and Γ is the general Dirac matrix. From the vector $(\Gamma = \gamma_{\mu})$ and axial vector $(\Gamma = \gamma_{\mu}\gamma_{5})$ currents and corresponding charges came the algebra of $\mathrm{SU}(2) \times \mathrm{SU}(2)$, chiral symmetry and the Adler-Weisberger relation. After these successes, it seems sensible to find the largest possible algebra that could be abstracted from this current quark model and which could be used to classify single particle states. A proper discussion of this question leads us to "infinite momentum symmetries" or equivalently to "light-like charges". Fortunately, these have been discussed by Ruegg in his lectures at this school [7] and we therefore shall only summarize the conclusions.

In order to define a sensible approximate symmetry, in which the generators take single particle states to single particle states, and in which one avoids the problems of "vacuum break-up" and Coleman's theorems, one must use *not* the equal time charges

$$F_{\alpha} = \int J_{\alpha} d^4x \, \delta(x^0)$$

but the "light-like charges"

$$\hat{F}_{\alpha} = \int J_{\alpha} d^4x \, \delta(x^0 + x^3).$$

These are the so-called "good charges at infinite momentum".

In the quark model they have the following properties

(1) They close on an $SU(6)_w$ algebra

$$[\hat{F}_\alpha, \hat{F}_\beta] = C_{\alpha\beta\gamma}\hat{F}_\gamma.$$

This is usually referred to as $SU(6)_{w,\ \text{currents}}$.

(2) They annihilate the vacuum

$$\hat{F}_\alpha|0\rangle = 0$$

thereby avoiding problems associated with the pair states produced by non-conserved equal time charges.

(3) They have a larger stability group on the null plane, $x^+ = \frac{1}{\sqrt{2}}(x^0+x^3) = 0$, than the equal time charges have on the spacelike surface $t = 0$; namely

$$[K_3, \hat{F}] = 0,\ [P_\perp, \hat{F}] = 0,$$

$$[P^0+P^3, \hat{F}] = 0,\ [E_\perp, \hat{F}] = 0,$$

where K_i, and P_i are the familiar Poincaré boost and displacement generators, respectively, and the E boosts

$$E_1 = K_1+J_2,$$

$$E_2 = K_2-J_1$$

generate transformations within the null plane.

Thus we see that the maximal extension of current algebra leads uniquely to an $SU(6)_w$ algebra which *may* be used as an approximate symmetry algebra of one particle states. Furthermore, the generators $\hat{F}_\alpha$ are related to integrals of measurable current operators. Since we now have two $SU(6)_w$ algebras — one the experimentally observed symmetry of the resonance spectrum, $SU(6)_{w,\ \text{constituents}}$ — and the other the obvious generalization of current algebra, $SU(6)_{w,\ \text{currents}}$ — it is natural to suppose that the two are related! This brings us to the Melosh transformation.

4. *The Melosh transformation*

From the experimentally observed approximate $SU(6)_w$ multiplet structure we deduced the existence of a set of $SU(6)_w$ generators, $\hat{W}_\alpha$, that transform a one-particle state to another resonance state, within the same $SU(6)_w$ multiplet

$$\hat{W}_\alpha|1\rangle \approx |1'\rangle.$$

However, we have no knowledge as to whether the $\hat{W}_\alpha$ operators are physically measurable. On the other hand, from current algebra we know of the existence of a set of operators, $\hat{F}_\alpha$, that also generate an $SU(6)_w$ algebra, that are the integrals of measurable currents, and which can be used to classify states into approximate $SU(6)_w$ multiplets. Furthermore, by the current algebra postulate, physical charges and currents have simple transformation properties under the $\hat{F}_\alpha$'s. For example, the pion axial charge Q_π transforms as a $\underset{\sim}{35}$ with W-spin one, and z-component zero,

$$(Q_\pi)_{\hat{F}} \sim \underset{\sim}{35}(\pi;\ W = 1\ W_z = 0).$$

Similarly, the good component of the electromagnetic current ($J^+_{\text{e.m}} = J^0 + J^3$) may be shown to transform as a $\underset{\sim}{35}$ with W-spin zero.

