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ON THE QUARK SELF ENERGY AND A NEW WAVE EQUATION FOR THE MESONS

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A model of coloured quarks interacting through non Abelian vector gauge potentials is analyzed. Conditions are described under which the self consistently determined quark mass equals infinity. A covariant bound state equation for color singlet mesons is derived which is *three* dimensional. The Regge trajectories predicted by such a model cannot rise indefinitely.

1. *Introduction*

The recent discovery of the narrow 1^- resonances at Brookhaven and at SLAC has provoked a vast amount of speculation and model building designed to explain this new phenomenon. There is as yet no general consensus as to what the correct group for the strong interactions must be, but there are several strong candidates. This article focuses on dynamical problems facing some of the theories which attempt to underwrite these simple phenomenological models [1].

Hadronic model building conveniently begins with a set of fundamental Fermion variables $\chi(x)$ interacting via vector gauge potentials $A_a^\mu(x)$ (with corresponding field strengths $F_a^{\mu\nu}(x)$). The perennial problem facing such theories, however, has always been the "mass zero character" of the Yang Mills field. No one has ever seen massless strongly interacting particles, and gauge invariance prohibits the introduction of explicit mass terms for the vector degrees of freedom. It will prove useful to divide gauge models into two classes according to how they deal with this "infrared problem". Class one theories we call superconductor models.

2. *Superconductor models*

The name is borrowed from solid state physics and connotes in this context only that some scalar operators of the theory develop non vanishing vacuum expectation values. The local (but not the global) gauge symmetry is spontaneously broken, the

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vector degrees of freedom become effectively massive and the infra-red difficulty disappears. Typical models are the original Han-Nambu model having SU(3)′ × SU(3)″ as the basic group and the MIT bag model (SU(N) × SU(3) (color)). In the latter case one widens the superconductor notion to include the simultaneous coexistence of regions of normal and superconducting phase.

Class two theories we call Kondo type models.

3. Kondo models

Here one explicitly assumes that the local gauge symmetry is *not* spontaneously broken, the IR divergences are welcomed as a virtue rather than being regarded as a vice, it being hoped that they are responsible for the non-observability of the quark and ultimately for gluon self-confinement as well. As the complementary aspect to the asymptotic freedom enjoyed by such theories Glashow termed this possibility "infrared slavery". The rest of this article is concerned with studying under what conditions (if at all) such IR quark self trapping can obtain. It is, furthermore, not our intenstion to discuss phenomenological implications of any specific model; however the strong group G_s we will have in mind will be G_s = SU(N) × SU(3) (color) [2]. The "experimental package" of bare facts our theory will be required to explain is taken to be:

(*i*) The physical, renormalized free quark mass $m = \infty$. *There is no spectrum of single quark states.* If finite mass quarks are discovered, the model is to be abandoned.

(*ii*) The lesson learned from deep inelastic electron proton scattering is that quarks "inside" hadrons (what precisely is meant by "inside" will be defined later) behave as if their effective mass is small, of the order of *a typical hadron mass.*

(*iii*) The physical low lying hadrons are color singlets.

(*iv*) There can be no strong, long range correlation effects between the observed hadrons. Strongly interacting mass zero particles have never been detected. Any such long range colored gluon effects must be dynamically suppressed.

4. The Kondo effect

As a field theory is defined by the totality of its set of coupled Green's functions, "solution" of the theory (in the absence of a small expansion parameter) must proceed via Ansatz and the demand for selfconsistency. In practice one guesses likely behaviour for one (or several) Green's function(s), assesses the influence of that guess (by using the equations of motion) on other functions and then tries to prove that the whole procedure is self-consistent. Of course one can not make random guesses. There must be a unifying physical picture that suggests Ansatze and guides in making approximations. We now discuss the solid state phenomenon which gave this section its name — the Kondo mechanism [3].

