THE M.I.T. BAG MODEL*

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The M.I.T. bag model for hadron structure is reviewed.

1. Introduction

The model invented to account for the properties of elementary particles, which I will discuss, was developed at M.I.T. [1–4]. It describes the particles as composite systems with their internal structure being associated with quark and gluon field variables. In this respect it is quite conventional. We cannot relate the internal quark structure of hadrons to particles, since that would restrict the description to a non-relativistic framework. We must account for the internal structure with fields. The same fields as the ones used in conventional relativistic field theories are used in our model. However, since the fields with which we describe the substructure of the hadron will belong only to the substructure of a particle, we do not hang the field variables on all points of space as in ordinary field theory. We hang the field variables only on the subset of points which are inside of an extended particle. We call this set of points a "bag". Hence the terminology: M. I. T. Bag Model.

Since when one quantizes the amplitudes of a field, one associates the quantized amplitude with the creation and destruction operators for particles — the "quarks" are such objects in our theory. However, these constituent particles will be present only "inside" the hadron, since the creation and destruction operators are built from fields which exist only in the interior of the hadron.

Such fields as we are describing are familiar in the physics of material objects. For example, phonon fields and spin wave fields exist only on the interior points of pieces of matter and it would make no sense to think of these fields existing in all of space. We accept this easily because we recognize that fields of this type are collective variables related to the microscopic variables which characterize the material, and are "useful"

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since they describe the important dynamical parameters when the material is excited in some collective motion. However, we might find that this sort of phenomenon might also be true for what we have been accustomed to think of as "empty" space. When we locally disturb this medium, we may excite a "collective" motion of some underlying "stuff" which is present everywhere in its ground state. This collective motion will then be described by fields: which we will take to be the quark-gluon fields. The localized excitation will be a "particle", a hadron. The physics of such an extended object will then be the physics of a finite object containing the " collective fields" in our case, the quark and gluon degrees of freedom. If this picture is valid, it would make no sense to associate these variables with all of space.

2. *The bag equations*

The problem [2] is to construct a set of equations which mathematically describes such a situation and then to extract quantitative results from them. We have the picture of the hadron as given in Fig. 1. The surface of the hadron must be flexible if we are not

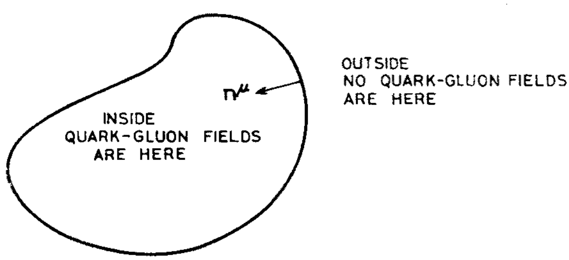

Fig. 1

to violate causality. We indicate the normal to the surface at a given point by n^μ as in Fig. 1. n^μ is a unit space-like vector. In the instantaneous rest frame of the surface point, n^m is the ordinary space normal, and n^0 is zero.

Let us suppose that the quark field is $q_a(x)$, where $q_a(x)$ is a Dirac field. The index a stands for color, (SU(3) color) and flavor (SU(3) flavor) where flavor stands for the usual SU(3) quantum numbers, u, d, s. We may later wish to supplement these with more flavors such as charm, etc. The field $q_a(x)$ is associated only with the points *inside* the hadron. It is zero outside, by definition. The local flux of these quantum numbers in the interior is

$$j^\mu_{ab}(x) = \bar{q}_a(x)\gamma^\mu q_b(x). \tag{2.1}$$

If the quantum numbers are not to be lost through the surface, then it is necessary that

$$n_\mu j^\mu_{ab}(x) = \bar{q}_a(x)\gamma \cdot n q_b(x) = 0$$

on the surface. Now, $(i\gamma \cdot n)^2 = 1$, so that $i\gamma \cdot n$ has eigenvalues ± 1. Let us assume that

$$i\gamma \cdot n q_a(x) = q_a(x) \tag{2.2}$$

on the surface (as we approach from interior points). Then it follows from (2.2) that

$$\bar{q}_a(x)\, i\gamma\cdot n = -\bar{q}_a(x)$$

and thus

$$in_\mu j^\mu_{ab}(x) = \bar{q}_a(x) i\gamma\cdot n q_b(x) = \bar{q}_a(x) q_b(x) = -\bar{q}_a(x) q_b(x).$$

Therefore,

$$n_\mu j^\mu_{ab}(x) = 0$$

and

$$\bar{q}_a(x) q_b(x) = 0$$

are consequences of (2.2). The boundary condition $i\gamma\cdot n q_a = q_a$ on a space like surface is consistent with the Dirac equation, $q_a = 0$ is not. Hence, in general, $q_a(x)$ will be discontinuous across the surface, since $q_a \equiv 0$ outside. Now let us calculate what energy and momentum will flow through the surface if (2.2) is imposed upon it. For simplicity, let us assume that the Dirac field obeys the free equation

$$\left(\gamma^\mu \frac{1}{i}\frac{\partial}{\partial x^\mu} + m\right) q_a(x) = 0 \tag{2.3}$$

inside. Then the stress tensor which describes momentum and energy flow inside is

$$T^{\mu\nu}_{\text{Dirac}}(x) = \sum_a \left[-\frac{i}{2}\bar{q}_a(x)\gamma^\mu \frac{\partial}{\partial x_\nu} q_a(x) + \frac{i}{2}\frac{\partial \bar{q}_a(x)}{\partial x_\nu}\gamma^\mu q_a(x)\right], \tag{2.4}$$

and

$$\partial_\mu T^{\mu\nu}_{\text{D}}(x) = 0.$$

However, if we want conservation of the total energy and momentum of the hadron, none should flow through the surface. This flow would be given by $n_\mu T^{\mu\nu}_{\text{D}}$ evaluated on the surface. Now

$$n_\mu T^{\mu\nu}_{\text{D}} = \sum_a \left[-\frac{i}{2}\vec{q}_a\gamma\cdot n \frac{\partial q_a}{\partial x_\nu} + \frac{i}{2}\frac{\partial \bar{q}_a}{\partial x_\nu}\gamma\cdot n q_a\right]$$

which using (2) becomes

$$n_\mu T^{\mu\nu}_{\text{D}} = \sum_a \left\{\tfrac{1}{2}\bar{q}_a \frac{\partial q_a}{\partial x_\nu} + \tfrac{1}{2}\frac{\partial \bar{q}_a}{\partial x_\nu} g_a\right\}$$

$$= \tfrac{1}{2}\frac{\partial}{\partial x_\nu}\left(\sum_a \bar{q}_a(x) q_a(x)\right).$$

We have already found that $\bar{q}_a q_a = 0$ on the surface, and hence its derivative must lie along the normal,

$$\frac{\partial}{\partial x^\nu}\left(\sum_a \bar{q}_a q_a\right) = n_\nu 2P_{\text{Dirac}}.$$

We therefore find that

$$n_\mu T_{\mathrm{D}}^{\mu\nu} = n^\nu P_{\mathrm{D}}. \tag{2.5}$$

We recognize P_{D} as a "pressure" on the surface, since in the instantaneous rest frame of the surface, the momentum flow is normal to the surface and is given by P_{D}. We can call P_{D} the Dirac pressure.

Because we want to conserve energy and momentum within the hadron we must provide a pressure to balance P_{D}. We postulate that the total energy momentum tensor for hadrons is

$$\begin{aligned} T^{\mu\nu}_{\text{Hadron}} &= T_{\mathrm{D}}^{\mu\nu} - g^{\mu\nu}B, \quad \text{inside} \\ &= 0, \qquad\qquad\quad \text{outside}. \end{aligned} \tag{2.6}$$

In (2.6), B is a universal constant with the dimensions of pressure (E/V), or in units where $\hbar = c = 1$, $B^{1/4}$ has the dimension of mass.

