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TWO VERSUS EIGHT GLUONS

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In this paper we discuss three puzzles of the quark model, formulated recently by Lipkin and answered by him on the ground of the three-triplet quark model with eight vector gluons which had, in fact, already been solved some years ago in the three-triplet quark model with two vector gluons. The lower symmetry of the latter model is sufficient to answer Lipkin's three questions but does not lead inevitably to the "symmetric quark model".

In this paper we discuss three puzzles of the quark model, formulated recently by Lipkin [1] and answered by him on the ground of the three-triplet quark model with eight vector glouns [2, 3, 4] which had, in fact, already been solved some years ago in the three-triplet quark model with two vector gluons [5]. The lower symmetry of the latter model is sufficient to answer Lipkin's three questions but does not lead inevitably to the so called "symmetric quark model" [3]. The two-gluon coupling can be embedded into the eight-gluon coupling which, as shown by Lipkin [1], provides the validity of the "symmetric quark model". All arguments here as well as in Lipkin's paper are based on the assumption that quark interactions within hadrons are dominated by one-gluon-exchange static forces. In particular, one-gluon annihilation forces appearing in the case of quark-antiquark interactions are neglected.

Let us first quote Lipkin's puzzles:

1. "*The triality puzzle*: With attractive interactions, why are three quarks and an antiquark not bound more strongly than a baryon or two quarks and an antiquark bound more strongly than a meson?"

2. "*The exotics puzzle*: Why are there no strongly bound exotic states of zero triality, like those of two quarks and two antiquarks or four quark and an antiquark?"

3. "*The diquark or meson-baryon puzzle*: Why is the quark-quark interaction just enough weaker than the quark-antiquark interaction so that the diquarks near the meson mass are not observed, but three quarks systems have masses comparable to those of mesons? If the quark mass is very heavy, one quark-antiquark interaction must compensate

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for two quark masses in a meson while three quark-quark interactions compensate for three quark masses in a baryon. This suggests that the quark-quark interaction is accidentally exactly half the strength of the quark-antiquark interaction."

These three questions were answered some time ago [5] by an observation that three-triplet quarks provide us with a natural mechanism of "saturation in three" of interquark strong forces if those are given by the following coupling of quarks and two vector gluons X_3^μ and X_8^μ:

$$g\bar{q}\,\frac{\sqrt{3}}{2}(-\lambda_8'+i\lambda_3')\gamma_\mu q\,\frac{1}{\sqrt{2}}(-X_8^\mu+iX_3^\mu)+h.c.$$

$$= g\sqrt{\frac{3}{2}}\,(\bar{q}\lambda_3'\gamma_\mu qX_3^\mu+\bar{q}\lambda_8'\gamma_\mu qX_8^\mu), \tag{1}$$

where λ_a' are the Gell-Mann λ's acting on the SU'(3)-triplet indices of q recently called colour indices [6]. The complex "quark charge" [5]

$$Z = \frac{\sqrt{3}}{2}(-\lambda_8'+i\lambda_3'), \tag{2}$$

appearing in (1), defined generally as

$$Z = \sqrt{3}\,(-F_8'+iF_3'), \tag{3}$$

describes the diagonal part of the SU'(3) generators F_a' and, therefore, we will adopt for it the term colour [6]. Its eigenvalues Z_A are

$$Z_1 = \frac{-1+i\sqrt{3}}{2},\quad Z_2 = \frac{-1-i\sqrt{3}}{2},\quad Z_3 = 1 \tag{4}$$

and have the properties

$$Z_1^3 = Z_2^3 = Z_3^3 = 1,\quad Z_1+Z_2+Z_3 = 0. \tag{5}$$

Our notation for X_3^μ and X_8^μ suggests that they are components 3 and 8 of an SU'(3) octet, but at the moment we assume nothing about that. We do assume, however, that they are neutral with respect to the colour Z. Then the coupling (1) is invariant under rotations around axes 3 and 8, giving conservation of Z.