$$(J^+_{\text{e.m.}})_{\hat{F}} \sim \underset{\sim}{35}(W = 0).$$

Thus, the states transform simply under the $\hat{W}$'s and the currents under the $\hat{F}$'s. Are the $\hat{W}_\alpha$ and the $\hat{F}_\alpha$ related? Gell-Mann suggested that they are related by a unitary transformation V [8]

$$\hat{W} = V\hat{F}V^{-1}.$$

Why can we not set $V = 1$ and identify the two sets of generators? — i. e. identify the current and constituent quarks? There are two "phenomenological" reasons why not:

(*i*) The identification $V = 1$ leads ot the prediction that the nucleon anomalous moments are zero [9]

$$\mu_{\text{N}}^{\text{A}} = 0,$$

whereas experimentally they are at least as large as the Dirac moments.

(*ii*) If $V = 1$, we can calculate the value of the axial vector coupling constant

$$|G_{\text{A}}/G_{\text{V}}| = 5/3 \sim 1.7,$$

whereas experiment yields near to 1.2. Both these results suggest that $V = 1$ is not a good *phenomenological* approximation to the world we live in — but can we find a purely *theoretical* reason for $V \neq 1$? In fact we can, and the argument is sketched below [10]. It depends crucially on the properties of the lightlike charges $\hat{F}_\alpha$.

Consider the matrix elements of the electromagnetic current between the members of the ($\underset{\sim}{35}$; $L^P = 0^-$) multiplet containing the π and the ϱ. In particular, consider the matrix elements of J^+ between ϱ^+ meson states

$$\langle \varrho^+; \boldsymbol{P}', \lambda' | J^+_{\text{e.m.}} | \varrho^+; \boldsymbol{P}, \lambda \rangle.$$

$\lambda(\lambda')$ is the eigenvalue of W_z in the initial (final) state and the spin states have been constructed from K_3 and E boots (light-like helicity states) that commute with the $\hat{F}_\alpha$'s. If $V = 1$ then current algebra demands J^+ to have $W = 0$ and thus we predict

$$\langle \varrho^+; \boldsymbol{P}', \lambda' | J^+ | \varrho^+; \boldsymbol{P}, \lambda \rangle \propto \delta_{\lambda\lambda'}.$$

Applying these constraints leads to the following condition on the three form factors describing the process

$$F_1(t) = F_2(t) = F_3(t) = 0$$

for $t \neq 0$. A quite minimal analyticity assumption would then require

$$F_1(0) = 0,$$

in contradiction with the charge normalization

$$F_1(0) = 1.$$

Thus, the hypothesis $V = 1$ is too strong to allow any sensible dynamics and must therefore be rejected.

In his thesis, Melosh [11] attempted to motivate the form of V by a study of the free quark model. After correct criticism from de Alwis and Stern [12], Melosh [13] then produced a new form of transformation motivated by spin arguments. (Since this form has similar algebraic properties to his first transformation the distinction between the two is not often stressed [14].) More recently Bell and Ruegg [15] have extended Melosh's discussion from the purely free quark framework to include a potential. These developments are discussed by Ruegg at this school.

In the remainder of these lectures we shall take a phenomenological point of view in that we shall not attempt to deduce the form of the Melosh transformation theoretically. Rather, we shall take the simple algebraic predictions of the form suggested by Melosh and compare this structure with experiment. This scheme has the most general SU(6) structure of any of the many dynamical SU(6) models available yet contains the minimal dynamical assumptions.

