AN INHOMOGENEOUS THICK PLATE IN GENERAL RELATIVITY

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The explicit form of the interior and exterior metric is found for a special but reasonable case of an inhomogeneous mass distribution in a thick plane plate.

Many objects existing in nature resemble plane disks more than spheres. The gravitational field of an infinitely thin plane is presented in [1]. The explicit form of the metric generated by a thick plate has been found in [2] under the assumption of a homogeneous mass distribution. It is clear, however, that such a mass distribution in the plate represents a very rough approximation to the mass distribution in real objects possesing plane symmetry. Mass density in real objects usually decreases from the centre to the boundary of the disk. Thus it is not without physical importance to try to find the explicit form of the gravitational field generated by a thick plane plate in which the mass density distribution behaves more realistically compared to the $\varepsilon = \text{const.}$ case.

The metric tensor of a plane plate with mass density $\varepsilon(x)$ is determined in [2] by

$$b(x) = C \exp \left\{ -\tfrac{3}{4} \kappa \int \varepsilon(x) a(x) dx \right\} \tag{1}$$

and by the function $a(x)$ for which

$$\frac{da}{dx} = 1 + \frac{3}{4} \kappa \varepsilon(x) a^2(x) \tag{2}$$

holds under the assumption

$$\lim_{x \to 0^+} a(x) \to -\infty. \tag{3}$$

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The most simple form of $a(x)$ satisfying the condition (3) is

$$\tilde{a}(x) = -\frac{D}{x}, \tag{4}$$

where D is a positive constant. Then we get

$$\varepsilon(x) = \varepsilon(0) - \frac{\varepsilon(0)}{D} x^2, \qquad \varepsilon(0) \equiv \frac{4}{3\kappa D}. \tag{5}$$

It is obvious that (5) gives, in our sense, a better description of mass distribution in real objects. As in the case of spherical symmetry, the central mass density is a free parameter [3].

For our "parabolic mass distribution" we can then find the explicit form of the interior and exterior metric

$$g_{\alpha\beta}(x) = \begin{pmatrix} 1 & & & \\ & [G(p, q, x)]^{4/3} & & \\ & & [G(p, q, x)]^{4/3} & \\ & & & -[G(p, q, x)]^{-2/3} \end{pmatrix}, \tag{6}$$

$$G(p, q, x) = \operatorname{sgn} x \, (1 - \tfrac{3}{4} \kappa\varepsilon(0) p \cdot x + \tfrac{3}{4} \kappa\varepsilon(0) \cdot p^2) e^{-3/8\kappa\varepsilon(0) \cdot q^2},$$

where $p = 0$, $q = x$ inside the plate and $p = q = L$ outside the plate of the thickness $2L$. The exterior solution has a physical singularity if

$$x = \frac{4}{3\kappa\varepsilon(0) \cdot L} + L \equiv x_{(s)} . \tag{7}$$

holds. Because $\varepsilon(0) > 0$ we always have $x_{(s)} > L$. Thus for this inhomogeneous mass distribution there also exists a physical singularity, as expected.

Although this is a very simple example we can say that the method presented above is the only feasible method of obtaining a wide variety of explicit and exact solutions. This follows from the fact that equation (2) is the Riccati equation.

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