TWO-DIMENSIONAL DESCRIPTION OF THE VIBRATIONAL MOTION IN ACTINIDE NUCLEI

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Stiffness and mass parameters as well as the energies of the vibrational states in some actinide nuclei have been calculated. Only quadrupole and hexadecapole degrees of freedom have been included. The calculations have been performed in the vicinity of the first minimum of the energy surface, by making use of three different approaches: Mottelson-Nilsson-Bés-Szymański, Strutinsky and microscopic ones. The results obtained in the calculations have been compared.

1. Introduction

Several authors performed (see for example Ref. [2–5]) calculations including quadrupole and hexadecapole degrees of freedom to describe some static nuclear properties. Such approach was very successful in the explanation of nuclear ground state deformations, nuclear masses, stability etc., as well as some features of the fission process. In the present paper we apply the two-dimensional approach to the description of nuclear vibrational motion. The energy of vibrational states can be expressed by eigen-oscillations of the system in the framework of the theory of small oscillations.

This energy is proportional to the square root of the expression containing the ratio of the appropriate stiffness and mass parameters characterizing the motion. The stiffness parameters can be calculated using two different approaches:
(*i*) from the curvature of the energy surface, and
(*ii*) from the microscopic [1, 2] calculations based on the collective description of the vibrations obtained from the multipole plus pairing treatment.

The energy surface has been calculated either by adding the energies of the occupied single-particle levels in deformed field (Mottelsson-Nilson-Bés-Szymański (MN–BS) method) [6, 7], or by the use of Strutinsky [8] presciption. The mass parameters are determined from the microscopic method only.

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One can see that the physical assumptions underlying the two methods described above are different. Therefore, the ground state shapes obtained from minimizing the total sum of single-particle energies are different than those obtained from the shell correction method (the first give much worse agreement with experiment, except for quadrupole case; see Ref. [9] and Table I in our paper, as an example). Also the comparison between the total energies obtained from MN–BS procedure and from the approach based on the quadrupole interaction between nucleons indicates that the agreement between them occurs only in the stationary points (see Ref. [10]).

This is the fact of special interest while studying the vibrational excitations. The aim of the present paper is to calculate the vibrational energies of the lowest quadrupole and hexadecapole excitations by making use of the different methods applied to the calculations of stiffness parameters.

2. *Stiffness and mass parameters*

We consider the total energy of the collective motion in the case of harmonic approximation. It is expressed by the classical formula containing the static and dynamic terms

$$E = E_0(\alpha_{20}, \alpha_{40}) + \tfrac{1}{2} \sum_{i,j=2,4} C_{\alpha_i\alpha_j}(\alpha_i - \alpha_{i0})(\alpha_j - \alpha_{j0}) + \tfrac{1}{2} \sum_{i,j=2,4} B_{\alpha_i\alpha_j}\dot{\alpha}_i\dot{\alpha}_j, \tag{1}$$

where E_0 is the static energy of the equilibrium state, α_i are collective coordinates considered either as multipole moments or as parameters describing the nuclear shape, α_{i0} are their equilibrium values, $C_{\alpha_i\alpha_j}$ and $B_{\alpha_i\alpha_j}$ are stiffness and mass parameters, respectively.

a. Static calculations of stiffness parameters

From Eq. (1) one can simply derive the expressions for stiffness parameters. They can be written in the equilibrium point in the following form

$$C_{\alpha_i\alpha_j} = \left.\frac{\partial^2 E}{\partial\alpha_i \partial\alpha_j}\right|_{\substack{\alpha_i=\alpha_{i0} \\ \alpha_j=\alpha_{j0}}}. \tag{2}$$

In order to obtain these parameters we should know in principle the behaviour of the total energy surface. In practice we calculate E only in a few points around equilibrium in order to determine the derivatives.

The static energy of the nucleus has been calculated in two ways:

(*i*) According to Mottelsson-Nilsson [6] prescription including pairing forces what was done by Bés and Szymański [7] (MN–BS method). The deformation energy was calculated by adding the energies of single-particle levels with the inclusion of the pairing interaction between nucleons in the deformed field with constant volume

$$E^{\mathrm{MN-BS}} = \sum_{\nu=1}^{Z} 2v_\nu^2 e_\nu - \frac{\Delta_p^2}{G_p} - G_p \sum_{\nu=1}^{Z} v_\nu^4 + \sum_{\nu=1}^{N} 2v_\nu^2 e_\nu - \frac{\Delta_n^2}{G_n} - G_n \sum_{\nu=1}^{N} v_\nu^4, \tag{3}$$

where e_ν are single-particle energies obtained from Nilsson Hamiltonian H

$$H^{\mathrm{N}} = \frac{1}{2}\hbar\omega_0(\varepsilon_2, \varepsilon_4)\left[-\Delta_\varrho + \frac{2}{3}\varepsilon_2 \frac{1}{2}(2\partial^2/\partial\zeta^2 - \partial^2/\partial\xi^2 - \partial^2/\partial\eta^2)\right.$$

$$\left. + \varrho^2\left(1 - \frac{2}{3}\varepsilon_2 P_2(\cos\theta_t) + 2\varepsilon_4 P_4(\cos\theta_t)\right) - 4\kappa\frac{\overset{\circ}{\omega}_0}{\omega_0}\vec{l}_t\cdot\vec{s} - 2\mu\kappa\frac{\overset{\circ}{\omega}_0}{\omega_0}(\vec{l}_t^{\,2} - \langle\vec{l}_t^{\,2}\rangle)\right] \tag{4}$$

(notation as in Ref. [2]), $\Delta_{\mathrm{p}}, \Delta_{\mathrm{n}}, G_{\mathrm{p}}, G_{\mathrm{n}}$ and v_ν are the energy gaps, the pairing force strengths and pairing factors, respectively, connected with BCS [12] description of pairing interaction (considered for protons and neutrons separately). The Coulomb correction has been also included (see Ref. [3]).