We can formalize this by writing

$$T_{\mathrm{H}}^{\mu\nu} = \theta_B(x)\,(T_{\mathrm{D}}^{\mu\nu} - g^{\mu\nu}B) \tag{2.6^1}$$

where $\theta_B(x)$ is 1 on the space time points occupied by the hadron and zero otherwise. In this case

$$\frac{\partial\theta_B(x)}{\partial x^\mu} = n_\mu \delta_{\mathrm{S}}(x)$$

where $\delta_{\mathrm{S}}(x)$ is a surface δ function. We find

$$\partial_\mu T_{\mathrm{H}}^{\mu\nu} = \delta_{\mathrm{S}}(x) n_\mu (T_{\mathrm{H}}^{\mu\nu} - g^{\mu\nu}B)$$

or using (2.5)

$$\partial_\mu T_{\mathrm{H}}^{\mu\nu} = \delta_{\mathrm{S}}(x)\,(P_{\mathrm{D}} - B) n^\nu.$$

We now set

$$P_{\mathrm{D}} = B \tag{2.7}$$

on the surface. Since, according to (2.5),

$$P_{\mathrm{D}} = \tfrac{1}{2} n \cdot \frac{\partial}{\partial x}\left(\sum_a \bar{q}_a q_a\right)$$

(2.7) is the same as

$$B = \tfrac{1}{2}\, n \cdot \frac{\partial}{\partial x}\left(\sum_a \bar{q}_a q_a\right). \tag{2.8}$$

If (2.8) holds on the surface, then the stress tensor (2.6^1) defined throughout all space time is conserved so the total energy momentum vector P^μ is a constant where

$$P^\mu = \int d^3x T_{\mathrm{H}}^{0\mu},$$

and the integral extends over all space in any Lorentz system. We see that in view of (2.6^1)

$$P^\mu = \int_{\mathrm{Bag}} d^3x (T_{\mathrm{D}}^{0\mu} - g^{0\mu} B).$$

We find that $-Bg^{\mu\nu}$ contributes to P^0, the energy, a term of the form BV where V is the volume occupied by the fields at any time. It makes no contribution to the momentum. The boundary condition (2.2) may be imposed on any space-like surface. (2.8) is the contour map of a space-like surface. Hence if we take as our equations (2.2), (2.3) and (2.8), we have a complete set of equations to determine the field $q_a(x)$ inside of the hadron, and also the equation of the space-like surface of the hadron. Because (2.8) is a local equation, causality will not be violated. Since the surface variables do not enter into the energy with any time derivatives, the surface variables are *not* new dynamical degrees of freedom. They are simply given as functions of the interior field $q_a(x)$.

For simplicity we have considered the example of a free Dirac field, but a realistic model must involve interacting fields. This is not because bags filled with free fields are non-interacting particles. That is, bags can fission, so the bag theory is an interacting theory from the start. It is because a realistic theory must not involve particles which carry the quantum numbers of a quark. A bag with one quark in it would be a physical quaιk. It is easy to introduce an interaction between the quarks which will prevent the existence of hadrons with the quantum numbers of a quark. We accomplish this by coupling the "color" variables of the quark to a "colored" vector field in the Yang-Mills fashion. We shall introduce the colored vector field in exact analogy to the quark field — that is it will be a variable associated with the inside of a hadron. We can call a hadron containing colored quarks and gluons a "mixed bag". If we define

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu,$$

where f_{abc} are the structure constants of SU(3) color, and we also couple the color vectors to the quarks, then

$$\partial_\mu F_a^{\mu\nu} = j_a^\nu = g(\bar{q}\lambda_a \gamma^\nu q + f_{abc} F_b^{\mu\nu} A_{c\mu}). \tag{2.9}$$

The condition analogous to $i\gamma \cdot n q_a = q_a$ for the color vector fields is

$$n_\mu F_a^{\mu\nu} = 0, \text{ on the surface.} \tag{2.10}$$

The total stress tensor is conserved [2] when

$$B = \tfrac{1}{2} n \cdot \frac{\partial}{\partial x}\left(\sum_a \bar{q}_a q_a\right) - \tfrac{1}{4} \sum_a F_{a\mu\nu} F_a^{\mu\nu}. \tag{2.11}$$

The color current (2.9) can be defined over all space with the aid of the quantity $\theta_B(x)$ defined following (2.6[1]), thus let

$$J_a^\mu(x) = \theta_B(x) j_a^\mu(x).$$

It is clear that

$$\partial_\mu J_a^\mu(x) = n_\mu j_a^\mu(x) \delta_S(x) = 0,$$

so $J_a^\mu(x)$ is conserved on all space time points. Hence the color operator C_a is a global constant where

$$C_a = \int d^3x J_a^0(x) = \int d^3x \theta_B(x) j_a^0(x). \tag{2.12}$$

If we use (2.9), (2.12) becomes

$$C_a = \int d^3x \theta_B(x) \partial_m F_a^{m0}(x)$$

and integrating by parts, with $F^{\mu\nu} \equiv 0$ outside,

$$C_a = -\int d^3x \partial_m \theta_B(x) F_a^{m0}(x) = -\int d^3x \delta_S(x) n_m F_a^{m0}(x).$$

Since $F^{\mu\nu}$ is antisymmetric,

$$C_a = -\int d^3x \delta_S(x) n_\mu F_a^{\mu 0}(x) = 0 \tag{2.13}$$

in virtue of (2.10). Hence all solutions to our theory are automatically color singlets by a simple application of Gauss' law.

Thus, the coupling of the quarks to the color gluon field automatically succeeds in producing absolute color confinement. It is easy to understand why this is so by looking

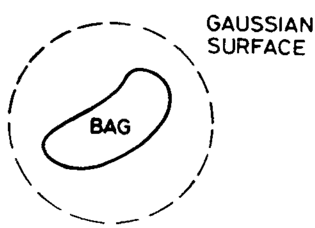

Fig. 2

at Fig. 2. Since the color fields are zero outside of the bag no color flux can penetrate a Gaussian surface which encloses the bag. Hence, C_a must vanish. It is also easy to see why a colorless bag cannot fission into oppositely colored bags by considering Fig. 3.

The mean gluon field strength in the neck is given by Gauss as gC_a/A. Hence the total gluon field energy in the neck is

$$\frac{1}{2}\left(\frac{C_a}{A}\right)^2 g^2 AL = \frac{1}{2} g^2 \frac{C_a^2}{A} L.$$

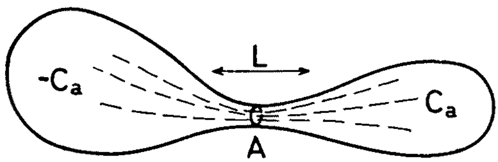

Fig. 3

This diverges when $A \to 0$, so fission (when $C_a \neq 0$) is forbidden. Color singlets must remain color singlets.

3. *Simple estimates of the parameters and consequences of the Bag Model*

a) Ground states

One would expect that in the ground state, the hadrons would be composed of quarks moving in the lowest mode in a bag [3]. We look for solutions where the shape does not change as a function of time. We assume that the gluon coupling constant is small enough that we can use perturbation theory to explore its consequences. Hence the quark field will obey the free Dirac equation. We believe that Bjorken scaling is only possible if the quark mass is low. We shall take it to be zero.

If we assume that in its rest frame the hadron is spherical, we shall find that it is possible to find such solutions to (2.2), (2.3), and (2.8). For if the bag is spherical, and the surface is static (2.2), (2.3) and (2.8) become (with $m = 0$)

$$\left(\vec{\gamma} \cdot \frac{1}{i} \vec{\nabla} + \gamma^0 \frac{1}{i} \frac{\partial}{\partial t}\right) q_a(x) = 0, \tag{3.1}$$

$$-i\gamma \cdot \hat{r} q_a = q_a, \quad \text{at} \quad r = R, \tag{3.2}$$

$$B = -\frac{1}{2} \frac{\partial}{\partial r}\left(\sum_a \bar{q}_a q_a\right) = P_{\mathrm{D}}, \quad \text{at} \quad r = R. \tag{3.3}$$

We shall proceed by finding the general solutions to (3.1), (3.2). This is the problem of the normal modes of a Dirac field confined in a spherical cavity with radius R. The quantization will proceed by quantizing the amplitudes of these normal modes, since this is the proper way to provide that the constants of motion Q_{ab} will obey the appropriate commutation rules.

The ground states of hadrons will be the states in which the quarks (and antiquarks) all occupy the mode with the lowest frequency. This mode has the frequency $\omega = \frac{2.04}{R}$ [3]. It further has the property that $\bar{q}q$ is independent of angles. Therefore if all the quarks (and/or antiquarks) occupy this mode the Dirac pressure P_D defined by (3.3) is independent of angles and time and therefore can be put equal to B as in (3.3) to determine the radius of the state. That is, we have found solutions to (3.1) through (3.3).

The mass of the hadron is given by evaluating

$$P^0 = M = \int d^3x(T_D^{00}+B). \tag{3.4}$$

If we compute (3.4), we find

$$M = n\frac{2.04}{R} + \frac{4\pi}{3}BR^3, \tag{3.5}$$

where in the first term n is the total number of quarks and antiquarks which occupy the lowest mode. The condition $P_D = B$ is equivalent to minimizing (3.5) with respect to R to obtain an equation for R. Thus,

$$0 = \underset{\substack{\uparrow \\ \text{integrated Dirac pressure}}}{-n\frac{2.04}{R^2}} \quad \underset{\substack{\uparrow \\ \text{integrated } B \text{ pressure}}}{+4\pi BR^2}. \tag{3.6}$$

If we solve (3.6), we find

$$R = \left(\frac{1}{4\pi B}\right)^{1/4}(n2.04)^{1/4}, \tag{3.7}$$

As promised, we find that the equation for the bags surface defines the surface as a function of the quark field operators (n). On substituting (3.7) into (3.5) we obtain

$$M_n = \tfrac{4}{3}(4\pi B)^{1/4}(n2.04)^{3/4} \tag{3.8}$$

for the mass of a n quark-antiquark hadron. We see that the "size" of hadron is given in terms of M_n by

$$R = \tfrac{4}{3}(2.04)n\frac{1}{M_n}.$$

Thus, the bag theory already explains why a proton ($n = 3$) is large,

$$R_p = \frac{8.16}{M_p}.$$

It is because R is a large $\left(\frac{8.16}{3}n\right)$ multiple of the Compton wavelength ($1/M$) that it is justified to treat the particle as an extended object and neglect in first approximation the quantum fluctuations associated with its center of mass motion.