Imagine at time $t = 0$ a high energy X ray digging a hole, a localized spin $\frac{1}{2}$ impurity, deep in the conduction band of a metal (Fig. 1) and ask the question: "is this state likely to persist in time as localized spin $\frac{1}{2}$?" Kondo materials are defined to be those whose

Hamiltonian contains a piece $\delta H = g\boldsymbol{s}\cdot\boldsymbol{S}(\boldsymbol{x}, t)$ where $\boldsymbol{s}$ represents the impurity spin, and $\boldsymbol{S}(\boldsymbol{x}, t)$ denotes the *long wavelength*, collective spin waves in the solid. The spin waves carry spin and as such are capable of transporting spin out of the initial region of localization. Seen in another light, states with impurity spin up (down) and an arbitrary number of spin wave quanta, due to the long wavelength nature of $\boldsymbol{S}(\boldsymbol{x}, t)$ are degenerate in energy. Thus the passage of time induces an uncontrollable averaging over the up(down) configurations available to the impurity resulting in a (locally) observed value of zero. Such an averaging is not expected unless the bosonic degrees of freedom are *infrared.* Conclusion:

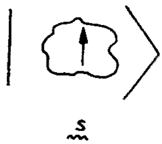

Fig. 1

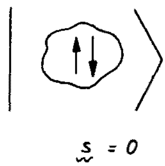

Fig. 2

a state, to be long lived must be an eigenstate of all elementary processes available to the system. A localized (particle like) spin $\frac{1}{2}$ impurity does not satisfy this criterion.

Imagine now the more difficult experiment in which not a single hole is created, but an "exciton" having a spin up and a spin down tightly bound in a state of total spin zero. (Fig. 2). Just as above we can ask whether this "particle" is likely to persist in time as a localizable state. Because the exciton has $\boldsymbol{s} = 0$ it decouples from the long wave length spin waves (which couple to spin) and is therefore insulated from the IR mechanism which doomed a single isolated spin impurity to "disappear". Thus we expect that $\boldsymbol{s} = 0$ excitons will persist as long lived particle like states.

The relevance of the Kondo phenomenon for our quark colored gluon model is drawn by making the identifications

Spin $\frac{1}{2}$ impurity ⟷ colored quark
Solid ⟷ physical vacuum
Spin waves $\boldsymbol{S}(\boldsymbol{x}, t)$ ⟷ colored Y.M. gluons
$\boldsymbol{s} + 0$ excitons ⟷ color singlet hadrons.

5. *The quark self-energy*

Questions concerning the quark spectrum must be addressed to the equation for the quark self-energy.

$$G^{-1}(p) = m_0+\gamma p+ig^2\int\frac{(dk)}{(2\pi)^4}\gamma^\mu t_a G(p-k)\Gamma_b^\nu(p-k, p)D_{\mu\nu}^{ab}(k). \tag{1}$$

It will be convenient to define $G^{-1}(p) = A(p^2)\{\gamma p+m(p^2)\}$. The first Slavnov identity [4] guarantees that the gluon propagator can be written

$$D_{ab}^{\mu\nu}(k) = \delta_{ab}\left\{\xi\frac{kk}{(k^2)^2}+\left[g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right]\frac{d(k^2)}{k}\right\} \tag{2}$$

with ξ the covariant gauge parameter and $d(k^2)$ an invariant *dimensionless* function. Using the physics of the Kondo mechanism as a guide we write down Ansätze for $d(k^2)$ and $\Gamma_b^\nu(p-k,p)$:

(*i*) The physical vector gluon spectrum extends down to zero mass so we set

$$d(k^2) = \left(\frac{\Omega^2}{k^2 - i\varepsilon}\right)^{\lambda(g^2)} + 1, \tag{3}$$

where $\lambda(g^2)$ parametrizes the strength of the cut at $k^2 = 0$ and will be assumed positive. The "1" in Eq.(3) is intended as a rough estimate of the expected ultraviolet behaviour of the gluon propagator [5].

(*ii*) The coupling in the limit of long wave lengths persists (infrared spin waves can still cause spin flipping) which we take to mean

$$\Gamma_b^\nu(p-k, p) \to \gamma^\nu t_b. \tag{4}$$

The final technicality to be resolved is the question of gauge. We refer the reader to Ref. [6] for details. Suffice it to say that the dependence on the invariant function $A(p^2)$ can be eliminated (near the quark mass shell) with the eigenvalue equation for the quark mass becoming

$$m(p^2)|_{-p^2=m^2} = m. \tag{5}$$

As a function of m, Ω and an infrared cut off μ [7], we obtain

$$1 = \frac{3g^2C_0}{(4\pi)^2}\frac{\Gamma(\lambda)}{\Gamma(1+\lambda)}\begin{cases}\left(\dfrac{\Omega^2}{m^2}\right)^\lambda B(1+\lambda, 1-2\lambda) & 0 < \lambda < \frac{1}{2} \\[2ex] \left(\dfrac{\Omega^2}{\mu^2}\right)\dfrac{\mu}{m}C(\lambda) & \frac{1}{2} < \lambda.\end{cases} \tag{6}$$