Since this approach is connected with the description of nuclear shape, we chose parameters α_i as deformation parameters ε_i. Therefore, formula (2) assumes the following form

$$C_{\varepsilon_i\varepsilon_j} = \left.\frac{\partial^2 E^{\mathrm{MN-BS}}}{\partial\varepsilon_i\partial\varepsilon_j}\right|_{\substack{\varepsilon_i=\varepsilon_{i0}\\ \varepsilon_j=\varepsilon_{j0}}} \quad \text{for } i,j = 2,4. \tag{5}$$

(*ii*) By making use of Strutinsky [8] method. The potential energy of the nucleus is calculated as the sum of the energy of the homogeneously charged liquid drop and the Strutinsky shell correction connected with single-particle aspects of nuclear structure. In comparison with the previous method the smooth part of the energy is extracted and replaced by corresponding term calculated in the liquid drop model. The pairing forces are also included. Consequently the following expression for the energy is obtained

$$E^{\mathrm{Strut}} = E^{\mathrm{LD}} + \Delta E^{\mathrm{Strut}} + \Delta P, \tag{6}$$

where E^{LD} is the liquid drop energy given by Weizsäcker type formula, ΔP is the pairing correction term considered as the result of a substraction of the sum of single-particle energies without pairing interaction between nucleons from the similar sum including pairing interaction

$$\Delta P = E^{\mathrm{MN-BS}} - \left(\sum_{\nu=1}^{Z/2} 2e_\nu - \frac{G_{\mathrm{p}}}{2}Z + \sum_{\nu=1}^{N/2} 2e_\nu - \frac{G_{\mathrm{n}}}{2}N\right). \tag{7}$$

Finally, the shell correction term can be written as

$$\Delta E^{\mathrm{Strut}} = 2\left(\int_{-\infty}^{\varepsilon_{\mathrm{F}}} G(e)ede - \int_{-\infty}^{\varepsilon_{\mathrm{F}}} g(e)ede\right), \tag{8}$$

where

$$G(e) = \sum_\nu \delta(e - e_\nu) \tag{8a}$$

denotes the sharp distribution of single-particle levels and

$$g(e) = \frac{1}{\gamma\sqrt{\pi}} \sum_{\nu} f_{\text{corr}} \exp\left(-\frac{e-e_\nu}{\gamma}\right)^2 \tag{8b}$$

denotes the smeared-out level density (for wider discussion see for example Ref. [9]).

For stiffness parameters we use again formula (2)

$$C^{\text{Strut}}_{\varepsilon_i\varepsilon_j} = \left.\frac{\partial^2 E^{\text{Strut}}}{\partial\varepsilon_i\partial\varepsilon_j}\right|_{\substack{\varepsilon_i=\varepsilon_{i0}\\ \varepsilon_j=\varepsilon_{j0}}} \quad \text{for } i,j = 2,4. \tag{9}$$

Another way for obtaining the stiffness as well as the mass parameters is the microscopic method.

b. Microscopic calculations of stiffness and mass parameters

The principles of microscopic calculations are presented in Ref. [1] while the details of the calculations are described in Ref. [2]. This method is based on the assumption of independent motion of the nucleons in the single particle deformed potential (in our case it is the Nilsson potential) and of their interaction by the pairing and multipole-multipole forces. Small oscillations around equilibrium point as well as the adiabaticity of the motion are assumed. From the microscopic calculations we obtain the following formulae for stiffness and mass parameters

$$C^{\text{Micro}}_{Q_iQ_j} = \tfrac{1}{2}\sigma_1^{ij} - \kappa_i\delta_{ij}, \tag{10}$$

and

$$B^{\text{Micro}}_{Q_iQ_j} = \tfrac{1}{2}\hbar^2 \sum_{k,l=2,4} \sigma_1^{ik}\Sigma_3^{kl}\sigma_1^{lj} \tag{11}$$

for $i,j = 2,4$, where

$$\Sigma_n^{ij} = \sum_{\mu,\nu} \frac{\langle\mu|\hat{q}_i|\nu\rangle\langle\nu|\hat{q}_j|\mu\rangle}{(E_\mu+E_\nu)^n}(u_\mu v_\nu + u_\nu v_\mu)^2 \tag{12}$$

for $n = 1,3$; $i,j = 2,4$. In this expression E_ν denotes the quasiparticle energy, $\langle\mu|\hat{q}_i|\nu\rangle$ is matrix element of the single-particle quadrupole $\hat{q}_2$

$$\hat{q}_2 = 2\mu^2 P_2(\cos\theta), \tag{13}$$

or hexadecapole $\hat{q}_4$ (notation as in Ref. [2])

$$\hat{q}_4 = \varrho^2 P_4(\cos\theta_t) \tag{14}$$

moment between Nilsson states and u_ν, v_ν are BCS pairing coefficients. The quantity σ_1^{ij} denotes the matrix inverse with respect to Σ_1^{ij} and κ_i are quadrupole ($i = 2$) and hexadecapole ($i = 4$) force strengths.

