DUAL PARTON MODEL AND THE PROCESS $\pi^+n \to p\omega$

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The differential cross section for the process $\pi^+n \to p\omega$ has been determined on the basis of the dynamical dual model of hadrons as proposed in an earlier paper. It is shown that the theoretical prediction is in excellent agreement with the experimental results. Also, it can nicely explain the fact that there is no dip in the differential cross section. Moreover, it is shown that the large value of the density matrix element ϱ_{00} in the Gottfried-Jackson frame, as observed in experiments, can be interpreted in a nice way.

1. *Introduction*

Two significant themes have recently emerged in hadron physics. The recognition of duality concept in strong interactions and the emergence of parton idea in deep inelastic scattering are the two interesting ideas which play crucial role in determining the structure of hadrons as well as the dynamics of strong interactions. Apparently, the parton model is in contradiction with the concept of duality. However, the parton-parton interaction has been put forth recently in dual-parton view (Olesen and Nielsen 1970) or the fishnet diagram interpretation of the dual resonance model (Sakita and Virasoro 1970). Again, Bloom and Gilman (1970) suggested that the substantial part of the scaling curves for the structure functions is in fact built up from resonances. Indeed, in a previous paper (Bandyopadhyay et al. 1972), we showed that a five-parton model for nucleon can be constructed in such a way that the deep inelastic scattering can be assumed to be contributed by resonances in conformity with the concept of duality. In this five-parton model, partons are taken to be spin 1/2 point-like constituents with integral charges. Here proton is considered to be composed of 5 spin 1/2 partons with one having positive charge +1 while all the others are neutrals. To get a good fit for the *e*n scattering the neutron is assumed to be

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a mixture of the following two different configurations (*i*) all partons are neutral and (*ii*) two of them have charges $+1$ and -1 and the remaining three are neutral. Also, in our recent paper (Bandyopadhyay and De 1972) we showed that duality can be incorporated from the very dynamics of strong interactions in view of this five-parton model provided we assumed that any two spin 1/2 point-like constituents (parton antiparton pair) can form a π-meson cluster in the structure of baryons and the strong interaction actually involves the interaction of this π-meson with the incident hadron. Indeed, a suitable form factor can be introduced so that the clustering effect will not alter the scaling behaviour in the high energy limit (Okumura 1971). In view of this, we assumed that a nucleon can be represented as $N = (\pi\pi c)$, where π is composed of any two spin 1/2 constituents and c represents the unbound parton (Bandyopadhyay and De 1972). Specifically, we can write the configuration for proton as $(\pi^+\pi^0 c)$ and for neutron $\alpha(\pi^0\pi^0 c)+\beta(\pi^+\pi^- c)$ where α and β denote the weightage of the respective configurations. This model of strong interaction satisfies the requirement of duality in MB scattering in the sense that both the s- and t-channel amplitudes are contributed by the same meson (*viz.*, ϱ-meson). Indeed, in this scheme, we can take that a baryonic resonance occurs when a π-meson cluster in the structure of a baryon is excited to the level of ϱ. The most interesting aspect of this interpretation of duality is that the inconsistencies which crop up in the $B\bar{B}$ scattering in the naive quark model are removed in this scheme as this interaction also involves the $\pi\pi$ interaction, where all the pions are in the structure of the baryons (Bandyopadhyay and De 1972). It was also shown elsewhere (Bandyopadhyay and De 1973) that this model can interpret the occurrence or non-occurrence of dips in differential cross sections of various processes in consistency with the experimental results, where it is assumed that dip is a resonance effect (Imachi et al. 1970). This also follows from the requirement of duality and the finite energy sum rule (FESR) that the t-channel amplitude at high energy is contributed by the s-channel resonances at low energy.

In this note, we shall quantitatively analyse the scattering process $\pi^+ n \to p\omega$ in view of the above dynamical dual model of strong interactions. It is found that the differential cross section as well as the obtained energy dependence is in excellent agreement with experiments. Also, this can explain the characteristic features of the process such that the non-occurrence of dip at $-t = 0.6$ GeV^2 and the nonvanishing of the density matrix-element ϱ_{00} in the Gottfried-Jackson frame, the facts which are in contradiction with the simple Regge pole model with ϱ-exchange.