The last of the properties (5) is the neutralization condition for the colour Z leading via (1) to the "saturation in three" of interquark strong forces, in a similar way as the electromagnetic coupling provides the "saturation in two" of electron-proton Coulomb forces in the atom (for the electric charge Q we have there $Q_1^2 = Q_2^2 = 1$ and $Q_1+Q_2 = 0$ instead of (5)). In fact, one-gluon-exchange static forces between quarks q_A and q_B (of colours Z_A and Z_B) or between quark q_A and antiquark $\bar{q}_B$ (of colours Z_A and $Z_B = -Z_B$) are given by the interaction energies [5]

$$V_{AB} = \begin{Bmatrix} 2 \text{ for } A = B \\ -1 \text{ for } A \neq B \end{Bmatrix}\frac{g^2}{4\pi}\,\frac{e^{-m_X r}}{r}, \tag{6}$$

or

$$V_{A\bar{B}} = \begin{cases} -2 \text{ for } A = B \\ \ \ 1 \text{ for } A \neq B \end{cases} \frac{g^2}{4\pi} \frac{e^{-m_X r}}{r}, \tag{7}$$

respectively, where m_X is the mass of gluons X_3^μ and X_8^μ (we assume equal masses for both). Formula (6) describes attraction/repulsion between two quarks of different/equal colour indices A and B, whereas (7) gives attraction/repulsion between a quark and an antiquark of equal/different colour indices A and B.

Formulae (6) and (7) can be written down together in the form

$$V_{ij} = v_{ij}\left(\frac{\lambda'_{3i}}{2}\frac{\lambda'_{3j}}{2} + \frac{\lambda'_{8i}}{2}\frac{\lambda'_{8j}}{2}\right), \qquad v_{ij} = 6\,\frac{g^2}{4\pi}\,\frac{e^{-m_X r_{ij}}}{r_{ij}}, \tag{8}$$

where λ'_{ai} are λ'_a or $-\lambda'_a$ acting on colour indices of the i-th quark or antiquark, respectively. Thus, for a system of n quarks or antiquarks we have the following interaction energy:

$$V(n) = \frac{1}{2}\sum_{i\neq j} v_{ij}\left(\frac{\lambda'_{3i}}{2}\frac{\lambda'_{3j}}{2} + \frac{\lambda'_{8i}}{2}\frac{\lambda'_{8j}}{2}\right). \tag{9}$$

If the dependence of v_{ij} on the individual particles i and j is neglected in space expectation values ($\bar{v}_{ij} = \bar{v}$), then the average interaction energy of n quarks or antiquarks is given by the formula

$$\bar{V}(n) = \frac{1}{2}\bar{v}\sum_{i\neq j}\left(\frac{\lambda'_{3i}}{2}\frac{\lambda'_{3j}}{2} + \frac{\lambda'_{8i}}{2}\frac{\lambda'_{8j}}{2}\right) = \frac{1}{2}\bar{v}\left(F_3'^2 + F_8'^2 - \frac{1}{3}n\right) = \frac{1}{6}\bar{v}(|Z|^2 - n), \tag{10}$$

where

$$F'_a = \sum_i \frac{\lambda'_{ai}}{2}, \qquad Z = \sqrt{3}\sum_i\left(-\frac{\lambda'_{8i}}{2} + i\,\frac{\lambda'_{3i}}{2}\right) = \sum_i Z_i, \tag{11}$$

Z_i being the colour of the i-th quark or antiquark, i. e. $Z_i = Z_A$ if the i-th particle is q_A or $Z_i = -Z_A$ if the i-th particle is $\bar{q}_A$. It follows from (10) that

$$\bar{V}(n)_{Z=0} < 0, \qquad |\bar{V}(n)_{Z=0}| \geqslant |\bar{V}(n)|. \tag{12}$$