5. $SU(6)_w$ *phenomenology*

5.1. π decays

To apply these ideas of $SU(6)_w$ symmetry to pionic decays $A \to B+\pi$ one must resort to using PCAC in order to approximate the decay amplitude by a *single particle* matrix element

$$\langle B\pi|A\rangle \underset{\text{PCAC}}{\sim} \langle B|Q_\pi|A\rangle.$$

The states A and B transform simply under $SU(6)_{\text{w, constituents}}$ but Q_π transforms simply under $SU(6)_{\text{w, currents}}$. The Melosh transformation must now be used to find how Q_π transforms under the $\hat{W}$'s — i. e. under $SU(6)_{\text{w, constituents}}$. Melosh's suggestion was that the Q_π remains in a $\underset{\sim}{35}$ — but transforms as the sum of two representations

$$(Q_\pi)_{\hat{W}} \sim \alpha\{\underset{\sim}{35};\, W = 1\ W_z = 0;\, L_z = 0\}+\beta\{\underset{\sim}{35};\, W = 1\ W_z = \pm 1;\, L_z = \mp 1\},$$

i. e. $(Q_\pi)_{\hat{W}} \sim \alpha\{\underset{\sim}{35};\ (\text{“}\pi\text{”})\}+\beta\{\underset{\sim}{35};\ (\text{“}A_1\text{”})\}$.

In principle, there could be many other terms present (e. g. exotic $\underset{\sim}{405}$ representations) but this form is clearly the simplest phenomenological expression of the fact that V *cannot* be unity.

Application of this model to Baryon resonance decays is now straightforward. For decays into the ground state multiplet $\{\underset{\sim}{56}, 0^+\}$

$$A(J^A\lambda: LS^A) \to B(\{\underset{\sim}{56}, 0^+\}; S_B\lambda) + \pi$$

we predict the following structure

$$g_\lambda = \sum_{L_z} (LL_z S^A\lambda - L_z | J^A\lambda)\,(S^B\lambda 1 - L_z | S^A\lambda - L_z)$$

$$\times \left\{\begin{matrix} \underset{\sim}{56} & \underset{\sim}{35} \\ (\underset{\sim}{N}^B, 2S^B+1) & (\underset{\sim}{8}, 3) \end{matrix}\middle| \begin{matrix} \underset{\sim}{M}^A \\ (\underset{\sim}{N}^A, 2S^A+1) \end{matrix}\right\}_\alpha \left\{\begin{matrix} \underset{\sim}{N}^B & \underset{\sim}{8} \\ B & \pi \end{matrix}\middle| \begin{matrix} \underset{\sim}{N}^A \\ A \end{matrix}\right\}_\alpha a^{L_z},$$

$\underset{\sim}{M}^A$, $\underset{\sim}{N}^A$ and S^A denote the SU(6), SU(3) and SU(2) representations of state A: $\underset{\sim}{N}^B$ and S^B the SU(3) and SU(2) representations of state B. The first two Clebcsh-Gordan coefficients correspond to the LS coupling of the SU(6)$\otimes$O(3) wavefunction and W-spin conservation, respectively. Then follows an SU(6) coupling coefficient [16] and an SU(3) isoscalar factor: α denotes a sum over F and D type couplings in the case of octets. The amplitudes a^{L_z} are the unknown reduced matrix elements that are parameters in this model.

Firstly, we wish to emphasize the fact that this model is much more predictive than SU(3). For decays of the $Y=1$ states of the $\{\underset{\sim}{70}; 1^-\}$ to the $\{\underset{\sim}{56}; 0^+\}$, the 21 independent SU(3) coupling constants are related in terms of known SU(6) coupling coefficients and only *two* unknown parameters! Secondly, before we can apply the model to data, some choices must be made as to how best to incorporate obvious SU(3) symmetry breaking effects — such as the mass differences of π, K and η — when we extend the model to include Y* decays. Furthermore, since our SU(6) multiplets are by no means mass degenerate, the question of barrier factors is important for numerical agreement. For example, Gilman, Kugler and Meshkov [17] retain a factor $(M_A^2 - M_B^2)$ in the amplitude arising from PCAC, and use the *same* barrier factor for decays with *different* partial waves. Phenomenologically, some dependence on 3-momentum p which varies with the partial wave angular momentum l, such as p^l, seems to give best agreement [18, 19]. Finally, since we have N* and Y* states of the same spin and parity in the $\{\underset{\sim}{70}, 1^-\}$ multiplet, there is the possibility of mixing between the pure SU(6) states. All of these details are discussed more fully in the literature, along with detailed discussion of the data and full details of the fits [19, 20]. Here, we concentrate instead on highlighting some general features of these SU(6)$_w$ fits which are largely independent of these problems.