2. *The scattering process $\pi^+ n \to \omega p$ in the forward region*

Following the model discussed above we assume that the configuration of a neutron is given by $\alpha(\pi^0\pi^0 c)+\beta(\pi^+\pi^- c)$ and the process $\pi^+ n \to p\omega$ is depicted according to the following scheme.

$$\pi^+ + n = \pi^+ + \begin{Bmatrix} \pi^0 \\ \pi^0 \\ c \end{Bmatrix} \to \begin{Bmatrix} \varrho^+ \\ \pi^0 \\ c \end{Bmatrix} \to \begin{Bmatrix} \pi^+ \\ \pi^0 \\ c \end{Bmatrix} + \omega = p\omega. \tag{1a}$$

$$\pi^+\mathrm{n} = \pi^+ + \left\{\begin{matrix}\pi^- \\ \pi^+ \\ c\end{matrix}\right\} \rightarrow \left\{\begin{matrix}\varrho^0 \\ \pi^+ \\ c\end{matrix}\right\} \rightarrow \left\{\begin{matrix}\pi^0 \\ \pi^+ \\ c\end{matrix}\right\} + \omega = \mathrm{p}\omega. \tag{1b}$$

Evidently, the main interaction here is the $\pi\pi \rightarrow \pi\omega$ via ϱ-exchange and from the requirement of duality, we assume that the t-channel ϱ-exchange contribution is equivalent to the s-channel resonance contribution where it is assumed that the formation of a ϱ in the structure of a nucleon gives rise to a nucleon resonance (Bandyopadhyay and De 1972). From this assumption we now write the amplitude for the process $\pi^+\mathrm{n} \rightarrow \mathrm{p}\omega$ as follows:

$$A(\pi^+\mathrm{n} \rightarrow \mathrm{p}\omega) = \tfrac{2}{3}\, A(\pi\pi \rightarrow \pi\omega)\, \tfrac{1}{2}\, T(s, t). \tag{2}$$

Here $1/2\ T(s, t)$ denotes the factor which arises from the structural rearrangement of partons involved in duality diagrams in the process $\pi^+\mathrm{n} \rightarrow \mathrm{p}\omega$. Indeed, this factor may be considered to come from the overlap integrals of the wave functions or the form factor of hadrons as the composite structure of the nucleon (Imachi et al. 1971). The factor 2/3 has

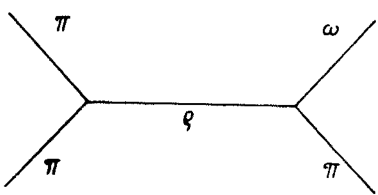

Fig. 1. Feynman diagram for the process $\pi\pi \rightarrow \pi\omega$ via ϱ-exchange

been introduced in Eq. (1) because of the fact that of the three pion states π^+, π^0, and π^- in the structure of the target neutron, the incident π^+ interacts only with the two states π^0 and π^- as is revealed in Eq. (1).

To calculate the cross section for the process $\pi\pi \rightarrow \pi\omega$, we consider here the following Feynman diagram (Fig. 1) with a certain form factor.

For the $\varrho\pi\pi$ and $\omega\varrho\pi$ interactions we take the conventional Lagrangians

$$L_{\mathrm{I}} = i g_{\varrho\pi\pi} \varrho_\mu \left(\bar{\pi} \frac{\partial \pi}{\partial x_\mu} - \pi \frac{\partial \bar{\pi}}{\partial x_\mu} \right), \tag{3}$$

$$L_{\mathrm{II}} = i g_{\omega\varrho\pi} \varepsilon_{\mu\nu\lambda\sigma} \varepsilon^{\omega}_{\mu} k_\nu \varepsilon^{\varrho}_{\lambda} q_\sigma, \tag{4}$$

where k and q are the four-momenta of ω and ϱ-mesons, respectively. The differential cross section, when calculated according to this diagram (Fig. 1), gives in the c. m. system for large s

$$\frac{d\sigma}{dt} = \frac{\pi}{2} \left(\frac{g^2_{\omega\varrho\pi}}{4\pi} \right) \left(\frac{g^2_{\varrho\pi\pi}}{4\pi} \right) \frac{1}{m^2_\varrho - t}\, F(t). \tag{5}$$

Here $F(t)$ determines a suitable form factor normalised as $F(0) = 1$. Taking $g^2_{\varrho\pi\pi}/4\pi = 2.5$ and $g^2_{\omega\varrho\pi}/4\pi = (0.41 \pm 0.09)/m^2_\pi$ (Dashen and Sharp 1964) we find

$$\frac{d\sigma}{dt} = 15 \cdot 10^{-27} \frac{1}{m_\varrho^2 - t} F(t)\,\text{cm}^2/\text{GeV}^2. \tag{6}$$

To calculate the cross section for the process $\pi N \to N\omega$, we note that one of the colliding particles is a π-meson cluster in the composite structure of the nucleon. It may be recalled here that in this configuration scheme for nucleons, the number of fundamental spin 1/2 constituent is assumed to be five, of which any two spin 1/2 constituents form a π-meson cluster. Thus, the c. m. energy E' of the colliding $\pi\pi$ system may be assumed to be given by the relation $E' = 2/5E$, where E is the c. m. energy of the scattering system πN.