C_0 is the Casimir operator defined by $\sum_{a=1}^{8} t_a t_a = C_0 1$ and $C(\lambda)$ a real constant. From Eq. (6) it is clear that for $\lambda > \frac{1}{2}$ (in the limit $\mu \to 0$) the IR divergence is sufficiently strong to force the renormalized mass m to infinity. This is the regime for which total confinement occurs. If, on the other hand, $\lambda < \frac{1}{2}$ there is no dependence of m on the IR cut off! The quark mass in such a theory remains finite, mass zero gluon effects are expected to be non-negligible and therefore on physical grounds such a model is to be rejected [8]. From now on we assume that $\lambda(g^2) > \frac{1}{2}$, where the degree of divergence of the quark mass is given by

$$m(\mu) = \left(\frac{1}{\mu}\right)^{2\lambda-1}. \tag{7}$$

Physically what has happened here is that the real part of the quark self-mass has received an infinite contribution from soft Y.M. radiation.

Conclusion If $\lambda(g^2) < \frac{1}{2}$

(*i*) $\chi(x)$ produces states of infinite energy when acting on the vacuum.

(*ii*) An arbitrary operator carrying color quantum numbers we expect to suffer the same fate as the Y.M. fields must couple universally to color wherever it be found localized and in whatever form.

(*iii*) Color singlets (the candidates for our physical hadrons) decouple from the massifying soft radiation and can have finite energy.

There remains a serious problem however. While it is true that $m \to \infty$ removes "free quarks" from further consideration what about the confinement of the (assumed) massless gluons? As already mentioned in the introduction zero mass effects between strongly interacting particles would make this model unusable for hadron physics. But this question can only be resolved by first spelling out the details of the hadron structure implied by the model, and then looking to see if unwanted effects can appear. Let us begin by studying the mesons.

The mesons

The first feature we notice is that since $G(p)$ is effectively zero all Bethe Salpeter equations are homogeneous, as is to be expected if no free constituent states can be produced. Defining the bound state wave function $\Phi_P(x) = \langle P | \left(\chi\left(\frac{x}{2}\right)\bar{\chi}\left(\frac{x}{2}\right)\right)_+ |0\rangle$ one normally solves the equation depicted in Fig. 3. To understand physically what must happen in the limit $m \to \infty$ recall the meaning of the relative time variable appearing in $\Phi_P(x)$. At $t = -\frac{x^0}{2}$ the operator $\bar{\chi}$ acts on the vacuum and produces a "particle". *After the elapse of* x^0 *seconds*

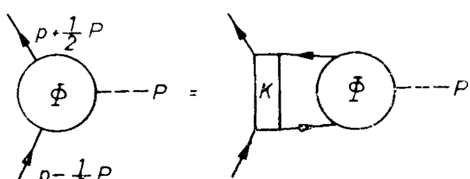

Fig. 3

the operator χ acts to produce the companion "anti particle"; the objects proceed to interact and evolve into a bound state. Thus x^0 measures the length of time one constituent can exist by itself before it's bound state partner is even created. There is no logical objection to this possibility when the constituent mass is finite. However, as $m \to \infty$ it becomes impossible to create one of the partners alone, thus leading us to expect that in some sense $\Phi_P(x) \approx 0$ unless $x^0 \approx 0$, or said another way that the internal configuration space becomes essentially three dimensional. It can be shown [9] that we define the proper time $\tau = -(xP)/M(= x^0$ in the rest frame of $P^\mu)$ then in the limit $m \to \infty$ $\Phi_P(x)$ assumes the form