It seems that the hexadecapole moment

$$q_4 = 8r^4 P_4 (\cos\theta) \tag{15}$$

ought to be used when calculating collective vibrational energies. We have taken, however, the expression $q_4 = \varrho^2 P_4 (\cos\theta_t)$ bearing in mind that the single-particle Nilsson potential (Eq. (4)) contains the corresponding term.

One can see that in the expression

$$q_4 = 8r^4 P_4 (\cos\theta) = \varrho^4 (aP_4 (\cos\theta_t) + bP_2 (\cos\theta_t) + cP_0 (\cos\theta_t)) \tag{16}$$

(where a, b and c are constant for a given set of deformation parameters) there is no term like Nilsson's $\varrho^2 P_4 (\cos\theta_t)$ operator. This correspondence is needed in the procedure of calculating κ_4 parameter (described in Sec. 3).

The description of motion in microscopic approach is given in terms of multipole forces. In order to make comparison with quantities obtained before ($C^{\mathrm{MN-BS}}_{\varepsilon_i\varepsilon_j}$ and $C^{\mathrm{Strut}}_{\varepsilon_i\varepsilon_j}$), we transform parameters $B^{\mathrm{Micro}}_{Q_iQ_j}$ and $C^{\mathrm{Micro}}_{Q_iQ_j}$ to ε_2 and ε_4 coordinates. We obtain

$$B^{\mathrm{Micro}}_{\varepsilon_i\varepsilon_j} = \sum_{k,l=2,4} B_{Q_kQ_l} \frac{\partial Q_k}{\partial\varepsilon_i} \frac{\partial Q_l}{\partial\varepsilon_j}, \tag{17}$$

where Q_k are the total mass multipole moments, obtained on the base of BCS procedure

$$Q_k = 2 \sum_{\nu} \langle \nu | \hat{q}_k | \nu \rangle v_\nu^2. \tag{18}$$

Similar relations are valid for stiffness parameters $C^{\mathrm{Micro}}_{\varepsilon_i\varepsilon_j}$. Formula (10) for stiffness tensor contains the force strength parameters. Therefore, in order to proceed with calculations we have to determine their values.

3. The quadrupole and hexadecapole force strengths

In order to determine κ_2 and κ_4 parameters we compare two different descriptions of the deformed system. We start with a two-body Hamiltonian

$$H^{\mathrm{TB}} = H_0 - \frac{\kappa_2}{2} \hat{q}_2 \hat{q}_2 - \frac{\kappa_4}{2} \hat{q}_4 \hat{q}_4. \tag{19}$$

Applying the Hartree method we obtain one-body Hamiltonian out of Eq. (19)

$$H^{\mathrm{OB}} = H_0 - \kappa_2 Q_2 \hat{q}_2 - \kappa_4 Q_4 \hat{q}_4, \tag{20}$$

where $\hat{q}_2$ and $\hat{q}_4$ are single-particle multipole moments (Eqs (13) and (14)) whereas Q_2 and Q_4 are their total multipole moments (Eq. (18)).

Now, according to Mottelson [13] prescription, we identify this Hamiltonian with Nilsson Hamiltonian (see Eq. (4)), requiring that the corresponding terms standing in front of $\hat{q}_2$ and $\hat{q}_4$ operators are identical (see remark connected with Eqs (15) and (16)). If we consider H_0 as Nilsson Hamiltonian taken in the equilibrium point and the remaining terms as describing the deviation from the equilibrium we obtain [15] the following expressions for force strength parameters (after comparison with the Nilsson Hamiltonian taken

in a given point, beyond equilibrium)

$$\kappa_2 = \frac{M\omega_0^2}{6}\left(\frac{\partial Q_2}{\partial \varepsilon_2}\right)^{-1}\left(1+\frac{1}{3}\varepsilon_2\right)\left(1-\frac{2}{3}\varepsilon_2\right), \tag{21}$$

$$\kappa_4 = -\hbar\omega_0\left(\frac{\partial Q_4}{\partial \varepsilon_4}\right)^{-1}. \tag{22}$$

The following relation has been used

$$\hat{q}_2 = 2r^2 P_2(\cos\theta) = \frac{\hbar}{M\omega_0}\frac{1}{\left(1+\frac{1}{3}\varepsilon_2\right)\left(1-\frac{2}{3}\varepsilon_2\right)}\left(2\varrho^2 P_2(\cos\theta_t)+\frac{2}{3}\varepsilon_2\varrho^2\right)$$

in order to compare Eq. (20) with Nilsson potential. We have not investigated the coupling between the two kinds of nucleons.

For the methods of calculating multiple force strength parameters see also Ref. [1, 10, 13, 14].