We see that the mass of these hadrons is independent of the spin of the quarks. Hence a proton ($J = \frac{1}{2}$) and Δ ($J = \frac{3}{2}$) are degenerate, and a ϱ ($J = 1$) and π ($J = 0$) also have the same mass. We also see that the ratio of mesons ($n = 2$) to the masses of baryons ($n = 3$) is given by

$$\frac{M_2}{M_3} = (\tfrac{2}{3})^{3/4} = 0.74.$$

This is alright for ϱ/p, but the fact that ϱ and π have the same mass is rather bad.

If we determine B by fitting M_3 in (3.8) to the mass of the proton (0.940 GeV) we find $B^{1/4} = 96$ MeV. If we fit to the mass of the Δ (1.232 GeV) we get $B^{1/4} = 126$ MeV. Therefore, at this stage we can say that $B^{1/4} \simeq 110$ MeV$\pm 15\%$. We shall find later that when the effects of SU(6) breaking are taken into account that $B^{1/4} \simeq 145$ MeV$\pm 5\%$.

Of course since we have wave functions for the quarks we can also compute other quantities. These all turn out to be rather encouraging. Since we shall discuss these quantities in much greater detail below, we shall defer this, with one exception. If one computes the β decay G_A, one finds $G_A/G_V = 1.1$ instead of 5/3 as in the non-relativistic quark model. The experimental number is 1.24. If we give the quark a finite rest mass m, and allow it to increase, G_A/G_V increases until when $m \to \infty$, the non-relativistic result is obtained. This is why we have assumed that the quarks are massless in our zeroth approximation.

b) Regge trajectories

We next wish to consider what happens when the bag contains a lot of angular momentum, and we assume that it acquires the shape indicated in Fig. 4. In both the cases of the baryon and the meson the flux in the tubular bag is the same. This will lead

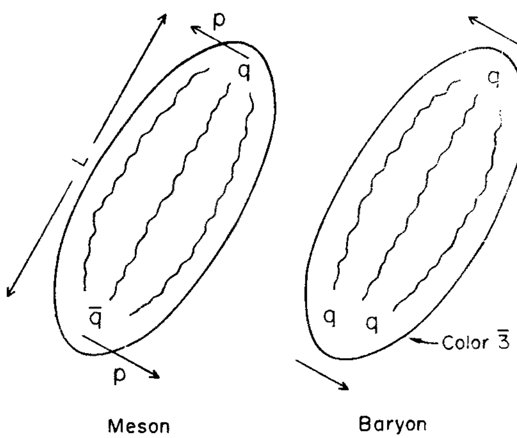

Fig. 4

to a mass spectrum for mesons and baryons which is asymptotically the same. We shall assume that the tube rotates uniformly with frequency ω and that the ends move with the velocity of light, i.e. $\omega L/2 = 1$. This is so that we get the maximum angular momentum associated with a given mass (the leading trajectory). At a distance x from the axis the tube moves with velocity $v = x2/L$. The color electric flux in the bag is

$$AE_a = g\lambda_a \tag{3.9}$$

where A is the cross section at the relevant point. The linear boundary condition, $n_\mu F_a^{\mu\nu} = 0$, on the side surfaces, or the simple application of relativity implies that

$$\vec{B}_a = \vec{v} \times \vec{E}_a \tag{3.10}$$

at a point along the surface. We shall assume that the dominant field pressure on the sides is produced by the flux lines and not the quark fields. We shall subsequently show that this is consistent. In this case the surface equation is

$$B = \tfrac{1}{2} \sum_a (E_a^2 - B_a^2)$$

which by (3.9) and (3.10) gives

$$B = \tfrac{1}{2} \frac{g^2}{A^2} (1-v^2) \tfrac{16}{3} ,$$

where $\sum_a \lambda_a^2 = \frac{16}{3}$. Therefore, the cross section at a point where the surface moves with velocity v is

$$A = \sqrt{\tfrac{8}{3}} \frac{g}{\sqrt{B}} \sqrt{1-v^2} . \tag{3.11}$$

We can now compute the total energy contained in the color flux lines and the volume energy. Thus

$$E_{\text{flux}} = \int d^3r \tfrac{1}{2} \sum_a (E_a^2 + B_a^2) = \int_0^{L/2} dx \frac{g^2}{A} \tfrac{16}{3} (1+v^2)$$

$$= \sqrt{\tfrac{8}{3}} Lg \sqrt{B} \int_0^1 dv \frac{1+v^2}{\sqrt{1-v^2}} , \tag{3.12}$$

where we have used (3.10) and (3.11). Further,

$$BV = 2B \int_0^{L/2} dxA = \sqrt{\tfrac{8}{3}} Lg \sqrt{B} \int_0^1 dv \sqrt{1-v^2} . \tag{3.13}$$

Therefore,

$$E_{\text{flux}} + BV = \sqrt{\tfrac{8}{3}} Lg \sqrt{B} \int_0^1 dv \frac{2}{\sqrt{1-v^2}} . \tag{3.14}$$

We finally must compute the energy of the quarks at the ends. We assume that the quarks are moving with momentum and energy p, so

$$E_{\text{quarks}} = 2p.$$

The angular momentum of the quarks will be

$$J_{\text{quarks}} = pL,$$

and the angular momentum of the color flux lines will be

$$\vec{J}_{\text{flux}} = \int d^3r \vec{r} \times \sum_a \vec{E}_a \times \vec{B}_a.$$

If we use (3.10), we find $\vec{E}_a \times \vec{B}_a = \vec{v}(E_a)^2$, and then if we employ (3.9) and (3.11) we find

$$J_{\text{flux}} = \sqrt{\tfrac{8}{3}}\, L^2 g \sqrt{B} \int_0^1 dv \frac{v^2}{\sqrt{1-v^2}}.$$

The total angular momentum is then

$$J = J_{\text{quarks}} + J_{\text{flux}} = pL + \sqrt{\tfrac{8}{3}}\, L^2 g \sqrt{B} \int_0^1 dv \frac{v^2}{\sqrt{1-v^2}}.$$

Therefore, the energy of the quarks is

$$E_{\text{quarks}} = 2p = 2\frac{J}{L} - \sqrt{\tfrac{8}{3}}\, Lg \sqrt{B} \int_0^1 dv \frac{2v^2}{\sqrt{1-v^2}}. \tag{3.15}$$

Consequently, the total energy of the fields and volume is gotten from (3.14) and (3.15), so

$$E = 2\frac{J}{L} + \sqrt{\tfrac{8}{3}}\, Lg \sqrt{B}\, 2 \int_0^1 dv \sqrt{1-v^2} = 2\frac{J}{L} + \sqrt{\tfrac{8}{3}}\, Lg \sqrt{B} \frac{\pi}{2}. \tag{3.16}$$

The boundary condition at the ends which corresponds to L being time independent is obtained by minimizing E with respect to L for a fixed total angular momentum J. Hence, we find

$$2J = \sqrt{\tfrac{8}{3}}\, g \sqrt{B} \frac{\pi}{2} L^2. \tag{3.17}$$

When we substitute (3.17) into (3.16) we find that $E = M$ can be written in the form

$$J = \alpha' M^2.$$

Thus, we have an asymptotically linear trajectory with the slope

$$\alpha' = \tfrac{1}{16} \sqrt{\tfrac{3}{2}} \frac{1}{\pi^{3/2}} \frac{1}{\sqrt{B}} \frac{1}{\sqrt{\alpha_c}},$$

where $\alpha_c = g^2/4\pi$ is the color gluon coupling constant. If we substitute in the best values for B and α_c which are gotten by fit ing the low lying hadron states (see below), $(B)^{1/4} = 146$ MeV, and $\alpha_c = 0.55$, we get

$$\alpha' = 0.88\ (\text{GeV})^{-2}$$

in remarkable agreement with the experimental value of 0.9 $(\text{GeV})^{-2}$.