$$\Phi_P(x) = e^{-im\tau}\varphi_M(\bar{x}), \tag{8}$$

with $\bar{x}^\mu = \left[g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2}\right]x_\nu$; $\bar{p}^\mu = \left[g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2}\right]P_\nu$ and $\bar{x}P = \bar{p}P = 0$. The four dimensional internal Minkowski space $\{x^\mu\}$ has shrunk to a hypersurface in that space given by $\{\bar{x}^\mu; xP = 0\}$. Furthermore if one expands $\varphi_M(\bar{x})$ in Dirac bilinears

$$\varphi = 1\varphi^S + \gamma_5\varphi^{PS} + \gamma^\mu\varphi^V_\mu + \gamma^\mu i\gamma_5\varphi^A_\mu + \tfrac{1}{2}\sigma^{\mu\nu}\varphi^T_{\mu\nu} \tag{9}$$

only φ^{PS} and φ^v remain non zero in the $m \to \infty$ limit. The amplitude $\varphi = \gamma_5\varphi^{PS} + \gamma^\mu\varphi^v_\mu$ satisfies

$$(\tfrac{1}{2}M + \gamma\bar{p})\varphi_M(\bar{x})(\tfrac{1}{2}M + \gamma\bar{p}) = \frac{C_0 g^2}{i} D_{\alpha\beta}(\bar{x})\gamma^\alpha\varphi_M(\bar{x})\gamma^\beta, \tag{10}$$

where we have written the r.h.s. in the ladder approximation for definiteness, but the whole derivation goes through for the general kernel K. We now summarize our results:

(*i*) Equation (10) is covariant, but essentially three dimensional.

(*ii*) For $\lambda > \frac{1}{2}$, $m \sim (\infty)^{2\lambda-1}$.

(*iii*) The vanishing of φ^S, φ^A, φ^T in the $m \to \infty$ limit has provided us with the states expected from the naive non-relativistic quark mnemonic, namely ϱ and π.

(*iv*) SU(6) is intrinsically broken as φ^{PS} and φ^v_μ satisfy different (though similar) equations. This is due to the vector nature of the colored gluons.

(*v*) Not only quarks (i.e. the $\underset{\sim}{3}$ representation) have infinite mass, but rather all localizable color carrying states as well [10].

(*vi*) The appearance of the mass $\frac{1}{2}M$ in the effective inverse quark propagator $\frac{1}{2}M + \gamma\bar{p}$ "inside" the mesons is at least an heuristic suggestion that the effective quark mass $m_{\text{aff}} = \frac{1}{2}$ the meson mass. This will be important for studies of the deep inelastic structure functions.

(*vii*) It can be shown from Eq. (10) that the (pion) amplitude $\varphi^{PS}(\bar{x})$, in the rest frame of the vector P^μ obeys

$$\left\{-\nabla^2 + \left[-\frac{C_0 g^2}{i} g_{\alpha\beta}D^{\alpha\beta}(\boldsymbol{x})\right]\right\}\varphi^{PS}(\boldsymbol{x}) = -\tfrac{1}{4}M^2\varphi^{PS}(\boldsymbol{x}). \tag{11}$$

Despite it's Schroedinger like appearance it is to be emphasized that we have *not* made a non-relativistic approximation. It is well known that in the weak binding limit (set $P^0 = 2m + \varepsilon$, $\varepsilon/m \ll 1$), that is to say for calculating bound states which lie just beneath the constituent threshold (Fig. 4) the non-relativistic assumption is justified and one

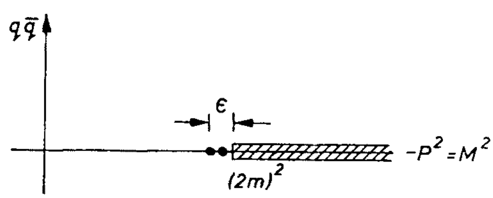

Fig. 4

obtains (as in the case of positronium) the equation

$$\left\{\frac{\mathbf{p}^2}{m} - g^2 \frac{1}{4\pi} \frac{1}{|\boldsymbol{x}|}\right\} \psi_\varepsilon(\boldsymbol{x}) = \varepsilon \psi_\varepsilon(\boldsymbol{x}). \tag{12}$$

Note, however, the explicit appearance of the constituent mass m. It is m that provides the dimensional scale for the weakly bound spectrum. By contrast, in equation (11) the scale is fixed by the mass Ω contained in $D^{\alpha\beta}$ and is applicable for strong binding, i.e. when the mass deficit is of the order of the constituent mass itself.