4. The energies of the quadrupole and hexadecapole states

In order to obtain the energies of the lowest quadrupole and hexadecapole vibrational states in two-dimensional plane $(\varepsilon_2, \varepsilon_4)$ we investigate the normal modes of the motion. Therefore, we transform the total energy of the vibrating nucleus (see Eq. (1))

$$E = E_0(\varepsilon_{20}, \varepsilon_{40}) + \tfrac{1}{2}\sum_{i,j=2,4} C_{\varepsilon_i\varepsilon_j}(\varepsilon_i-\varepsilon_{i0})(\varepsilon_j-\varepsilon_{j0}) + \tfrac{1}{2}\sum_{i,j=2,4} B_{\varepsilon_i\varepsilon_j}\dot{\varepsilon}_i\dot{\varepsilon}_j \tag{23}$$

to the canonical form. The energies of the vibrational state are given by the eigen-oscillations of the system

$$E_{\text{vib}} = \hbar\omega_{\text{vib}}, \tag{24}$$

which leads, in our case, to the following expressions

$$E_2 = \hbar\sqrt{\frac{-B-\sqrt{B^2-4AC}}{2A}}, \tag{25}$$

$$E_4 = \hbar\sqrt{\frac{-B+\sqrt{B^2-4AC}}{2A}}, \tag{26}$$

where

$$A = B_{\varepsilon_2\varepsilon_2}B_{\varepsilon_4\varepsilon_4} - B_{\varepsilon_2\varepsilon_4}^2,$$

$$B = 2C_{\varepsilon_2\varepsilon_4}B_{\varepsilon_2\varepsilon_4} - C_{\varepsilon_2\varepsilon_2}B_{\varepsilon_4\varepsilon_4} - C_{\varepsilon_4\varepsilon_4}B_{\varepsilon_2\varepsilon_2},$$

$$C = C_{\varepsilon_2\varepsilon_2}C_{\varepsilon_4\varepsilon_4} - C_{\varepsilon_2\varepsilon_4}^2.$$

The energies of the vibrational states have been calculated by making use of formulae (25) and (26) with the mass parameters obtained from Eq. (17) and with the stiffness parameters calculated according to MN–BS (Eq. (5)), Strutinsky (Eq. (9)) and microscopic methods.

5. *Results and discussion*

The following details of the computations underlie our procedure:

(*i*) The matrix elements of the Nilsson Hamiltonian have been calculated between the states with the same major quantum number N. The remaining elements have not been included. The oscillator shells were included up to $N_{max} = 9$ and the usual Nilsson model parameters were used:

$$\kappa_p = 0.0577, \quad \mu_p = 0.650,$$

$$\kappa_n = 0.0635, \quad \mu_n = 0.325.$$

(*ii*) In the calculations of the matrix elements of single-particle multipole moments (Eq. (12)) between Nilsson states also the states with $\Delta N = 2$ have been taken into account.

(*iii*) Pairing interaction has been accounted for by the BCS procedure with the pairing forces strength independent of deformation.

The BCS equations were solved within Z or N levels and the pairing forces strengths were

$$G_p = 20.02 \text{ MeV/A}, \quad G_n = 14.1 \text{ MeV/A},$$

for protons and neutrons, respectively.

(*iv*) In the calculation of the Strutinsky shell correction the correction factor f_{corr} (Eq. (8b)) was taken as a polynomial of order six [3] and the smearing parameter $\gamma = 0.8\,\hbar\omega_0$.

The numerical calculations have been performed for a few nuclei in actinide region around the first minimum of the energy surface in 16 grid points, by using

4 values of ε_2 ($\varepsilon_2 = 0.17,\ 0.19,\ 0{,}21,\ 0.23$) and

4 values of ε_4 ($\varepsilon_4 = -0.06,\ -0.05,\ -0.04,\ -0.03$).

The equilibrium deformations obtained from MN–BS (Eq. (3)) and Strutinsky (Eq. (6)) energy surfaces are presented in Table I. One can see that these methods give similar results only for quadrupole distortion while in the hexadecapole case the MN–BS values are about two times bigger than Strutinsky ones.

TABLE I

The equilibrium deformations calculated using Mottelson–Nilsson–Bés–Szymański and Strutinsky methods

Z	N	ε_{2eg}		ε_{4eg}	
		BS	STRUT	BS	STRUT
92	142	0.198	0.199	−0.022	−0.049
92	144	0.206	0.208	−0.020	−0.042
92	146	0.213	0.213	−0.017	−0.036
94	144	0.216	0.216	−0.020	−0.039
94	146	0.222	0.229	−0.017	−0.029

For the partial explanation of this descrepancy see Fig. 1. The values of the hexadecapole MN—BS equilibrium deformations calculated with the inclusion of the coupling between the single-particle levels ($\Delta N = 0$ and $\Delta N = 2$) by B. Nilsson [5] and our values without the coupling (only $\Delta N = 0$) are compared. One can see that the inclusion of coupling