There are two types of SU(6)$_w$ predictions — amplitude signs and magnitudes.

(1) Amplitude signs [21]

For N_1^* and N_2^* in the same SU(6) multiplet the model predicts the relative signs of inelastic amplitudes i. e. A_1/A_2 where

$$A_1 \text{ for } \pi N \to N_1^* \to \pi\Delta,$$

$$A_1 \text{ for } \pi N \to N_2^* \to \pi\Delta.$$

Thus, a quick check of the number of non-trivial sign predictions for the $\{\underset{\sim}{70}, 1^-\}$ and $\{\underset{\sim}{56}, 2^+\}$ decays yields a total of 16 in agreement with the $SU(6)_w$ model and, at present, no obvious sign discrepancies [20]. Suffice it to say that 2^{16} is a large number!

(2) Magnitudes

The detailed fits to experimental amplitudes $\sqrt{xx'}$ are generally quite good (see Refs [19] and [20]) and here we demonstrate this in two ways:

(*i*) In order to assess the significance of the $SU(6)_w$ fits we have attempted to fit data using random numbers (normalized between ± 1) in place of the SU(6) Clebsch-Gordan coefficients in our theory predictions. (We of course keep the SU(3) coefficients.) If good fits could be obtained in this way, the success of SU(6) would obviously be much less convincing. The results of such fits to $\{\underset{\sim}{56}, 2^+\}$ decay are shown in Fig. 5. The SU(6) coefficients gave a χ^2 of 34 compared to the best "random fit" with a χ^2 of 94. We must therefore conclude that the SU(6) agreement is indeed significant.

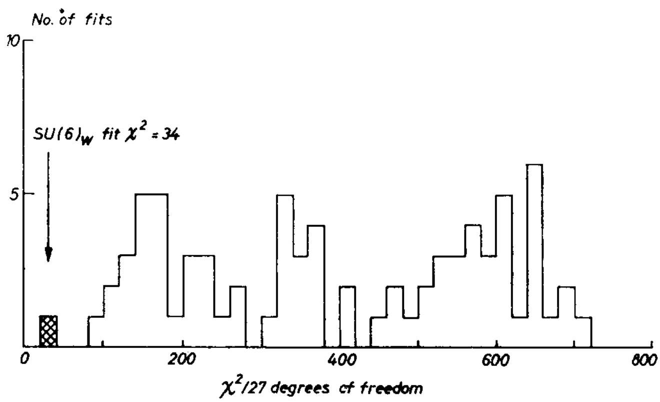

Fig. 5. Random number fits to the $\{56, 2^+\}\,\pi$ decays

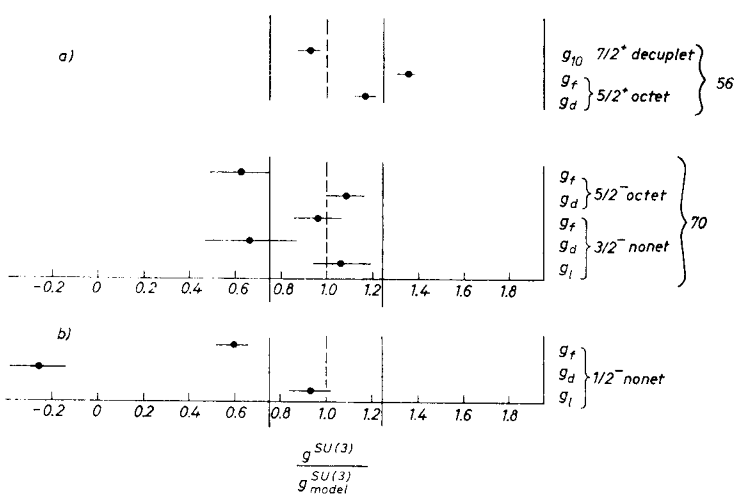

Fig. 6. Comparison of SU(3) couplings vs SU(6) predictions

(*ii*) It is interesting to compare the results of our $SU(6)_w$ analysis of the $\{\underset{\sim}{70}, 1^-\}$ and $\{\underset{\sim}{56}, 2^+\}$ multiplets with more familiar SU(3) analyses. In Fig. 6a we attempt to give an idea of the relative success of these two types of fits by plotting the ratio