Note that one can formally write Eq. (11) in the form Eq. (12). Dividing by $\frac{1}{4}M$ we find

$$\left\{-\frac{\nabla^2}{\frac{1}{4}M} + \left[-\frac{4C_0 g^2}{iM} g_{\alpha\beta} D^{\alpha\beta}(x)\right]\right\} \varphi^{\mathrm{PS}}(x) = -M\varphi^{\mathrm{PS}}(x), \tag{13}$$

suggesting by analogy $m_{\mathrm{eff}} = \frac{1}{4}M$. In any event we can anticipate $m_{\mathrm{eff}} \lesssim M$.

Before discussing qualitatively the solutions to Eq. (11), let us say what we can about mass zero gluon effects.

Long range forces

Graphs involving "single gluon emission" (i.e. the appearance of a single gluon propagator) can be dealt with immediately. The process $\underset{\sim}{1} \to \underset{\sim}{1} + \underset{\sim}{8}$ (Fig. 5.) is forbidden by group theory. The process $\underset{\sim}{1} \to \underset{\sim}{8} + \underset{\sim}{8}$ (Fig. 6.) is likewise irrelevant as hadronic octet states are infinitely massive. The first potentially dangerous process is depicted in Fig. 7., i.e.

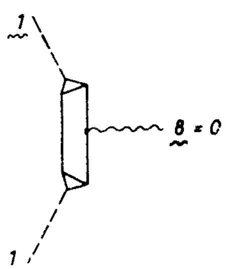

Fig. 5

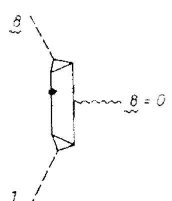

Fig. 6

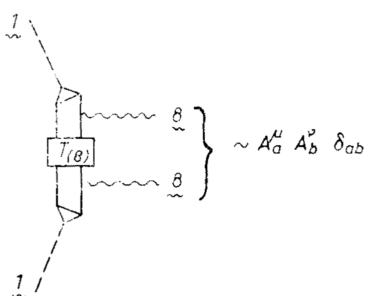

Fig. 7

correlated two gluon emission into an overall *color singlet state*, when the total energy and momentum siphoned off by the gluons tends to zero. But the emission of an infinitesimal amount of momentum can not change what was a highly localized hadronic ($\underset{\sim}{1}$) state

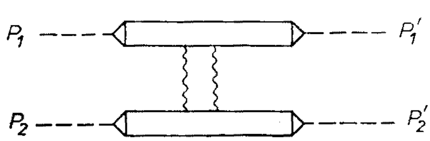

Fig. 8

into a delocalized one. But after the emission of the soft gluon ($\underset{\sim}{8}$) one then has a *localized* octet state which we have argued to have infinite energy, and hence the amplitude for the emission of soft radiation is suppressed. But this then means that potentially long range effects originating in graphs as in Figure 8 will be absent. Thus at least on the level of this heuristic argument the same mechanism that is responsible for the elimination of physical quarks and other color carrying hadronic states succeeds in suppressing mass zero gluon effects. One still feels a bit uneasy that $q\bar{q}$ annihilation into multi gluons could render the amplitude in Fig. 7 non zero and ultimately cause the instability of what are supposed to be stable mesons. We do not as yet have an answer to this question. In any event the above arguments are certainly not the last word on gluon confinement, but encourages us to take the next step and investigate the meson spectrum.

The meson spectrum

As we are interested in general features we will concentrate on $\varphi^{PS}(x)$ because it is simpler, but entirely analogous arguments apply for $\varphi^{V}_{\mu}(x)$. Write Eq. (11) in the form

$$\{-\nabla^2+I(r)\}\varphi(r)=\varepsilon\varphi(r), \tag{14}$$

where

$$I(r) = -3g^2C_0\frac{\Omega^{2\lambda}}{\pi^2}\frac{\Gamma(1-\lambda)}{\Gamma(1+\lambda)}\frac{1}{4^{1+\lambda}}|r|^{2\lambda-2}-\frac{3g^2C_0}{4\pi^2}\frac{1}{r^2} \tag{15}$$

and we have identified $\varepsilon = -\frac{1}{4}M^2$. The term containing the Γ functions is the IR contribution, the $\frac{1}{r^2}$ comes from the "1" in Eq. (3) and may be termed the UV contribution to the "potential". It is to be emphasized that any property of the spectrum that depends on the UV term having precisely $\frac{1}{r^2}$ behaviour should be treated with caution. The UV behaviour of $D_{\alpha\beta}(k^2)$ should be calculable in this model and is more likely than not to turn out different from pure $1/r^2$.