Thus, we can see that one-gluon-exchange static forces following from coupling (1) are globally attractive and are saturated in a system of n quarks or antiquarks if the system is neutral with respect to the colour Z. For instance, in qqq systems these forces are attractive and saturated for $q_1q_2q_3$ (the "saturation in three"), whereas in qq and $q\bar{q}$ systems they are attractive and not saturated for q_1q_2, q_2q_3, q_3q_1, and attractive and saturated for $q_1\bar{q}_1$, $q_2\bar{q}_2$, $q_3\bar{q}_3$. Both properties are much desirable in the quark model of baryons and mesons. In the case of qqq systems, the saturation condition leads via Fermi statistics (we assume equal masses for quarks of different colours) to the SU′(3)-scalar bound states $\varepsilon_{ABC}q_Aq_Bq_C$ if we consider qqq states to be symmetric under permutations of $SU(6)\times O(3)_L$ degrees of freedom as it is assumed in the "symmetric quark model" [3]. However, the "symmetric quark model" does not follow necessarily from the saturation condition. Similarly, in the case of $q\bar{q}$ systems the saturation condition does not imply the SU′(3)-scalar bound states $q_A\bar{q}_A$ although the bound states $q_1\bar{q}_1$, $q_2\bar{q}_2$, $q_3\bar{q}_3$, degenerated in mass, are implied. One can argue that switching on annihilation forces for $q\bar{q}$ systems provides that the $q\bar{q}$ ground bound states are $q_A\bar{q}_A$ (cf. [5]).

Now, the answer to Lipkin's first puzzle follows from the comparison of masses of qqq and $q\bar{q}$ bond states with masses of possible $qqq\bar{q}$ and $q\bar{q}q$ bound states. Denoting by m_q, m_B, m_M the quark, baryon and meson mass, respectively, we obtain from (10)

$$3m_q+\bar{V}(q_1q_2q_3) = 3\left(m_q-\frac{\bar{v}}{6}\right) = m_B, \tag{13}$$

$$2m_q+\bar{V}(q_A\bar{q}_A) = 2\left(m_q-\frac{\bar{v}}{6}\right) = m_M \tag{14}$$

and

$$4m_q+\bar{V}(q_1q_2q_3\bar{q}_A) = 4m_q-3\frac{\bar{v}}{6} = m_B+m_q, \tag{15}$$

$$3m_q+\bar{V}(q_A\bar{q}_Aq_B) = 3m_q-2\frac{\bar{v}}{6} = m_M+m_q, \tag{16}$$

where $A, B = 1, 2, 3$. So, the coloured states $qqq\bar{q}$ and $q\bar{q}q$, if bound at all, are bound more weakly and have much higher masses than baryons and mesons. In our approximation they are rather the scattering states $qqq+\bar{q}$ and $q\bar{q}+q$. We assume here that $m_q \gg m_B \simeq m_M$.

To discuss Lipkin's second puzzle we find out from (10) that

$$5m_q+\bar{V}(q_1q_2q_3q_A\bar{q}_A) = 5\left(m_q-\frac{\bar{v}}{6}\right) = m_B+m_M \tag{17}$$

and

$$4m_q+\bar{V}(q_A\bar{q}_Aq_B\bar{q}_B) = 4\left(m_q-\frac{\bar{v}}{6}\right) = 2m_M. \tag{18}$$

Thus, the non-coloured exotic states $qqqq\bar{q}$ and $q\bar{q}q\bar{q}$ are in our approximation not bound states but rather scattering states $qqq+q\bar{q}$ and $q\bar{q}+q\bar{q}$.

In order to answer Lipkin's third puzzle we compare masses of qqq, $q\bar{q}$ and qq bound states. Since from (10)

$$2m_q+\bar{V}(q_Aq_B) = 2m_q-\frac{\bar{v}}{6} = \frac{m_M}{2}+m_q, \tag{19}$$

where $A \neq B$, we conclude that the coloured bound states qq are bound more weakly and have much higher mass than both baryons and mesons whose masses are comparable. Notice that $V(q_Aq_B) : V(q_A\bar{q}_A) = 1 : 2$, where $A \neq B$.

In contradistinction to quark states which can be SU′(3) scalars (if they are neutral with respect to the colour Z), gluon states cannot be SU′(3) scalars (being, however, always neutral with respect to Z) if the gluons X_3^μ and X_8^μ are components 3 and 8 of an SU′(3) octet. In this case coupling (1) is invariant under rotations in 3–8 plane, in addition to its former invariance under rotations around axes 3 and 8.