$$R = g_{\mathrm{Expt}}^{\mathrm{SU(3)}} / g_{\mathrm{Model}}^{\mathrm{SU(3)}}$$

for members of both these multiplets. The coupling constants $g_{\mathrm{Expt}}^{\mathrm{SU(3)}}$ are taken from independent analyses [22] of N*, Σ^* and Λ^* resonances in the various J^P multiplets. The $g_{\mathrm{Model}}^{\mathrm{SU(3)}}$ are calculated using the best values of the two SU(6) parameters a^{L_z} (L_z = 0 and 1) together with the predicted SU(6) D/F ratios. From the agreement in Fig. 6a where the results for the various ratios are seen to cluster around one, it is clear that our SU(6) fit (of 2 parameters to 55 data points for the $\{\underset{\sim}{70}, 1^-\}$ and 2 parameters to the 27 data points of the $\{\underset{\sim}{56}, 2^+\}$) fares quite well compared to the much less ambitious SU(3) fits. The errors shown are those quoted by the SU(3) analyses and are very probably underestimated. But, for the $\frac{1}{2}^-$ states in Fig. 6b all is not well. Our SU(3) predictions, however, were calculated *as if* the three $\frac{1}{2}^-$ resonances used in the SU(3) analysis were actually the SU(6) $^2\underset{\sim}{8}$ and $^2\underset{\sim}{1}$ states. A look at the SU(6) fit [19] gives an explanation for this failure. The states used in the SU(3) analysis, the N*(1520), Λ*(1670) and Σ*(1740) are best classified in our analysis as predominantly $^2\underset{\sim}{8}$, $^4\underset{\sim}{8}$ and $^2\underset{\sim}{10}$ respectively! *Thus an SU(3) analysis of these states is inappropriate.*

In summary then, we must conclude that there is strong evidence for $SU(6)_w$ structure in these pion decays. The amplitude signs may be categorized as follows [21]:

For the $\{\underset{\sim}{70}, 1^-\}$: "Anti $SU(6)_w$" (signs as when $a_{70}^0 = 0$).

For the $\{\underset{\sim}{56}, 2^+\}$: "$SU(6)_w$-like" (signs as when $a_{56}^1 = 0$).

Some final comments on the π-decays are in order here.

(1) Radial excitations: $\{\underset{\sim}{56}, 0^+\}_R$

These multiplets contain $^2\underset{\sim}{8}$ and $^4\underset{\sim}{10}$ states and there are two candidates for $^2\underset{\sim}{8}$ P11 nucleon resonance states.

(a) NP11(1430): The "Roper" resonance. The sign of the amplitude $\pi N \to \pi\Delta$ for the N(1430) unambiguously assigns it to a $\{\underset{\sim}{56}, 0^+\}$: other possible assignments in a $\{\underset{\sim}{70}, 0^+\}$ or a $\{\underset{\sim}{70}, 2^+\}$ yield the opposite sign to experiment. This conclusion is supported by an independent SU(6) analysis of photoproduction data [20].

(b) NP11(1750). Again the $\pi\Delta$ analysis and photoproduction indicate that if an SU(6) classification is at all appropriate, then this must belong to another $\{\underset{\sim}{56}, 0^+\}$ multiplet. Heusch and Ravndal [23], for example, with an explicit harmonic oscillator quark model spectrum in mind, assigned this to a $\{\underset{\sim}{70}, 0^+\}$ which is now ruled out by experiment.

Since both these states must be in 56 multiplets, and there is little, if any, evidence for any of the $\{\underset{\sim}{70}, 0^+\}$ and $\{\underset{\sim}{70}, 2^+\}$ states [24], the explicit energy level calculations of conventional harmonic oscillator quark models would appear to be in some trouble.