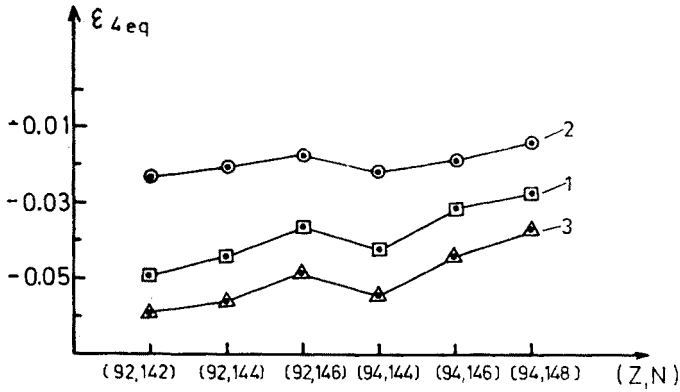

Fig. 1. Hexadecapole equilibrium deformations calculated from Mottelson–Nilsson–Bés–Szymański method including coupling between two shells (points denoted by *1*) and without coupling (points denoted by *2*). For comparison the corresponding values obtained from Strutinsky method are presented (points denoted by *3*)

between two shells in calculations of single-particle states influences strongly the values of hexadecapole equilibrium points. For comparison also Strutinsky's corresponding values are presented (as in Table I).

Thus, we shall use in our further calculations the Strutinsky equilibrium values and we shall not present the results connected with the hexadecapole distortion from the MN–BS method.

Table II presents the values of quadrupole and hexadecapole force strengths. The relation between stiffness parameters $C^{\text{Micro}}_{\varepsilon_2\varepsilon_2}$ and $C^{\text{Micro}}_{\varepsilon_4\varepsilon_4}$ and deformation parameters ε_2 and ε_4 can be deduced from Figs 2–5. It can be seen that the dependence of $C^{\text{Micro}}_{\varepsilon_2\varepsilon_2}$ and $C^{\text{Micro}}_{\varepsilon_4\varepsilon_4}$ on ε_2 is rather strong (Figs 2 and 3). While ε_2 changes from 0.17 to 0.23 the parameter $C^{\text{Micro}}_{\varepsilon_2\varepsilon_2}$ decreases by about 40% and $C^{\text{Micro}}_{\varepsilon_4\varepsilon_4}$ decreases by about 50% in com-

TABLE II

Force strength parameters for quadrupole (κ_2) and hexadecapole (κ_4) vibrational modes

Z	N	$\kappa_2\left[\text{MeV}\left(\frac{M\omega_0}{\hbar}\right)^2\right]$	κ_4 [MeV]
92	142	6.29	−4.49
92	144	5.94	−4.52
92	146	5.84 $\times 10^{-4}$	−4.38 $\times 10^{-2}$
94	144	6.15	−4.65
94	146	5.77	−4.45

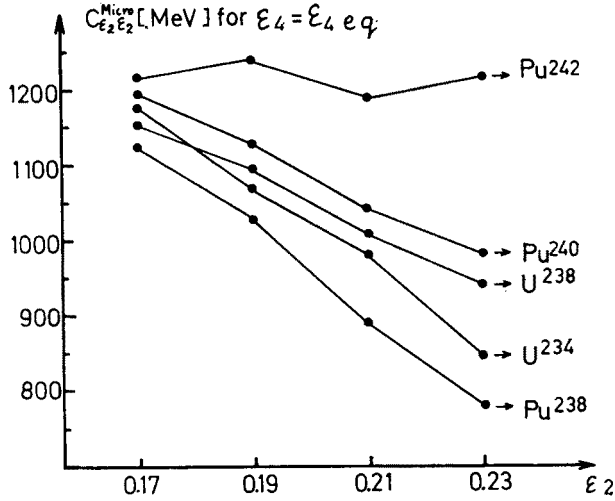

Fig. 2. The dependence of calculated microscopically stiffness parameters $C^{Micro}_{\varepsilon_2\varepsilon_2}$ on deformation ε_2 for ε_4 corresponding to the equilibrium value

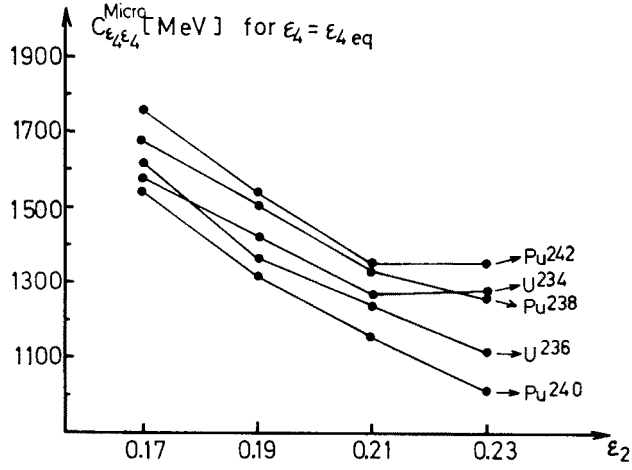

Fig. 3. The same as on Fig. 2 for $C^{Micro}_{\varepsilon_4\varepsilon_4}$ parameters

parison with values reached at $\varepsilon_2 = 0.17$. The dependence of stiffness parameters on ε_4 is much weaker. Namely, $C^{Micro}_{\varepsilon_2\varepsilon_2}$ decrease slightly (Fig. 4) and $C^{Micro}_{\varepsilon_4\varepsilon_4}$ increase about 10% with increasing of ε_4 from -0.06 to -0.03. The relations between stiffness and deformation parameters considered above are valid approximately for all investigated nuclei except ^{234}U and ^{242}Pu.