It is also interesting to remark that for the value of L given by minimizing (3.17), we find that $E_{\text{quarks}} = 0$, that is, the quark energy is small in comparison to J/L. All of the angular momentum of the long hadron is carried by the colored flux lines. As can be seen from (3.12) and (3.13), 3/4 of the energy is carried by the flux lines and 1/4 by the term BV. It is generally true in the bag model that 1/4 of the mass is carried by BV when the constituents are all massless. Since the quarks do not carry any appreciable energy, the bag model realizes a picture which is qualitatively the same as the dual string model for long hadrons with sufficiently high values of J. (That is, the string is not "weighted" on the ends.)

In the above calculation we assumed that the dominant field pressure on the sides of the long hadrons was contributed by the flux lines. This is true if the pressure of the quark field is negligible in comparison. The quark fields are localized near the ends with a wavefunction confined to a transverse dimension of the order $\sqrt{A}$. Hence the quark energy will be of the order $1/\sqrt{A}$. If this is small in comparison with the flux energy then the surface pressure will come dominantly from the flux. Thus our calculation is consistent if

$$E_{\text{flux}} \gg \frac{1}{\sqrt{A}}\,.$$

If we express this relation in terms of J and α_c using the formulas given above we find that

$$\alpha_c J \gg 1$$

is the condition of J and α_c. Thus, for any $\alpha_c \neq 0$, eventually the trajectory will become linear. In our case, the trajectory will certainly be linear when J is large enough so that our classical calculation makes sense.

c) Collisions of hadrons at high energies

I would now like to briefly discuss a picture for the scattering of hadrons which has been recently developed by Low [5]. He calls it a model of the bare pomeron.

At high energies in the impact picture we imagine the scattering of two bags as in Fig. 5. After the collision the bags assume the deformed configuration because the quarks

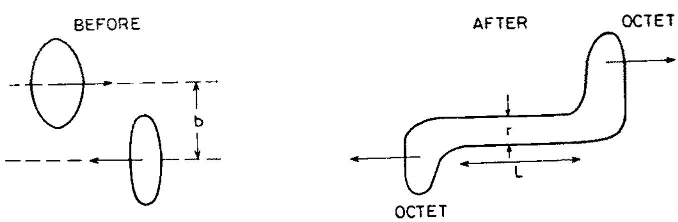

Fig. 5

have exchanged a single colored gluon. Therefore the fast moving quarks in both hadrons now form color octets and cannot separate since they are not color singlets. We see immediately that elastic scattering is impossible in lowest order. Therefore in general the scattering will be inelastic, and the elastic scattering will therefore be mainly diffractive, the shadow of the inelastic processes. All of this is in qualitative agreement with the empirically known properties of the elastic hadron scattering amplitude. Furthermore, we may calculate the elastic scattering by considering the second order process of two color gluon exchange between the quarks. Since the gluons are vector particles this gives rise to an elastic amplitude which is constant at high energies, and is purely imaginary, also in approximate agreement with what is found experimentally. It further confirms the purely diffractive character of the elastic scattering. We may therefore call this model a simple picture of the "bare" pomeron. There is no space for a discussion of the details of this work, for that one is referred to the paper of Low [5]. We would however like to mention some qualitative aspects of the inelastic processes associated with the picture of the scattering given in Fig. 5.

Just as in the picture for the high spin state of two quarks, the neck which is produced by the scattering will contain colored flux and an energy which is given by

$$\mathscr{E} = ALB + AL\tfrac{1}{2}E_a^2,$$

where the flux is given by

$$AE_a = g\lambda_a,$$

so

$$\mathscr{E} \cong ALB + AL\tfrac{1}{2}g^2\frac{1}{A^2}. \tag{3.18}$$

We determine A by minimizing (3.18) with respect to A to find

$$A \cong \left(\frac{g^2}{2B}\right)^{1/2}, \tag{3.19}$$

and

$$\mathscr{E} = 2L\sqrt{B}\left(\frac{g^2}{2}\right)^{1/2}.$$

The flux line will break by creating a pair of diquarks or gluons near the collision center, this will happen when

$$\mathscr{E} \cong 2\omega, \tag{3.20}$$

where ω is the minimum transverse energy of the quark-antiquark pair or gluon in one bag which is of the order of $\frac{1}{\sqrt{A}}$, thus

$$\frac{1}{\sqrt{A}} \cong L\sqrt{B}\left(\frac{g^2}{2}\right)^{1/2} \tag{3.21}$$

or inserting (3.19) we find

$$L \cong \frac{1}{B^{1/4}}\left(\frac{2}{g^2}\right)^{3/4}. \tag{3.22}$$

The time in the center of mass for the break to occur will be of the order of L and since this is independent of the energy of the particles, it will correspond to a long time in the laboratory system. This may possibly provide the basis for an understanding for the results of recent experiments on the scattering of baryons on nuclei [6].

Finally, we may estimate the inelasticity associated with the scattering. Let $\vec{p}'$ and E' be the momentum and energy of the rapidly moving quarks in one bag, and $\vec{q}$ and ω the momentum and energy of the slowly moving quark-antiquark or gluon in the bag after the neck has broken. Then the mass will be

$$M_1^2 = (E'+\omega)^2-(\vec{p}'+\vec{q})^2 \cong 2E'(\omega-\vec{v}\cdot\vec{q}) \cong 2E(\omega-\vec{v}\cdot\vec{q}), \tag{3.23}$$

where E is the energy of the incident hadron and $2E = \sqrt{2s}$. Then the initial stretched bag with mass $M_0 = \sqrt{2s}$ has split into two bags each of mass

$$M_1^2 \cong M_0(\omega-\vec{v}\cdot\vec{q}) \cong M_0 C_1^2,$$

where C_1 is of order unity. Since each of those bags will contain slow moving quarks in a color octet, the neck will continue to stretch until another break occurs. This break will again be similar to the first so

$$M_2^2 \cong M_1 C_2^2 \cong \sqrt{M_0}\, C_1 C_2^2.$$

This process will continue until M_n^2 is of the order of a typical hadron mass, that is,

$$m^2 \cong (M_0)^{\frac{1}{2(n-1)}} C_1^{\frac{1}{(n-1)}} C_2^{\frac{1}{(n-2)}} \ldots C_n^2.$$

Since $M_0 \cong \sqrt{2s}$, and $2(n-1) = N$ is the multiplicity we find that

$$k \log s \cong N,$$

where k is a factor of order unity.

Therefore, the bag model together with the color exchange interaction between quarks leads to logarithmic multiplicity for particle production in inelastic hadron scattering.

4. *Masses and other parameters of the light hadrons*

It will be our purpose in this part to refine the picture developed in the previous section [7]. However, we shall only be concerned with the masses and other static parameters of what we call the "light" hadrons. These are the pseudoscalar and vector meson nonets, the baryon octet and decuplet. In these hadrons the quarks all occupy the lowest mode in the spherical cavity.

To refine our calculation we shall include three major ingredients. The first concerns the quark mass. We shall break the SU(3) symmetry by assuming that the strange quark has a mass m_s different from the mass of the non-strange quarks, m. We shall find that no appreciable improvement on the description we finally obtain will be gotten by making the mass of the non-strange quarks different from zero. In any case we shall assume that m is small in units of $\frac{1}{R}$, where R is the radius of a typical hadron.

The second ingredient we shall include will concern the effects of the zero point motion of the fields occupying the hadron. In the previous section, when we calculated the energy of the constituent field of the hadron, we included only the energy of the occupied modes, the so-called valence quarks. Since the fields which occupy the hadron are quantized, they will also have a zero point energy associated with them which takes the form $\frac{1}{2}\sum\hbar\omega$ for the boson constituents and $-\frac{1}{2}\sum\hbar\omega$ for the fermion constituents. In conventional field theory these zero point energies are infinite, and "extensive", that is, they correspond to an infinite term proportional to $g^{\mu\nu}$ in the stress tensor. In conventional field theory the volume occupied by the fields does not change in any process, and hence this divergent term is discarded. In our model, the fields occupy a finite volume, and we see that an additional term proportional to $g^{\mu\nu}$ in the stress tensor will simply correspond to a redefinition of B. Since this effect is divergent, we shall adopt the standard procedure used to handle such divergences which is used in field theory. We shall introduce a cut-off. We shall then study the dependence of the zero point energy on the cut-off when it is taken to be large in comparison to any scale in our model. We incorporate cut-off dependent quantities into so-called renormalization constants. The cut-off insensitive results will be the reward. We shall find that in addition to the renormalization of B, the second effect of the zero point motion will be to add to the energy a cut-off independent term proportional to $\frac{1}{R}$ which is the same for all hadrons. It is this latter term which will produce another refinement of our previous mass formula.

The third ingredient which we shall include is the quark gluon coupling. We shall treat its effects in the lowest order of perturbation theory. It will produce an SU(3) non--invariant quark spin-spin interaction which will break SU(6) invariance. The spin-spin coupling will be non-SU(3) invariant because of the difference in the quark masses.