(2) Other SU(6) models for π-decays

(a) The $^3P_0$ model [25]. This model visualizes the decay as taking place via the creation of a quark-antiquark pair in a $^3P_0$ state, with vacuum quantum numbers (see

Fig. 7). In fact, if the magnitudes of the $L_z = 0$ and 1 terms are left free the SU(6) algebraic structure of this model may be shown to be identical to that of the Melosh $SU(6)_w$ model [26]. A slightly more explicit version of this model used by Le Yaouanc and co--workers [27] incorporates quark orbital wavefunctions for the states. Although yielding good agreement for the $\{\underset{\sim}{70}, 1^-\}$, the model predicts the "anti-$SU(6)_w$" signs for the $\{\underset{\sim}{56}, 2^+\}$ in contradiction with experiment.

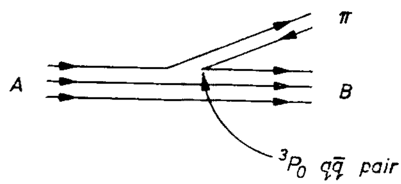

Fig. 7. $^3P_0$ model for π decay

(b) Explicit quark harmonic oscillator models. These models construct a detailed dynamics at the quark level: pion decays are characterized by a one-quark pion-emission operator H_π (see Fig. 8). For $L = 1 \to L = 0$ transitions, for example, H_π has the form

$$H_\pi = \alpha\sigma_z L_z + \beta\sigma_\pm L_\mp,$$

where the $L_{z,\pm}$ operators are orbital excitation operators. The model clearly yields the same algebraic structure as the Melosh $SU(6)_w$ model [28] but makes very specific predictions concerning the magnitudes of α and β for the various SU(6) transitions. In the version of Feynman, Kislinger and Ravndal [29], the anti-$SU(6)_w$ solution is again predicted [30] for the $\{\underset{\sim}{56}, 2^+\}$, in contradiction with experiment.

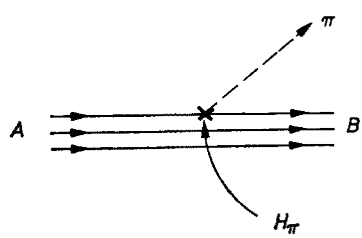

Fig. 8. Quark model of π decay

5.2. Photon transitions

Resonance photoproduction may be subjected to a similar algebraic $SU(6)_w$ analysis. For real photons there are at most two independent helicity amplitudes for resonance excitation: the amplitudes for a helicity $\frac{1}{2}$ proton to absorb $\lambda = \pm 1$ photons.

$$F_\pm \sim \langle N^* | J_\pm | N \rangle,$$

where $J_\pm$ represents the electromagnetic current operator corresponding to $\lambda = \pm 1$ photons. (For N*'s with $J = \frac{1}{2}$, there is only one amplitude). To formulate an $SU(6)_w$ model we must know the transformation properties of $J_\pm$ under $SU(6)_{w,\text{ constituents}}$. The Melosh transformation suggests the following structure [31, 32]

$$(J_\pm)_{\hat{W}} \sim A\left[\underset{\sim}{35}; \begin{matrix} W=0 \\ W_z=0 \end{matrix}; L_z=\pm 1\right] + B\left[\underset{\sim}{35}; \begin{matrix} W=1 \\ W_z=\pm 1 \end{matrix}; L_z=0\right]$$

$$+ C\left[\underset{\sim}{35}; \begin{matrix} W=1 \\ W_z=0 \end{matrix}; L_z=\pm 1\right] + D\left[\underset{\sim}{35}; \begin{matrix} W=1 \\ W_z=\mp 1 \end{matrix}; L_z=\pm 2\right]$$