Similarly, the different behaviour for ^{234}U and ^{242}Pu can be also observed in the case of studying mass parameters (Figs 6–9). The reason for this fact is not clear for us. Perhaps

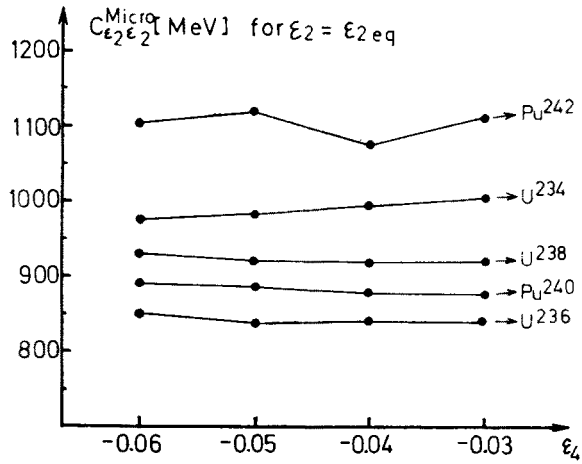

Fig. 4. The dependence of stiffness parameters $C^{Micro}_{\varepsilon_2\varepsilon_2}$ on ε_4 for ε_2 corresponding to the equilibrium value

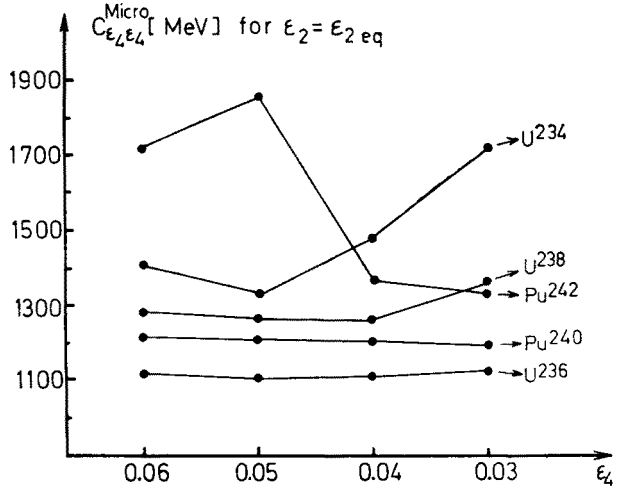

Fig. 5. The same as on Fig. 4 for $C^{Micro}_{\varepsilon_4\varepsilon_4}$

one of the single-particle levels with a strong ε_4 dependence appeared near the Fermi surface giving the stronger dependence of stiffness as well as mass parameters on ε_4 than ε_2, which is not valid for remaining nuclei. The mass parameters $B^{Micro}_{\varepsilon_2\varepsilon_2}$ and $B^{Micro}_{\varepsilon_4\varepsilon_4}$ are plotted versus deformation parameters ε_2 and ε_4 on Figs 6–9. One can see that $B^{Micro}_{\varepsilon_2\varepsilon_2}$ parameters depend strongly on ε_2. Their increase is approximately equal 50% while ε_2 increases from 0.17 to 0.23. In the remaining cases the dependence of mass parameters on deformation is much weaker. (This is not true for ^{234}U and ^{242}Pu as mentioned before).

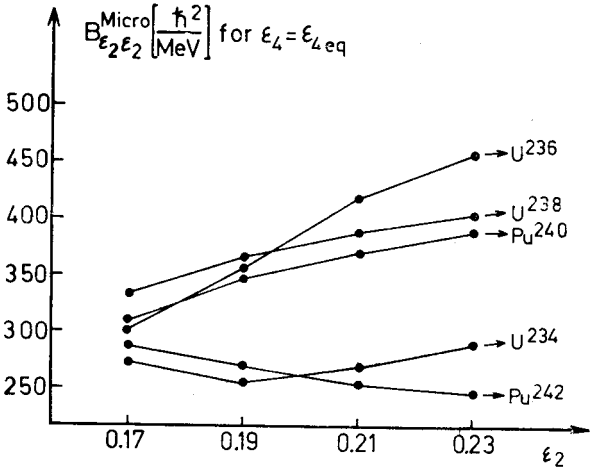

Fig. 6. The dependence of mass parameters $B^{Micro}_{\varepsilon_2\varepsilon_2}$ on ε_2 deformation presented for ε_4 equal to the equilibrium value

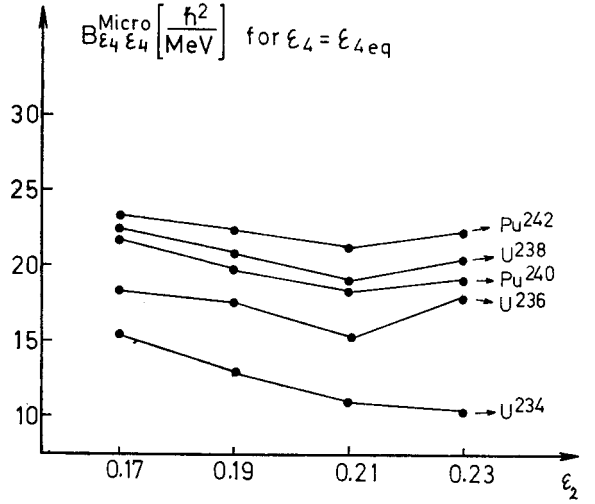

Fig. 7. The same as on Fig. 6 for $B^{Micro}_{\varepsilon_4\varepsilon_4}$ parameters