We consider the factor 1/2 $T(s, t)$ in Eq. (1) which is supposed to arise from the structural rearrangement of partons in duality diagrams. Indeed, Imachi et al. (1971) discussed the possible effects of this rearrangement in the energy dependence of various hadronic interactions. The contribution of the structural rearrangement factor in the process $\pi N \to N\omega$ can be calculated from the analysis of the following duality diagrams. It is noted that all these diagrams are nonplanar and satisfy $t - u$ duality and no planar diagram contributes to the process concerned. The rearrangement of partons involved in these diagrams (a), (b) and (c) contribute to the amplitude the following factors (Imachi et al. 1971)

$$\begin{aligned} T_a(s, t) = T_b(s, t) = T_c(s, t) &\cong [(p_a + p_d)^2]^{-\gamma}[(p_b + p_c)^2]^{-\gamma} \\ &= (-u)^{-2\gamma} \xrightarrow[s\ \text{large}]{} s^{-2\gamma}. \end{aligned} \tag{7}$$

The factor $[(p_i + p_j)^2]^{-\gamma}$ corresponds to the parton rearrangement from a hadron with momentum p_i to that with p_j and γ is a suitable constant.

It may be noted that there is a correspondence between $T(s, t)$ and the Regge amplitude T_{Regge} with strongly degenerate trajectories $\alpha(t)$ and residue $\beta(t)$ in the forward regions if we take $-2\gamma + 1 = \alpha(t)$. In fact, for the planar and nonplanar diagrams, the phase factor is $e^{2i\pi\gamma}$ and 1, respectively. Thus $T(s, t)$ can be written as

$$T(s, t) = (1 \pm e^{2i\pi\gamma})s^{-2\gamma} \tag{8}$$

and

$$T_{\text{Regge}}(P = \pm 1) = \frac{\beta(t)\,(1 \pm e^{-i\pi\alpha(t)})}{\sin \pi\alpha(t)}\, s^{\alpha(t)-1}. \tag{9}$$

It is noted that for the $\pi^+ n$ system we can have 2 competing channels $\pi^+ n \to p\omega$ and $\pi^+ n \to \pi^+ n$ for which we may construct 6 duality diagrams with n_R (number of rearranged partons) = 2, of which only 3 (Figs 2(a), (b) and (c)) contribute to the process $\pi^+ n \to p\omega$. So the factor 1/2 has been introduced in the rearrangement amplitude.

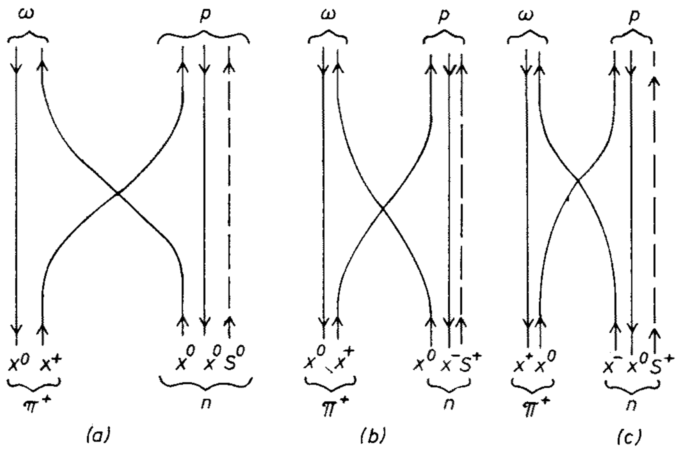

Fig. 2. Duality diagrams showing structural rearrangement of partons in the process $\pi^+\text{n} \to \text{p}\omega$. Here x^+, x^0 and x^- represent spin 1/2 partons and S represents the spectators

Substituting the value $T(s) = S^{-2\gamma}$ in Eq. (2) and taking $\gamma = 0.5$, we find the differential cross section for the process $\pi^+\text{n} \to \text{p}\omega$

$$\frac{d\sigma}{dt} = \left(\frac{5}{2}\right)^4 \frac{1}{9} \frac{\pi}{2} \left(\frac{g^2_{\omega\pi\varrho}}{4\pi}\right)\left(\frac{g^2_{\varrho\pi\pi}}{4\pi}\right) \frac{1}{m_\varrho^2 - t} \frac{1}{s^2} F(t)$$

$$= 66.67 \frac{1}{s^2} \frac{F(t)}{m_\varrho^2 - t} \text{ mb/GeV}^2, \tag{10}$$

with $F(0) = 1$. A good fit is obtained with $F(t) = \dfrac{1}{1+4|t|+8t^2}$ and is shown in Fig. 3.