The $SU(6)_w$ predictions are now a matter of straightforward Clebsch-Gordanery. Symbolically

$$\langle N^*|J_\pm|N\rangle = \sum_{\substack{A,B,\\C,D}} \begin{bmatrix} LS \\ C-G \end{bmatrix} \begin{bmatrix} W-\text{Spin} \\ C-G \end{bmatrix} \begin{bmatrix} SU(6) \\ \text{factor} \end{bmatrix} \begin{bmatrix} SU(3) \\ \text{factor} \end{bmatrix}$$

$$\times \begin{bmatrix} I-\text{Spin} \\ C-G \end{bmatrix} \langle N^*\|J_\pm\|N\rangle_{A,B,C,D}.$$

TABLE I

Photoproduction fits to {70, 1⁻} and {56, 2⁺}

a) {70, 1⁻}

	χ^2	A_{70}	B_{70}	C_{70}
1. Melosh $SU(6)_W$ Fit A_{70}, B_{70}, C_{70} free	25.4	8.3	2.2	4.2
2. Quark Model Fit $C_{70} = 0$	73.4	8.9	3.9	0
3. $^3P_0$ Model Fit $A_{70} = C_{70}$	47.3	6.6	0.6	6.6

b) (56, 2⁺)

	χ^2	A_{56}	B_{56}	C_{56}	D_{56}
1. Melosh $SU(6)_W$ Fit $A_{56}, B_{56}, C_{56}, D_{56}$ free	10.7	−6.7	−1.1	−6.8	4.6
2. Quark Model Fit $C_{56} = D_{56} = 0$	65.2	−4.8	−1.8	0	0
3. $^3P_0$ Model Fit $A_{56} = C_{56}$; $D_{56} = 0$	32.8	−5.9	−1.1	−5.9	0

The results of a detailed fit to $\gamma N \to \pi N$ analyses of the $\{\underset{\sim}{70}, 1^-\}$ and $\{\underset{\sim}{56}, 2^+\}$ resonances are shown in Table I [19, 20]. The conclusions may be summarized as.

For the $\{\underset{\sim}{70}, 1^-\}$: $A_{70} \neq C_{70}$; $C_{70} \neq 0$.

For the $\{\underset{\sim}{56}, 2^+\}$: $A_{56} \approx C_{56}$; $C_{56} \neq 0$; $D_{56} \neq 0$.

These results are sufficient to rule out other SU(6) models of photoproduction — the $^3P_0$ model [33], which predicts $A = C$ and $D = 0$, and the simplest versions of harmonic oscillator quark models [29, 34] which have $C = D = 0$. Furthermore, these harmonic oscillator quark models are unable to predict correctly the sign of the P11(1430) photoproduction amplitude. A critical appraisal of photoproduction analyses in terms of multipoles has recently been performed by Babcock and Rosner [35] who also make specific suggestions for reduction of uncertainties in the data.

Carlitz and Weyers [36] exploit the idea of an expansion of a non-local Melosh-type transformation in powers of a fundamental length, $a = 1/m$. This leads to predictions of the relative importance of the various $SU(6)_{w,\ \text{constituent}}$ terms in the transformed current operator. In particular, it leads to the prediction that the pionic transitions of the $\{\underset{\sim}{70}, 1^-\}$ should be "anti-$SU(6)_w$" whereas those of the $\{\underset{\sim}{56}, 2^+\}$ should be "$SU(6)_w$-like" — in agreement with experiment. In the application to photon transitions, the approach predicts that for the $\{\underset{\sim}{70}, 1^-\}$ transitions A_{70} and C_{70} should dominate, but for the $\{\underset{\sim}{56}, 2^+\}$ transitions B_{56} and D_{56} should give the dominant contributions. The $\{\underset{\sim}{70}, 1^-\}$ phenomenology is consistent with this picture, but although there *does* seem to be a need for a significant contribution from D_{56}, both A_{56} and C_{56} are also important. Perhaps this means that additional "exotic" pieces, such as those required by Osborn [37], are becoming important. Better data suggested by Babcock and Rosner [35] may help illuminate this situation.