It is obvious that under this assumption about X_3^μ and X_8^μ the coupling (1) can be embedded into the SU′(3)-invariant Yang-Mills coupling of coloured quarks and eight vector gluons X_a^μ forming an SU′(3) octet [2–4]:

$$g\sqrt{\tfrac{3}{2}}\,\bar{q}\lambda'_a\gamma_\mu q X_a^\mu, \tag{20}$$

where X_3^μ and X_8^μ are identical with the $Z = 0$ vector gluons appering in (1). To this coupling refers Lipkin's paper [1]. Now, the saturation condition leads automatically to the SU′(3)-scalar bound states $\varepsilon_{ABC}q_Aq_Bq_C$ which being antisymmetric in colour indices provide via Fermi statistics the validity of the "symmetric quark model". Similarly, the SU′(3)-scalar bound states $q_A\bar{q}_A$ become the only $q\bar{q}$ bound states.

However, as we have seen, it is not necessary to introduce the SU′(3)-invariant coupling (20) in order to allow for SU′(3)-invariant quark states. Moreover, even the SU′(3)-invariant coupling (20) does not guarantee automatically (in any evident way) the SU′(3)-invariance for all quark and gluon states. To this end the condition

$$F'_a\Psi = 0, \quad (a = 1, 2, \ldots, 8) \tag{21}$$

must be satisfied for all physical states Ψ. In order to quarantee, in addition, the imprisonment of all quarks and gluons inside physical hadrons, some stronger condition must be fulfilled, since (21) allows for appearing of separated quarks and gluons in globally SU′(3)-invariant asymptotic states.

It remains an open question as to whether, in the present stage of the theory, this imprisonment is to be assumed as an additional constraint for physical states or rather follows somehow [7–10] from the non-Abelian Yang-Mills theory with the coupling (20). There is also another possibility that coloured quarks $q_{\alpha A}$ are in fact dissociated into Gell–Mann–Zweig quarks q_α and new coloured scalar gluons c_A forming an SU′(3) triplet, which obey compensating "wrong" statistics, i. e. Bose for q_α and Fermi for c_A [11]. Because of the wrong statistics the particles q_α and c_A cannot be described by quantum fields satisfying usual axioms (including the asymptotic condition) and, therefore, must not appear in asymptotic states as separated particles. This possibility is perhaps supported by the well-known parton-model result that the fraction of neutral partons (gluons) inside the nucleon probed by deep inelastic lepton scattering is 50% if charged partons are assumed to be quarks with charges 2/3, −1/3, −1/3. Indeed, in the case of dissociated coloured quarks $q_{\alpha A}$, the quarks q_α and gluons c_A are in the proportion 50%: 50%.

Notice that in the case of dissociated coloured quarks $q_{\alpha A}$ the forces described by (6) or (7) act between gluons c_A and c_B (of colours Z_A and Z_B) or between a gluon c_A and an antigluon $\bar{c}_B$ (of colours Z_A and $\bar{Z}_B = -Z_B$). The saturation condition for these forces leads via Fermi statistics of c_A's to the SU′(3)-scalar bound states $\varepsilon_{ABC}c_Ac_Bc_C$, while q_α's satisfying Bose statistics must form states symmetric under $SU(6) \times O(3)_L$ degrees of freedom. Thus the "symmetric quark model" is now valid.

In conclusion, we can say that one-gluon-exchange forces in the two-gluon scheme are sufficient to answer Lipkin's three puzzles. They have, however, too low a symmetry to exclude by themselves some experimentally unobserved SU′(3)-non-scalar bond states qqq and $q\bar{q}$ which appear here and are unbound in the eight-gluon scheme. E. g. for $q\bar{q}$ system there appear, in addition to the SU′(3)-singlet, two unobserved components, 3 and 8, of the SU′(3)-octet which are unbound in the eight-gluon scheme. The unobserved qqq and $q\bar{q}$ bound states are excluded also in the two-gluon scheme if the dissociated-coloured-quark model is valid and $q\bar{q}$ annihilation forces are sufficient to remove the excited $q\bar{q}$ bound states (being the components 3 and 8 of the SU′(3)-octet).

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