We point out that Imachi et al. (1971) have shown from the finite energy sum rule that dip is a resonance effect and this is obtained only in planar diagrams (H-type) contribu-

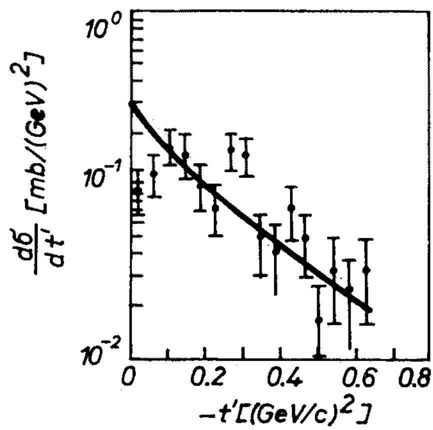

Fig. 3. The differential cross section $d\sigma/dt$ for the process $\pi^+\text{n} \to \text{p}\omega$ at laboratory pion energy 9 GeV/c

ting to the amplitude. As no planar diagram contributes to the process $\pi^+n \to p\omega$, we see that there should be no dip in this process in consistency with experiments.

Next we point out that although the process $\pi^+n \to p\omega$ is assumed to be contributed by ϱ-exchange here, yet the main contributing process is $\pi\pi \to \pi\omega$ via ϱ-exchange. Parity conservation requires odd angular momentum l in the $\varrho\pi$ system, angular momentum conservation limits the orbital angular momentum $l = 1$. It is to be noted here that the pion in the target proton is not free but is bound in a potential with certain orbital momentum (though we have neglected the interaction with the other constituents in the high energy limit). If this pion is assumed to be moving with $l = 1$ in the configuration of proton, then parity conservation as well as angular momentum conservation require the virtual ϱ meson in the $\varrho\pi$ system to have its orbital angular momentum $l = 0$. Evidently in this case the component m of the angular momentum along quantisation axis Δ (the momentum transfer vector) is zero. Hence only ϱ_{00} contributes i. e. $\varrho_{11} = \varrho_{1,-1} = \varrho_{10} = 0$ and $\varrho_{00} = 1$. Thus, we can explain the large value of the density matrix element ϱ_{00}^{G-J} ($\varrho_{00}^{G-J} = 0.62^{+0.13}_{-0.04}$ at 3.25 GeV/c in the range $0.7 \leqslant \cos\theta \leqslant 1$) observed in experiments. In fact, as the spin 1/2 partons in N will have some interference effect ϱ_{00} will not be exactly, 1, but it will have a substantial value as observed in experiments.

3. *Discussions*

We calculated the differential cross-section of the high energy scattering process $\pi^+n \to p\omega$ in the forward region according to the dual-parton model of hadrons and the results are found to be in excellent agreement with experiments. It may be added that the backward scattering process $\pi^+n \to \omega p$ can also be analysed according to this scheme when the rearrangement diagrams (Fig. 2) are modified in such a way that the spectator S also changes its side. Evidently, this will bring an additional factor $s^{-\gamma}$ to the rearrangement amplitude as follows from the discussions in the above section, and with $\gamma = 0.5$, we finally get the differential cross section in the backward region

$$\frac{d\sigma}{du} = 66.67 \frac{1}{s^3} \frac{F(u)}{m_\varrho^2 - u} \text{ mb/GeV}^2. \tag{11}$$

Thus, comparing Eq. (11) with Eq. (10), we predict for the ratio

$$\frac{\left(\frac{d\sigma}{dt}\right)_{t=0}}{\left(\frac{d\sigma}{du}\right)_{u=0}} = s. \tag{12}$$

However, as no data in the backward region is available until now, Eq. (12) cannot be tested at the present moment.

Finally, we observe that dual models so far discussed in literature pose very serious theoretical problems. Also the phenomenological dualam plitudes, such as the Veneziano model, are not devoid of theoretical inconsistency as it violates unitarity and also it does not fit two-body processes at all. In view of this, the good fit obtained for the process $\pi^+ n \rightarrow p\omega$ with all its characteristic features satisfied, the dynamical dual model as considered here seems to be worthy of further investigation.

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