Let us first discuss the consequences of introducing a quark mass. We must solve the Dirac equation

$$\left(\vec{\gamma}\cdot\frac{1}{i}\vec{\nabla}-\gamma^0\omega+m\right)q = 0 \tag{4.1}$$

with the boundary condition

$$-i\gamma\cdot\hat{r}q = q, \quad \text{at} \quad r = R. \tag{4.2}$$

If we study the frequency of the lowest mode, and we write

$$\omega = \left(m^2+\frac{x^2}{R^2}\right)^{1/2}, \tag{4.3}$$

then the momentum in units of $\frac{1}{R}$ is x where

$$x = x(mR)$$

is the solution of the transcendental equation

$$\tan x = \frac{x}{1-mR-(m^2R^2+x^2)^{1/2}} \cdot \tag{4.4}$$

In the previously studied massless limit, $x(0) = 2.04$. As $m \to \infty$, $x(mR) \to \pi$, which is the limit in which we obtain the non-relativistic Schrödinger equation. A graph of the solution

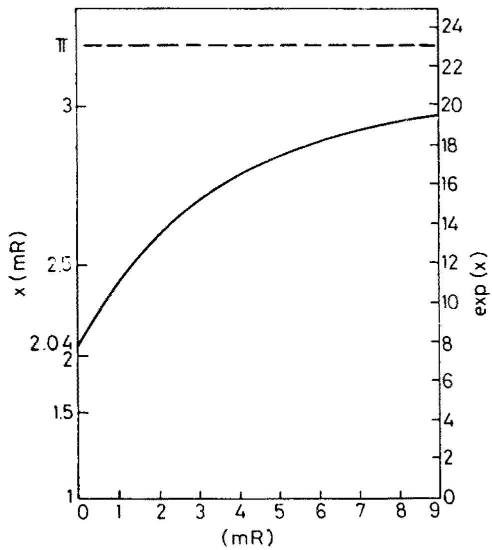

Fig. 6

of (4.4) as a function of mR is given in Fig. 6. Thus, with quark masses, the field energy of the occupied mode is now

$$n\left(m^2+\frac{x^2}{R^2}\right)^{1/2}+n_s\left(m_s^2+\frac{x_s^2}{R^2}\right)^{1/2}, \tag{4.5}$$

where n = number of u, d quarks (and, or antiquarks) and n_s refers to the strange quarks. With only such mass-kinetic energy effects, the strange states will be split from the non--strange, but Λ and Σ will remain degenerate.

Let us next consider the zero point energy associated with the quark and gluon fields [8]. We shall introduce a cut-off function $f_c(\omega/\Omega)$ where $f_c(0) = 1$. The finite results we shall obtain can be shown to be independent of the form of f_c and it is most convenient to use an exponential, $f_c = e^{-\omega/\Omega}$. Hence, we shall carry out the discussion for the cut-off zero point energy,

$$E_0 = \tfrac{1}{2}\sum \omega e^{-\omega/\Omega}. \tag{4.6}$$

We shall be interested in (4.6), when $\Omega \to \infty$, or when $(1/\Omega) \ll$ any relevant time scale. For convenience, we put $(1/\Omega) = \tau$, and consider (4.6) in the limit $\tau \to 0$. It will be amusing to first study a mathematical example, a one (space) dimensional field theory. This is provided, for example, by the dual string model. We shall present a rather simplified (probably over simplified) version.

Imagine a one dimensional bag, i.e. a string, which consists of a set of d scalar fields ϕ_i which correspond to the derivatives of the transverse coordinates of the string in $d+2$ dimensional space-time. They obey the one dimensional wave equation, $\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2}\right)\phi_i = 0$, and in the rest system of the center of mass, we imagine that the ends are at $x = 0$ and $x = L$ and here $\phi_i = 0$. The transverse modes are then associated with the wave functions $\sin(n\pi x/2)$ and frequencies $\omega_n = n\pi/L$. The cut-off zero point energy then has the form

$$E_0 = \frac{d}{2} \sum_{n=1}^{\infty} n \frac{\pi}{L} e^{-n\frac{\pi}{L}\tau}$$

$$= -\frac{d}{2} \frac{d}{d\tau} \left(\frac{1}{1-e^{-\pi\tau/L}}\right).$$

In the limit as $\tau = (1/\Omega) \to 0$,

$$E_0 \to -\frac{d}{2} \frac{d}{d\tau} \left(\frac{L}{\pi\tau} + \tfrac{1}{2} + \tfrac{1}{12} \frac{\pi\tau}{L} + \ldots\right)$$

$$= d \frac{L}{2\pi} \Omega^2 - \frac{\pi}{L} \frac{d}{24}.$$

The mass of the system is given by minimizing,

$$M = \sum_i \sum_{n=1}^{\infty} \frac{n\pi}{L} (a_n^i)^\dagger (a_n^i) + \left(B_0 + \frac{d}{2\pi} \Omega^2\right) L - \frac{\pi}{L} \frac{d}{24},$$

with respect to L. $a_n^{i\dagger} a_n^i$ corresponds to the occupation number of the n^{th} mode of the i^{th} transverse motion. We take $B_0 + d\frac{\Omega^2}{2\pi}$ to be B_{ren}, the renormalized string constant. When we minimize we get the mass formula

$$M^2 = 2\pi B_{\text{ren}} \left(\sum_i \sum_{n=1}^{\infty} n (a_n^i)^\dagger (a_n^i) - \frac{d}{24}\right).$$

This is the mass formula for the string. We see that the zero point energy produces a negative contribution of magnitude $-d/24$. It turns out that this constant must be equal to 1 in the string model in order to obtain a consistent theory of interacting strings. Hence, the famous requirement of 24 transverse dimensions or a 26 dimensional space time [9].

We are interested in the zero point energy of the quark and gluon fields in a sphere in three dimensions. This problem is presently being worked out. Since the solution is so far not available, we must content ourselves with an estimate based upon the only three dimensional geometry in which the calculation can be done analytically. This is the geometry of a slab.

For a vector field [10], there are two types of modes, transverse electric and transverse magnetic. For the color field boundary condition, $n_\mu F_a^{\mu\nu} = 0$, these are opposite to what they are for conducting plates. The frequencies are the same for both modes,

$$\omega = \left(k_T^2 + \left(\frac{n\pi}{L}\right)^2\right)^{1/2}. \tag{4.7}$$

The difference is that the value $n = 0$ is excluded in one case. $\left(\text{The case where the longitudinal wave function is } \sin\frac{n\pi x}{L}\right)$. In (4.7), k_T stands for the transverse momentum.

For the massless Dirac field, the frequencies are

$$\omega = \left(k_T^2 + (n+\tfrac{1}{2})^2\frac{\pi^2}{L^2}\right)^{1/2}.$$

Therefore, for the zero point energy, we obtain in these cases

$$\frac{E_0}{A} = \tfrac{1}{2}\left[2\sum_{n=1}^{\infty}\int\frac{d^2k_T}{(2\pi)^2}\left(k_T^2 + n^2\frac{\pi^2}{L^2}\right)^{1/2} e^{-\left(k_T{}^2 + \frac{n^2\pi^2}{L^2}\right)^{1/2}\tau} + \int\frac{d^2k_T}{(2\pi)^2}|k_T|e^{-|k_T|\tau}\right],$$

(Vector)

$$\frac{E_0}{A} = -\tfrac{1}{2}\,4\left[\sum_{n=0}^{\infty}\int\frac{d^2k_T}{(2\pi)^2}\left(k_T^2 + (n+\tfrac{1}{2})^2\frac{\pi^2}{L^2}\right)^{1/2} e^{-\left(k_T{}^2 + (n+\frac{1}{2})^2\frac{\pi^2}{L^2}\right)^{1/2}\tau}\right]. \tag{4.8}$$

(Dirac)

The factor 4 in the case of Dirac field is associated with the spin and antiparticle multiplicity. The transverse momentum integration can be done by putting $\omega e^{-\omega\tau} = \left(\frac{\partial}{\partial\tau}\right)^2\left(\frac{1}{\omega}e^{-\omega\tau}\right)$. If we carry out the integration, we then can sum over n just as in the one dimensional example. We find

$$\frac{E_0}{A} = \left(\frac{d}{d\tau}\right)^2\frac{1}{2\pi\tau}\left(\frac{1+e^{-\frac{\pi\tau}{L}}}{1-e^{-\frac{\pi\tau}{L}}}\right), \tag{4.9}$$

(Vector)

$$\frac{E_0}{A} = -4\left(\frac{d}{d\tau}\right)^2 \frac{1}{2\pi\tau}\left(\frac{e^{-\frac{\pi\tau}{2L}}}{1-e^{-\frac{\pi\tau}{L}}}\right). \tag{4.10}$$

(Dirac)

The factor $1+e^{-\frac{\pi\tau}{L}}$ in the numerator results from the two types of modes present in the vector case. The term with 1 coming because the $n = 0$ mode is included, $e^{-\frac{\pi\tau}{L}}$ because the $n = 0$ mode is excluded. The important thing to note is that the expression is even in τ. The same evenness is true in the Dirac case. In contrast if we had a scalar field present, we should have obtained an expression which was not even in τ, since a scalar field would be equivalent to having only one of the two types of modes present in the vector case. The point of making this remark about evenness in τ is seen if we study (4.9) or (4.10) in the limit as $\tau \to 0$ $(\Omega \to \infty)$. We obtain an expression of the form

$$\left(\frac{d}{d\tau}\right)^2 \left(\frac{A}{\tau^2} + B + C\tau^2 + \ldots\right) = 6A\Omega^4 + 2C.$$

So there is only a divergence proportional to Ω^4, *no lower order cut-off dependent terms arise*. On the other hand if the expressions were not even in τ a term proportional to Ω^3 would also be present. This occurs for scalar fields, but as we have seen, not for vector or Dirac fields.