5.3. Rho decays

Amplitudes for rho decays may only be treated within the framework of an $SU(6)_w$ scheme motivated by the Melosh transformation by relating them to matrix elements of the electromagnetic current. We therefore extrapolate the algebraic structure of $\lambda = \pm 1$ photon transitions from $q^2 = 0$ to $q^2 = M_\varrho^2$ and assume that the isovector portion approximates the $SU(6)_w$ structure of transverse rho meson decays. In this approach, since the π and ϱ decays of baryons involve assumptions about *different* current operators, their decay parameters need not be related in any obvious manner, unlike some other SU(6) models. For the longitudinal rho's, we have for the helicity amplitude F_0

$$F_0 \sim \langle A | J^+ | B \rangle,$$

where J^+ is the "good" $(J^0 + J^3)$ component of the electromagnetic current. Under $SU(6)_{w,\ \text{constituents}}$ J^+ is assumed to transform as [38, 20]

$$(J^+)_w \sim a^0\{\underset{\sim}{35}; W = 0 \quad L_z = 0\} + a^1\{\underset{\sim}{35}; W = 1\ W_z = \pm 1; L_z = \mp 1\}.$$

Thus in this Melosh approach there are many free parameters to fit the modest number of amplitudes measured in the Berkeley-SLAC isobar analysis [39]. Restricting the freedom by making a vector dominance calculation from the photoproduction parameter values leads to an acceptable fit, with no helicity zero rhos, although a significantly better fit is obtained if the longitudinal rho amplitude a_{56}^{0} is included. The results of this latter fit are shown in Table II. This is in contradiction to Faiman's suggestion [40] that the best fit is obtained with no longitudinal rho amplitudes.

TABLE II

Nϱ decays of the {70, 1⁻} and {56, 2⁺}

Resonance	Wave	Exp. value	Prediction
S11(~1500)	SS11ϱ_1	−.12±.0.8	−.15
D13(~1520)	DS13ϱ_3	−.32±.10	−.28
P13(~1730)	PP13ϱ_1	+.35±.10	+.17
F15(~1680)	FP15ϱ_3	−.27±.10	−.39
F35(~1860)	FP35ϱ_3	+.28±.08	+.04

The resonances below were not used in the fit since there are experimental problems in determining the sign of the resonant amplitudes. The signs in parentheses are those determined by a *T* matrix fit.

S31(~1610)	SS31ϱ_1	±.18(+)	+.36
D33(~1700)	DS33ϱ_3	±.20(−)	+.17
S11(~1660)	SS11ϱ_1	±.23(−)	−.19
D13(~1700)	DS13ϱ_3	~ 0 (+)	−.02
F37(~1920)	FF37ϱ_3	±.18(+)	−.05

In conclusion, we must warn against taking numerical values for the ϱ decay analysis too seriously. There are not enough well determined amplitudes to allow more than preliminary comparisons to be made. The more explicit quark models do, however, appear to predict some wrong signs [41], and this when coupled with the indications from the pion and photon transitions, would appear to foreshadow serious difficulties for these models in their present form.

6. *What next?*

For baryons, $SU(6)_w$ shemes have proved very successful both in the spectrum classification and for analysing π, γ and ϱ transitions. Experimentally, what is most urgently needed is more systematic data on the mesons. Firstly we must resolve the "A_1 crisis" and disentangle the resonance spectrum. This will also be of interest for the mixing patterns of the $I = 0$ mesons for different values of J^P. When we have the spectrum, the helicity structure of the $M^* \to M\pi$, $M^* \to M\gamma$ and $M^* \to M\varrho$ transitions will be vital in distinguishing between the different SU(6) models. Further knowledge of these "ordinary" mesons will undoubtedly aid our understanding of the new mesons discovered at Brookhaven and SLAC.

The theoretical challenge is to incorporate these $SU(6)_w$ successes into a realistic dynamical model, presumably based on quarks. The MIT Bag Model described by Johnson in his lectures at this school is one way to proceed. After trying for over ten years, we may be making progress on some of the theoretical questions!

The moral of these lectures should be clear. Although charm and colour schemes for the new particles have glamour, understanding — experimentally and theoretically — the spectrum of "normal" hadrons is of paramount importance for any real progress.

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