Although Eq. (14) has been written in Schroedinger form we are now presented with our first surprise. The derivation leading to Eq. (14) has also required that $\varepsilon = -\frac{1}{4}M^2 > 0$. This is so because $M^2 = -P^\mu P_\mu$, but the Equation (10) could only be derived from the original Bethe Salpeter equation (Fig. 3.) under the assumption that P^μ is time like, i.e. $M^2 > 0$. Thus Eq. (14) may have more solutions (those belonging to positive eigenvalues ε) than are physically meaningful.

$\frac{1}{2} < \lambda < 1$

The potential appears as in Fig. 9. Because it is very long range there will be a progression of discrete states having $\varepsilon_n < 0$ crowding closer and closer together as $\varepsilon_n \to 0$ and eventually merging into the continuum for $\varepsilon > 0$. But since $\varepsilon = -\frac{1}{4}M^2$ this would imply an infinity of low lying meson states — in flat contradiction with experiment. So for phenomenological reasons the domain $\frac{1}{2} < \lambda < 1$ is to be rejected. Note that this result is quite independent of precisely what the small r (UV) behaviour of $I(r)$ is.

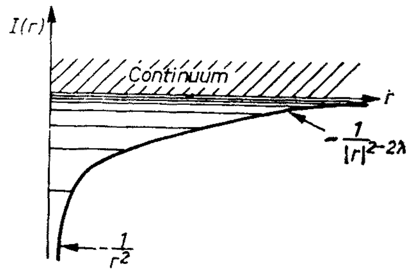

Fig. 9

$1 < \lambda$

Now the potential *grows with distance* (Fig. 10). Since $I(r)$ is spherically symmetric, if we set $\varphi(r) = Y_{lm}(\Omega)\frac{u^l(r)}{r}$ then

$$\left\{-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}+I(r)-\varepsilon_n\right\}u_n^l(r) = 0. \tag{16}$$

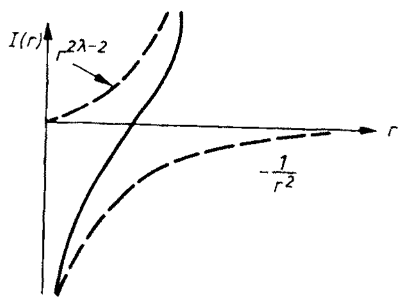

Fig. 10

Eq. (16) points out an important distinction between I's which behave as $\frac{1}{r^2}$ at $r \approx 0$ and I's which are less singular. If $I(r)$ is less singular the Schroedinger problem in Eq. (16) possesses a ground state with spectrum as in Fig. 11 — a finite number of discrete states having $\varepsilon_n < 0$. These are our mesons.

If $I(r)$ is at least as singular as $\frac{1}{r^2}$ then (Fig. 12) the Schroedinger problem of Eq. (16) does not possess a ground state in the usual sense. There is an infinite sequence of discrete states lying deeper and deeper in the potential [11]. One usually argues such behaviour to be unphysical and discards such potentials. Here that would be being a bit hasty as

$\varepsilon_n = -\frac{1}{4}M^2$. There is no reason to expect states of low mass to decay spontaneously into states of *higher* and *higher* mass.