Table III illustrate the vibrational energies of quadrupole E_2 (Eq. (25)) and hexadecapole E_4 (Eq. (26)) modes. Mass parameters used in the paper have been calculated from Eq. (17). The results presented in the corresponding columns differ in the method of calculating the stiffness parameters. The energies $E^{Micro}_{2(4)}$ have been calculated with stiffness microscopic parameters $C^{Micro}_{\varepsilon_i\varepsilon_j}$ while the energies $E^{Strut}_{2(4)}$ and E^{MN-BS}_2 with stiffness parameters obtained from the curvature of the Strutinsky ($C^{Strut}_{\varepsilon_i\varepsilon_j}$) and MN–BS ($C^{MN-BS}_{\varepsilon_i\varepsilon_j}$) energy surface, respectively.

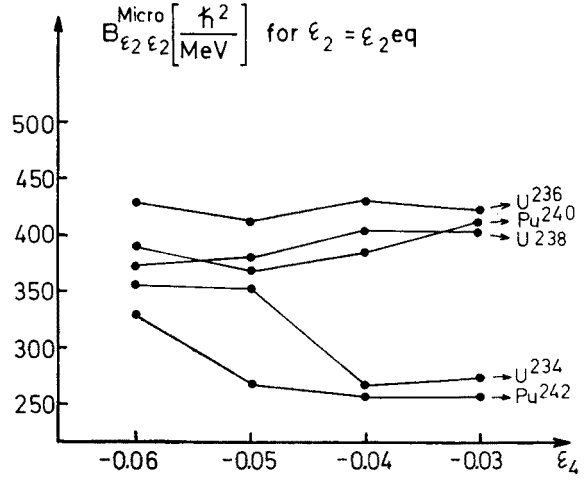

Fig. 8. The dependence of $B^{Micro}_{\varepsilon_2\varepsilon_2}$ mass parameters on ε_4 for ε_2 equal to the equilibrium value

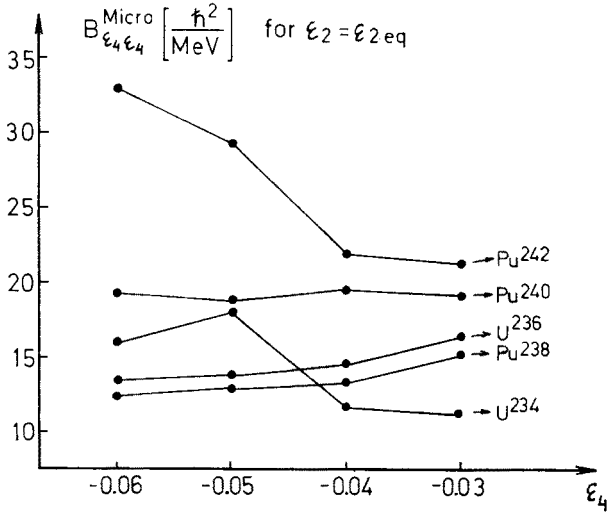

Fig. 9. The same as on Fig. 8 for $B^{Micro}_{\varepsilon_4\varepsilon_4}$ parameters

The comparison between corresponding columns including energies E_2 and E_4 shows a good agreement between the vibrational energies obtained from MN–BS, Strutinsky and microscopic methods in quadrupole case and from Strutinsky and microscopic methods in hexadecapole case.

The comparison with experimental values (E_2^{exp}) shows that the quadrupole energies are systematically much greater than the experimental ones, which can be partially explained by the fact of using simplified version of our numerical computation. The experimental values of hexadecapole vibrational energies are not known.

TABLE III

Quadrupole E_2 and hexadecapole E_4 vibrational energies calculated with stiffness parameters obtained in different ways (see the text)

Z	N	E_2 [MeV]				E_4 [MeV]	
		Micro	Strut	MN — BS	exp	Micro	Strut
92	142	1.78	1.84	1.93		10.4	11.3
92	144	1.49	1.44	1.52		10.1	11.2
92	146	1.57	1.43	1.53	0.993	9.2	10.1
94	144	1.45	1.46	1.54	0.943	10.4	11.9
94	146	1.58	1.25	1.52	0.863	9.8	10.3

The results obtained from microscopic calculations are strongly dependent on the multipole force strength parameters. This dependence is plotted on Figs 10 and 11. It can be deduced from Fig. 10 that the increase of κ_2 from 5×10^{-4} to 9×10^{-4} [MeV $(M\omega_0/\hbar)^2$]