It can be shown that the absence of the Ω^3 term is a general result, true for cavities with an arbitrary shape. Furthermore, the divergent term is the same for cavities of arbitrary shape, it corresponds to a divergence in the local stress tensor proportional to $+g^{\mu\nu}$, and as we have already remarked, will correspond to a renormalization of B.

If we work out the explicit expressions for (4.9) and (4.10) in the limit as $\tau \to \frac{1}{\Omega} \to 0$, we find

$$\frac{E_0}{V} = \Omega^4 \frac{3}{\pi^2} - \frac{\pi^2}{720}\frac{1}{L^4} \quad \text{vector field,}$$

$$\frac{E_0}{V} = -\Omega^4 \frac{6}{\pi^2} - \frac{7}{4}\frac{\pi^2}{720}\frac{1}{L^4} \quad \text{Dirac field.} \tag{4.11}$$

We have put $V = AL$, as the volume of a cross section of the slab.

To obtain a crude estimate of E_0 in the problem of interest, namely 8 colored vectors and 3 colors $\times$ 3 flavors of Dirac fields confined in a spherical cavity with radius R, we let $L \to R$ and $V = \frac{4\pi}{3} R^3$ above and thus obtain

$$E_0 = \Omega^4 \frac{3}{\pi^2}(8-6\times 9)V - \frac{\pi^3}{540}(8+\tfrac{7}{4}9)\frac{1}{R},$$

or for the renormalized zero point energy

$$E_0^{\text{ren}} = BV - \frac{1.36}{R}. \tag{4.12}$$

However, since we have not calculated E_0 for a spherical cavity, we shall merely put in an extra term of the form $-z/R$ in the expression for the energy of the confined quark and gluon fields and treat z as a phenomenological parameter. We shall find that we obtain the best fit to the mass spectrum with $z \approx 2$ which is in agreement with the sign and order of magnitude of the result obtained above.

The third ingredient which we incorporate with the improved calculation is the color vector fields interaction between the quarks [11], which we shall compute to lowest order in $g_c^2/4\pi = \alpha_c$.

Since the valence quarks all occupy the lowest mode in the cavity, their color currents are static and therefore the colored fields that they generate are static. The color fields will obey the boundary condition on the surface,

$$\hat{r} \cdot \vec{E}^a = 0,$$
$$\hat{r} \times \vec{B}^a = 0, \tag{4.13}$$

where $\vec{E}^a$ is the color electric fields, and $\vec{B}^a$ the color magnetic field. Clearly the first condition can be met only if the hadron is a color singlet. This was shown in general in Section 2.

The energy associated with the color gluon interaction can be separated into "color electric" and "color magnetic" energies and written in the form $\Delta E_{\mathrm{E}} + \Delta E_{\mathrm{M}}$, where

$$\Delta E_{\mathrm{E}} = \tfrac{1}{2} g_c^2 \sum_a \int_{\text{Bag}} d^3x \vec{E}^a(x) \cdot \vec{E}^a(x), \tag{4.14}$$

and

$$\Delta E_{\mathrm{M}} = -\tfrac{1}{2} g_c^2 \sum_a \int_{\text{Bag}} d^3x \vec{B}^a(x) \cdot \vec{B}^a(x). \tag{4.15}$$

These are calculated by solving Maxwell's equations using the total static currents of the valence quarks and the boundary conditions (4.13), on the surface. In doing this we should

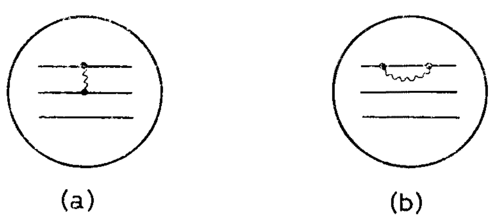

Fig. 7

remark on what additional approximations are involved. There can be expressed most clearly if we consider what diagrams correspond to the above forms for ΔE_{E} and ΔE_{M}. The relevant diagrams in this order are illustrated in Fig. 7 where the lines correspond to a typical valence quark. In the case of the self-energy diagram, the static current corresponds to keeping in the internal quark propagator only the lowest cavity mode.

Now, in ordinary atomic calculations, it would not be sensible to make such an approximation. However, in our problem we are forced to do this because if we do not include this part of the self-energy diagram, it is impossible to satisfy the boundary condition $\hat{r} \cdot \vec{E}^a = 0$ for the color electric field on the surface. We of course should then also take the rest of the self energy diagram and compute it. We shall not do this however. There is some justification for proceeding as we have. In contrast to the situation in atoms, the quarks in our system are moving relativistically. Chodos and Thorn [12] have found, when calculating the entire electromagnetic self-energy diagram in the case of massless quarks, that about 80% of the full result comes from just the static term where the quark remains in the lowest cavity mode. The problem that we have just referred to does not arise when we consider the color magnetic field. We therefore shall not include any of the self-energy diagram in our calculation. The reason that we have treated the magnetic term differently from the electric is that in the magnetic part of the self energy diagram the quark spin is opposite to what it was in the valence state. Hence, in the magnetic diagram, we have an intermediate state for the hadron where the total quark spins are different from the initial state. We shall soon find that this intermediate state will have a quite different energy as a consequence of the magnetic interaction with other valence quarks. Therefore, if we go to the next order, the quark current will not remain static. This situation does not arise in the case of the static color electric part of the self energy diagram. Thus, although it is not totally consistent, we feel the best physics will correspond to the approximation we have made. We have found that if treat the magnetic part of the self energy diagram in the same way as the electric, no qualitative difference in the final results are produced. The main effect is just to alter the values of the parameters which we fit. Thus, we now write for the final form of (4.14) and (4.15)

$$\Delta E_{\mathrm{E}} = \tfrac{1}{2} g^2 \sum_a \int d^3x \vec{E}^a(x) \cdot \vec{E}^a(x), \tag{4.16}$$

and

$$\Delta E_{\mathrm{M}} = -\tfrac{1}{2} g^2 \sum_{i \neq j} \sum_a \int d^3x \vec{B}_i^a(x) \cdot \vec{B}_j^a(x). \tag{4.17}$$

In (4.16), $\vec{E}^a(x)$ is the total color electric field of the valence quarks, and $\vec{B}_i^a(x)$ is the color magnetic field of the i^{th} quark.

Since the valence quarks all occupy the same mode, when the difference between quark masses are omitted, the color charge densities are all the same and hence $\vec{E}^a$ is proportional to the total color operator and hence vanishes. Therefore, in states where all the quarks are non-strange, or all are strange- (4.16) is zero. In states where there are both strange and non-strange quarks, (4.16) never amounts to more than about 5 MeV. Therefore, the main effect of the color gluon interaction is associated with the color-magnetic interaction (4.17). The color magnetic field is gotten by solving the equations

$$\vec{\nabla} \times \vec{B}_i^a = \vec{j}_i^a, \; r < R,$$

$$\vec{\nabla} \cdot \vec{B}_i^a = 0, \; r < R, \tag{4.18}$$

with the boundary condition on the surface

$$\hat{r} \times \sum_i \vec{B}_i^a = 0. \tag{4.19}$$

Here the current $j_i^a(x)$ is

$$\vec{j}_i^a(x) = q_i^\dagger \vec{\alpha} \lambda^a q_i \tag{4.20}$$

and $q_i(x)$ is the Dirac wave function for the valence quark.

We now solve these equations, compute ΔE_M, and we find an expression of the form

$$\Delta E_M = -\alpha_c \tfrac{1}{2} \sum_{i \neq j} \lambda_i \vec{\sigma}_i \cdot \lambda_j \vec{\sigma}_j \mu_{ij} \frac{1}{R}, \tag{4.21}$$

where μ_{ij} takes on three values, μ, μ', μ'' depending on whether we have a pair of u, d quarks, a u or d and s quark or a pair of s quarks, respectively. A graph of two functions $i(mR)$ and of $j(mR)$ is given in Fig. 8. It can be used in the case that u, d quarks are massless. In this case, $\mu = i(0) = j(0)$, $\mu' = i(m_s R)$, $\mu'' = j(m_s R)$.