The final point we wish to make is illustrated in Fig. 13. Let $I(r)$ generate the spectrum of Fig. 11 then for $l = 0$ we will have a *finite* number of states having $\varepsilon_n < 0$. The pion is that state lying highest but still below $\varepsilon = 0$. As we turn on the angular momentum barrier the effective potential in Eq. (16) shifts upward causing the pion to move to the

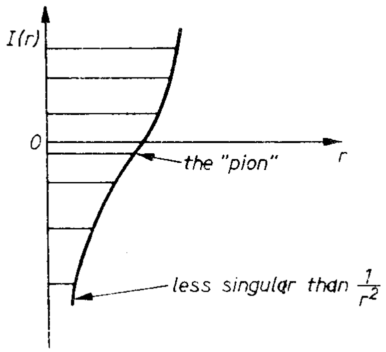

Fig. 11

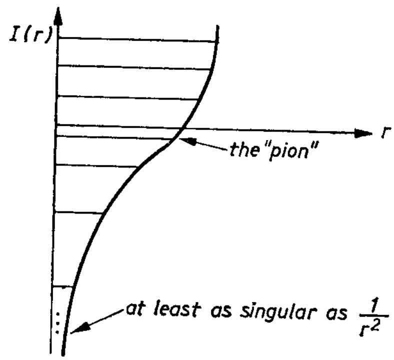

Fig. 12

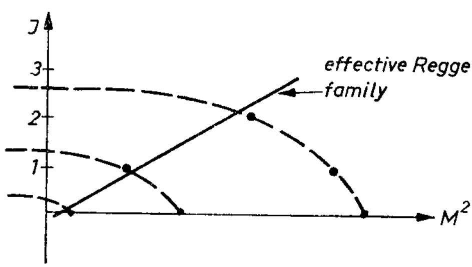

Fig. 13

left. It is easy to arrange the parameters such that by the time we have reached $l = 1$ what was the pion has passed into the unphysical region $M^2 < 0$. However, the "1st radial recurrence" of the π has moved *down* and looks now like an orbital excitation of π. Fig. 13 contains the author's conception of how in a model where the mathematical trajectories of the underlying Eq. (11) move (with increasing l) to the *left* one can still simulate a Regge like family structure in the $J-M^2$ plot. One conclusion is however unavoidable. Since in Fig. 11 there are only a finite number of "radial recurrences" of the π, effective Regge *trajectories can not rise indefinitely*. Three or four states may lie on a more or less straight line but the family then breaks off abruptly — *the trajectory simply ends*. For this model of the meson spectrum the effective linear trajectories are more a dynamical accident than anything else. One shuld perhaps ask the question: "do we necessarily expect the Regge concept employed in phenomenological data fitting to play a significant role at a more dynamical level?" The model described here suggests that the answer is "no".

Outlook

The model is clearly in its infancy. Before one can do explicit fitting to the Rosenfeld table, symmetry breaking of ordinary SU(3) (SU(4)?) must be included. This may be inserted via the bare mass term in Eq.(1). The explicit appearance of M^2 in Eq. (11) (and not M) means that symmetry breaking sum rules will involve M^2 for the mesons and not just M. An explicit model for meson meson scattering must be constructed and most important the proton inelastic structure functions must be calculated to see which if any results of the conventional parton model can be recovered. The phenomenology of the baryons, being more highly developed than for mesons, requires an extension of Eq. (10) (and/or Fig. 3.) to 3 bodies. That in itself will prove a formidable task.

REFERENCES

[1] See the companion article (this volume) of F. E. Close.
[2] This model is therefore o "charm" type.
[3] Kondo, *Adv. Solid State Phys.* **23**, 183 (1969).
[4] A. Slavnov, *Theor. Math. Phys.* **10**, 153 (1972); **10**, 99 (1972).
[5] This assumption about the UV behaviour is intended only to be "typical". Since our model is asymptotically free we should be able to *calculate* $d(k^2)$ for large k^2. However as we shall see the usual asymptotic freedom calculation can not be taken over directly as a renormalized parameter, the quark mass m, has gone to infinity.
[6] R. L. Stuller, ICTP/74/14, April 1975.
[7] The IR cut off is introduced by replacing $d(k^2)/k^2$ by $(\Omega)^\lambda/(k^2+\mu^2)^{1+\lambda}+1$ in all integrals. This device aids in ascessing the influence of the IR divergences.
[8] The regime $\lambda < \frac{1}{2}$ bears closer analogy to the original Kondo effect than $\lambda > \frac{1}{2}$. For $\lambda > \frac{1}{2}$ it is the ∞ shift in the real part of m that is significant.
[9] See Ref. [6] and R. L. Stuller, bo te published.
[10] We have not proven this here. See Ref. [6].
[11] K. M. Case, *Phys. Rev.* **80**, 797 (1950).