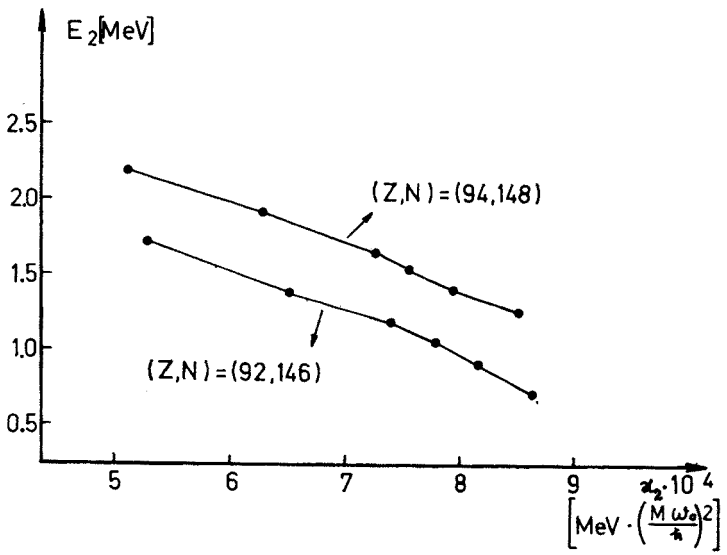

Fig. 10. The dependence of the quadrupole vibrational energy E_2 on the quadrupole strength parameter κ_2

causes the decrease of E_2 about 1 MeV. From Fig. 11 one can see that if κ_4 decreases from -2×10^{-2} to -7×10^{-2} [MeV] then E_4 increases about 4 MeV. The dependence of E_2 on κ_4 and E_4 on κ_2 is much weaker (it is not presented on the figures). The strong dependence of the vibrational energies on the multipole force strength parameters indicates that proper estimation of these parameters plays very important role in microscopic calculations.

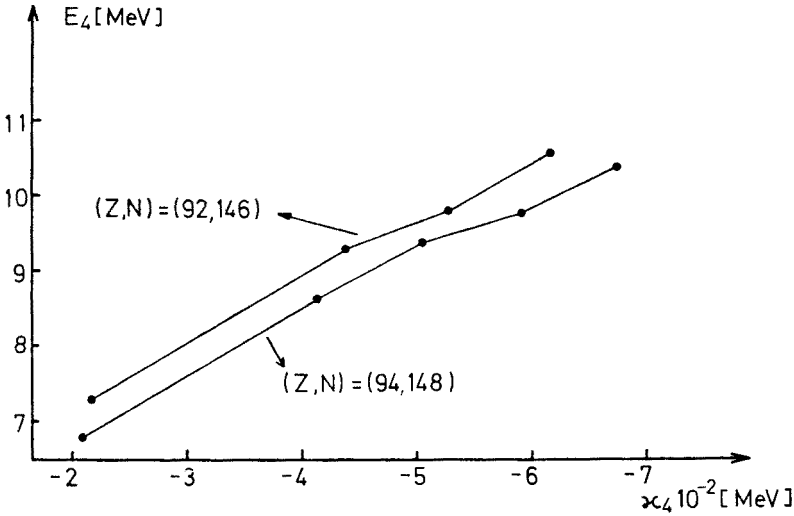

Fig. 11. The dependence of the hexadecapole vibrational energy E_4 on the hexadecapole strength parameter κ_4

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REFERENCES

[1] A. Bohr, B. R. Mottelson, *Lectures on Nuclear Structure and Energy Spectra*, to be published.
[2] A. Sobiczewski, Z. Szymański, S. Wycech, S. G. Nilsson, J. R. Nix, C. F. Tsang, C. Gustaffson, P. Möller, B. Nilsson, *Nucl. Phys.* **A131**, 67 (1969).
[3] S. G. Nilsson, C. F. Tsang, C. Gustaffson, A. Sobiczewski, Z. Szymański, S. Wycech, I. Lamm, P. Möller, B. Nilsson, *Nucl. Phys.* **A131**, 1 (1969).
[4] S. G. Nilsson, J. R. Nix, A. Sobiczewski, Z. Szymański, S. Wycech, C. Gustafsson, P. Möller, *Nucl. Phys.* **A115**, 545 (1968).
[5] B. Nilsson, *Nucl. Phys.* **A129**, 445 (1969).
[6] B. R. Mottelson, S. G. Nilsson, *Dan. Mat. Fys. Medd.* **1**, No 8 (1959).
[7] D. R. Bés, Z. Szymański, *Nucl. Phys.* **28**, 42 (1961).
[8] V. M. Strutinsky, *Nucl. Phys.* **A95**, 420 (1967).
[9] M. Brack, J. Damgaard, A. S. Jensen, H. C. Pauli, V. M. Strutinsky, C. Y. Wong, *Rev. Mod. Phys.* **44**, 2 (1972).
[10] M. Baranger, K. Kumar, *Nucl. Phys.* **A110**, 490 (1968).
[11] A. Bohr, *Dan. Mat. Fys. Medd.* **26**, No 14 (1952).
[12] J. Bardeen, L. N. Cooper, J. R. Schrieffer, *Phys. Rev.* **108**, 1175 (1957).
[13] B. R. Mottelson, *International School of Physics*, Varenna, 1960.
[14] Z. Bochnacki, *Phys. Lett.* **31B**, 175 (1969).
[15] A. Sobiczewski, private communication.