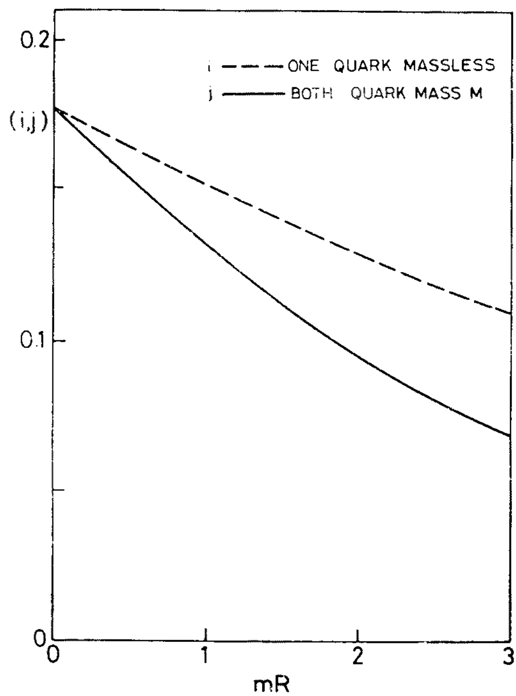

Fig. 8

We now must evaluate (4.21) in the various states of the mesons and baryons.

For a meson, we have a quark and antiquark in a color singlet state. λ_1^a is the color matrix of the quark and λ_2^a is the color matrix of the antiquark and they act on a colorless state,

$$\lambda_1^a + \lambda_2^a = 0,$$

so

$$\sum_a \lambda_1^a \lambda_2^a = \lambda_1 \cdot \lambda_2 = -\lambda^2 = -\tfrac{16}{3}. \tag{4.22}$$

Similarly, in the case of the baryon with 3 quarks,

$$\lambda_1 + \lambda_2 + \lambda_3 = 0. \tag{4.23}$$

We find by multiplying (4.23) by λ_1^a,

$$\lambda_1^2 + \lambda_1 \cdot \lambda_2 + \lambda_1 \cdot \lambda_3 = 0,$$

and the three similar equations obtained by multiplying (4.23) by λ_2 and λ_3, that

$$\lambda_1 \cdot \lambda_2 = \lambda_1 \cdot \lambda_3 = \lambda_2 \cdot \lambda_3 = -\tfrac{8}{3}.$$

Thus the meson and baryon are simple because the color "dot products" are diagonal.

We find as a consequence

$$\Delta E_{\mathrm{M}} = +\alpha_c \tfrac{16}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \frac{1}{R} \mu_\alpha$$

for mesons, where $\mu_\alpha = \mu$ for the mesons with no strange quarks (or antiquarks), $\mu_\alpha = \mu'$, if there is one strange quark (or antiquark), and $\mu_\alpha = \mu''$ for the meson with a strange quark and a strange antiquark. We see that the color magnetic interaction separates states with different total angular momenta.

In the case of baryons,

$$\Delta E_{\mathrm{M}} = \alpha_c \tfrac{4}{3} \sum_{i \neq j} \vec{\sigma}_i \cdot \vec{\sigma}_j \mu_{ij} \frac{1}{R}.$$

We see that ΔE_{M} depends upon the spin configuration as well as on whether or not there is a strange quark in the state. Hence, Σ will be split from the Λ, as well as different total spin states being split from each other.

In order to study the problem of possible exotic states, we have also evaluated the color magnetic interaction in non-strange baryons with more than three quarks. As many as twelve u, d quarks can occupy the lowest mode in a spherical hadron, since there are three colors, two spins, and two isospins. These can form color singlets in the cases of six, nine, and twelve quarks. Hence, there could be as many as three different sorts of exotic hadrons. We shall find that all of these are unstable against decay into nucleons. The width of these states would presumably be large since no quarks need be created for the decay. To evaluate (4.21) when more than three quarks form a color singlet is less straightforward than before since in this case the color dot products $\lambda_i \cdot \lambda_j$ are not diagonal. We instead consider any pair of quarks. The quark permutation operator P_{ij} should take on the eigenvalue-1 for any pair. P_{ij} can be written

$$P_{ij} = P^C_{ij} P^S_{ij} P^I_{ij},$$

where

$$P^C_{ij} = \tfrac{1}{3} + \tfrac{1}{2}\, \lambda_i \cdot \lambda_j$$

is the color permutation operator, and

$$P^S_{ij} = \tfrac{1}{2} + \tfrac{1}{2}\, \sigma_i \cdot \sigma_j, \qquad P^I_{ij} = \tfrac{1}{2} + \tfrac{1}{2}\, \tau_i \cdot \tau_j$$

are the spin and isospin permutation operators. Since for any permutation operator $P^2 = 1$, we may write for a state where $P_{ij} = -1$,

$$P^C_{ij} P^S_{ij} = -P^I_{ij}.$$

Using the explicit form for these, we find the identity

$$\tfrac{8}{3} + \lambda_i \cdot \lambda_j + \tfrac{2}{3}\, \sigma_i \cdot \sigma_j + 2\tau_i \cdot \tau_j = -(\lambda_i \cdot \lambda_j)(\sigma_i \cdot \sigma_j).$$

We may finally evaluate the color magnetic interaction by summing on $i \neq j$ and putting $(\sum_i \lambda_i)^2 = 0$ in the color singlet state, we find

$$\Delta E_n = \tfrac{4}{3}\, \alpha_c \frac{}{R}\, [n(n-6) + S(S+1) + 3I(I+1)], \tag{4.24}$$

where n is the number of non-strange quarks, and (S, I) are the total spin and total isospin of the state. It is interesting that this is negative only for the nucleon ($n = 3$, $S = \frac{1}{2}$, $I = \frac{1}{2}$). This is responsible for making all exotic $n = 6$ baryons unstable into decay into nucleons. The term $n(n-6)$ then makes the $n = 9$, and $n = 12$ states very unstable. We shall discuss the consequences of (4.24) below.

We now have all of the ingredients for our refined calculation of masses. We shall write for the mass of any hadron

$$M = \underbrace{\left[n\left(m^2 + \frac{x^2}{R^2}\right)^{1/2} + n_s \left(m_s^2 + \frac{x_s^2}{R^2}\right)^{1/2} \right]}_{\text{valence quark kinetic energy}} + \underbrace{\left[BV - \frac{Z_0}{R} \right]}_{\text{zero point energy}} + \underbrace{\Delta E_{\mathrm{M}} + \Delta E_{\mathrm{E}}}_{\text{color interaction energy}}, \tag{4.25}$$

where ΔE_{M} and ΔE_{E} have been defined above. ΔE_{M} and ΔE_{E} are evaluated in each state for the appropriate total quark spin and configuration of u, d and s quarks. We then minimize (4.25) with respect to R to determine the radius of the state, and evaluate M at the minimum to determine the mass.

The free parameters are B, m, m_s, Z_0 and α_c. We make no attempt to fit m, the mass of the non-strange quark. We simply have looked at two cases, $m = 0$, and $m \cong 100$ MeV. No appreciable difference results and for simplicity only the case $m = 0$ will be discussed here. The parameters B, Z_0 and α_c are chosen so as to fit the three heaviest states which involve no strange quarks, namely the Δ, N and ω. The ϱ and ω are degenerate in our approximation. The mass of the strange quark, m_s, is determined so that the mass of the Ω^-

TABLE I

Particle	M_{exp}	M_{bag}	R_0	Zero point energy $-Z_0/R$	BV	Quark kinetic energy	ΔE_M	ΔE_E
P	.938	.938	5.00	−.367	.234	1.226	−.155	0
Λ	1.116	1.105	4.95	−.371	.227	1.400	−.156	.005
$\Sigma^{(+)}$	1.189	1.144	4.95	−.371	.227	1.400	−.116	.005
$\Xi^{(0)}$	1.321	1.289	4.91	−.374	.222	1.572	−.136	.005
Δ	1.236	1.233	5.48	−.336	.308	1.119	.141	0
Σ^*	1,385	1.382	5.43	−.338	.301	1.292	.122	.005
Ξ^*	1.533	1.529	5.39	−.341	.293	1.465	.106	.005
Ω^-	1.672	1.672	5.35	−.343	.287	1.636	.092	0
ϱ	.77±.01	.783	4.71	−.390	.196	.868	.110	0
K^*	.892	.928	4.65	− 395	.189	1.039	.091	.004
ω	.783	.783	4.71	−.390	.196	.868	.110	0
ϕ	1.019	1.068	4.61	−.399	.183	1.207	.076	0
K	.495	.497	3.26	−.564	.065	1.407	−.415	.003
π	.139	.280	3.34	−.549	.070	1.222	−.462	0
	GeV	GeV	GeV^{-1}	GeV	GeV	GeV	GeV	GeV

$B^{1/4} = .145$ GeV, $Z_0 = 1.84$, $\alpha_c = .55$, $m_s = .279$ GeV.

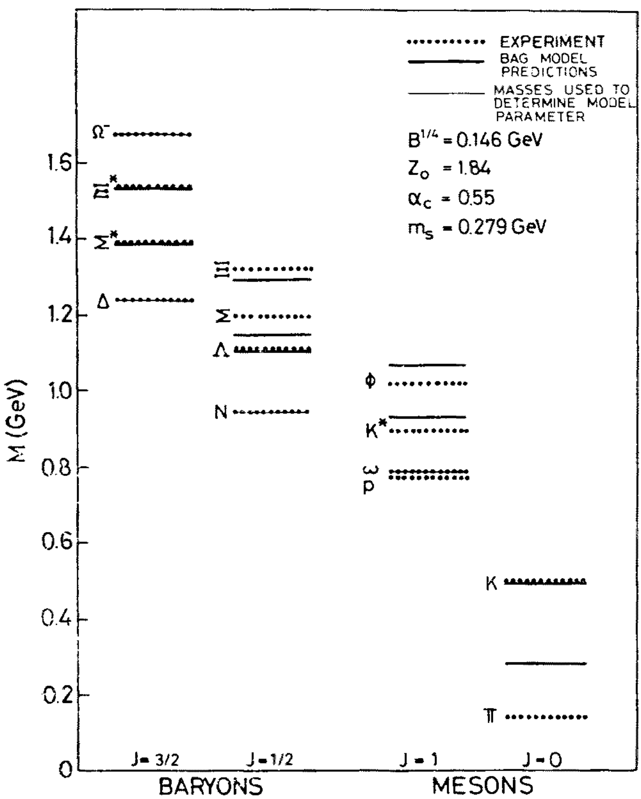

Fig. 9

is fit. With the parameters so determined, we can then calculate the masses of all the other light hadrons, a total of nine masses. As can be seen from Table I and Fig. 9, the results are in generally quite good agreement with the experimental numbers. No attempt was made to make a best fit.

We have omitted a calculation of the masses of the η and η' mesons which cause well known problems in the quark model. A detailed discussion of these particles and speculations about how to handle the problems they cause are contained in reference [4].

We have also evaluated other static parameters, magnetic moments, weak decay constants, etc. These change from our earlier results only because the mass of the strange quark differs from zero, and because the value of $B^{1/4}$ is more accurately fixed. The principal

TABLE II

Hadron	Magnetic moment μ/μ_p		
	Experiment	Bag model	SU(6)
N	$-.685$	$-2/3$	$-2/3$
Λ	$-.240\pm.021$	$-.255$	$-1/3$
$\Sigma^{(+)}$	$.93\pm.16$	$.97$	1
$\Sigma^{(0)}$	—	$.31$	$1/3$
$\Sigma^{(-)}$	$-.53\pm.13$	$-.36$	$-1/3$
$\Xi^{(0)}$	—	$-.56$	$-2/3$
$\Xi^{(-)}$	$-.69\pm.27$	$-.23$	$-1/3$

well tested result is the ratio of the Λ magnetic moment to that of the proton. The SU(6) prediction is $-\frac{1}{3}$. We find $\mu_\Lambda/\mu_\varrho = -0.26$ which is to be compared with the experimental value, -0.24 ± 0.02. The other ratios are given in Table II.

The mean square charge radius for the proton comes out to be 0.73 fm, in comparison with the experimental value, 0.88 ± 0.03 fm. Thus, our proton is somewhat too small. This is also reflected in the reduction of the g value for the proton from our previous result of 2.6 to 1.9. The reason can also be associated with the reduction in radius since the magnetic moment of a massless quark is equal to 0.2 R. In the case of the magnetic moment of the quark, the reason it is so small in units of R is because it is gotten by integrating the product of the large and small components of the Dirac wave function, and these are almost orthogonal. Any effect that would make the small component larger would produce a first order change in the magnetic moment. Since the color magnetic interaction is proportional to the square of the small component and increase in the small component would make α_c smaller since α_c was fit to produce the observed SU(6) splittings.

Finally, we should like to discuss the exotic resonances with more than three quarks. We can compute the mass of such hadrons which contain no non-strange quarks from the expression

$$M = n\frac{2.04}{R} + \frac{4\pi}{3}BR^3 - \frac{Z_0}{R} + \frac{\alpha_c\mu}{R}\frac{4}{3}\left[n(n-6)+S(S+1)+3I(I+1)\right],$$

since the color electric energy is zero when all quarks are massless. We use (4.24) for the color magnetic energy. When we minimize with respect to R we find for the mass of these hadrons

$$M = \tfrac{4}{3}(4\pi B)^{1/4}(2.04n - Z_0 + \tfrac{4}{3}\alpha_c\mu[n(n-6)+S(S+1)+3I(I+1)])^{3/4}. \qquad (4.26)$$

Using values of α_c, Z_0 and B which were fit to the case when $n = 3$, and expressing (4.26) in terms of the mass of the nucleon, we have

$$M = M_p\left[\frac{2.04n - 1.84 + 0.125[n(n-6)+S(S+1)+3I(I+1)]}{3.53}\right]^{3/4}. \qquad (4.27)$$

The possible values of S and I for the six quark bag are (3,0), (0,3), (2,1), (1,2), (1,0), (0,1). Therefore, it is interesting that the lowest value of M occurs for the case $S = 1$, $I = 0$, which are the quantum numbers of the deuteron. In this case we have

$$M_{(1,0)} = M_p\,(2.29).$$

On the other hand when $S = 0$ and $I = 1$, we find

$$M_{(0,1)} = M_p(2.37)$$

which is 80 MeV higher.

In the light of these results a recent paper of Fairley and Squires is of interest [13]. They suggest that the nuclear force may be regarded as a kind of chemical force between nucleon bags. Since, in contrast to the unstable lighter hadrons, the six quark bag can "decay" into two three quark bags without the necessity of creating new valence quarks, we might be also to view the interaction between two three quark as a "simple" fission process. If we assume that the fissioning of the six quark bag is a slow process on the time scale associated with the motion of the massless quarks, then we may adopt a Born–Oppenheimer picture where the mass of a deformed six quark bag is viewed as a potential. The six quark bag can lower its energy by deformation. We then take this energy as the effective potential which acts on the "deformation" parameter, which is in this case the relative separation between the two three quark bags into which the six quark bags can fission. In this way we get a "potential" which when the relative separation r is zero is 0.29 M_p higher than the energy of two free nucleons in the $S = 1$, $I = 0$ state, and has the value + 0.37 M_p in the $S = 0$, $I = 1$ state. At a relative separation of $2R_p$, where R_p is the radius of a nucleon, the potential is zero since the energy of the six quarks is now just $2M_p$. However, there is as well a region of *attraction*, when r is less than $2R_p$. For as two three quark bags approach each other and begin to overlap they lose volume, and hence the zero point energy goes down by amount $B\delta V$, where δV = loss of volume. At the same time the quark kinetic energies decrease since the quarks move in the larger volume of the combined particles. The crucial question is how deep the region of attraction is. Using the zeroth order version of the bag model, Fairley and Squires estimated that the depth of attraction was quite deep, a few hundred MeV. It is clear that one must now try to do a quantitative calculation of the potential using the refined version of the bag model including the color-gluon effects. If the region of attraction is deep enough, we will obtain an effective short range potential which is attractive at distances $\sim R_p$ but at short distances, repulsive, thus it will have the form that we associate with the nuclear force. It is interesting that the repulsion is finite so that we have a "soft" core. It is also interesting that the repulsion is lowest in the state where $S = 1$, $I = 0$ which is the only two baryon state which is bound (deuteron). However, the repulsion is only 80 MeV higher in the $S = 0$, $I = 1$ state, and this is qualitatively consistent with the existence of a virtual state with these quantum numbers in the two nucleon system.

5. *Conclusions*

We have seen that the bag model qualitatively yields a satisfactory unifying picture of many different aspects of hadron phenomenology.

A crucial role in this is played by the colored vector field, which we have argued accomplishes the following. It explains the quark statistics. It accounts for the absence of hadrons with quark quantum numbers. It is responsible for an asymptotically linear Regge trajectory with the same slope for $q\bar{q}$ mesons and three quark baryons. It leads to a qualitatively satisfactory picture of the bare pomeron. It gives rise to a spin-spin force between quarks which gives a quantitative understanding of the lowest mass meson and baryon multiplets. It explains why there are no stable higher quark baryons, and it gives at least a suggestive reason of why the deuteron is the lowest two baryon state. However, we were able to see that the color gluon interaction accomplishes all of the above because it was formulated in the context of a confined field model, that is, the bag model.

Editorial note. This article was proofread by the editors only, not by the author.

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