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SHORT-RANGE ΛNN CORRELATIONS AND THE Λ-PARTICLE BINDING IN NUCLEAR MATTER

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The contribution of the short-range ΛNN correlations to the binding energy of a Λ-particle in nuclear matter, $B_{\Lambda 3}$, is calculated with the help of the simplified method applied originally by Moszkowski in the pure nuclear matter problem. For the recent phenomenological ΛN potentials of Herndon and Tang, we obtain $B_{\Lambda 3} \simeq -4$ MeV.

1. Introduction

The binding energy of a Λ-particle in nuclear matter, B_Λ, is a quantity of considerable interest in the phenomenological analysis of the Λ-nucleon interaction $v_{\Lambda N}$. Several reaction matrix calculations, performed with ΛN interaction adjusted to Λ binding in light hypernuclei and to Λp scattering, have led to values of B_Λ much bigger than the empirical value of $B_\Lambda \simeq 30$ MeV (see, e.g., the review by Bodmer [1]). To get close to the empirical value of B_Λ one has to introduce into $v_{\Lambda N}$ a sizable hard core repulsion, a suppression of $v_{\Lambda N}$ in odd angular momentum states, and to consider the possibility of $\Lambda\Sigma$ conversion. The sizable hard core repulsion in $v_{\Lambda N}$ makes it important to estimate higher order contributions to B_Λ. Namely, the existing reaction matrix calculations of B_Λ have been performed within the two-hole-lines approximation, e.g., they have evaluated the terms proportional to the nuclear density ϱ.[1] In the presence of a sizable hard core, the terms proportional to ϱ^2 may become important.

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[1] One term proportional to ϱ^2, the rearrangement energy, has been considered in Ref. [2].

In the present paper, a simple estimate of $B_{\Lambda 3}$, the contribution to B_Λ proportional to ϱ^2, is given. In order to evaluate accurately $B_{\Lambda 3}$ it is, in principle, necessary to solve the three-body ΛNN problem in nuclear matter. (This path was followed in [3] where the higher order contributions to B_Λ in the presence of $\Lambda\Sigma$ conversion was investigated, however, for a ΛN interaction without a short-range repulsion.) Here, we shall apply a simplified method of estimating $B_{\Lambda 3}$, suggested originally by Moszkowski [4] for the pure nuclear matter problem. According to this method we split $v_{\Lambda N}$ and the nucleon-nucleon interaction v_{NN} into short- and long-range parts $v_{\Lambda N,S}$, $v_{NN,S}$ and $v_{\Lambda N,L}$, $v_{NN,L}$, the dividing line being made so that the short-range parts alone give zero scattering lengths [5]. With the short-range potentials alone the contribution to B_Λ proportional to ϱ vanishes. On the other hand, a conventional perturbation expansion in terms of the long-range potentials converges quite rapidly [5]. Thus, in estimating $B_{\Lambda 3}$ we consider only the short-range potentials. A more detailed discussion of the method is given in [4]. In the present note we simply extend the method of Ref. [4] to the case when one nucleon is replaced by a Λ-particle. This extension is presented in Section 2. The results obtained for $B_{\Lambda 3}$ are presented and discussed in Section 3.

2. *Calculation of* $B_{\Lambda 3}$

To calculate the short range ΛNN correlation energy $\varepsilon_{\Lambda NN}$, we apply the expression

$$\varepsilon_{\Lambda NN} = \langle\psi|H|\psi\rangle/\langle\psi|\psi\rangle \tag{2.1}$$

of Ref. [4], where H is the assumed ΛNN hamiltonian:

$$H = -\frac{\hbar^2}{2m_N}(\Delta_1+\Delta_2)-\frac{\hbar^2}{2m_\Lambda}\Delta_3+v_{NN,S}(r_{12})+v_{\Lambda N,S}(r_{13})+v_{\Lambda N,S}(r_{23}). \tag{2.2}$$

Particles 1 and 2 are nucleons, and 3 is the Λ-particle. We assume that the ΛN and NN interactions depend only on the interparticle spacings r_{13}, r_{23}, and r_{12}. Note that $\varepsilon_{\Lambda NN}$ includes both kinetic and potential contributions.

For the ΛNN wave function ψ we assume the form of a simple product wave function,

$$\psi = f_{\Lambda N}(r_{13})f_{\Lambda N}(r_{23})f_{NN}(r_{12}), \tag{2.3}$$

where the two-body correlation functions $f_{\Lambda N}$, and f_{NN} are S-state solutions of the two-body Schrödinger equations:

$$\frac{\hbar^2}{m_N}\Delta_{12}f_{NN}(r_{12}) = v_{NN,S}(r_{12})f_{NN}(r_{12}), \tag{2.4}$$

$$\frac{\hbar^2}{2\mu_{\Lambda N}}\Delta_{i3}f_{\Lambda N}(r_{i3}) = v_{\Lambda N,S}(r_{i3})f_{\Lambda N}(r_{i3}) \tag{2.5}$$

for $i = 1, 2$, where $\mu_{\Lambda N} = m_\Lambda m_N/(m_\Lambda+m_N)$. The boundary conditions imposed on the f functions are:

$$f_{\Lambda N}(r) \to 1, \quad f_{NN}(r) \to 1 \quad \text{for} \quad r \to \infty. \tag{2.6}$$

In view of the short range of the potentials $v_{\Lambda N,S}$ and $v_{NN,S}$ which excite nucleons predominantly to high momenta states, the exclusion principle for states occupied by other nucleons in the Fermi sea is not taken into account in Eqs (2.4) and (2.5). For the same reason the momenta of nucleons 1 and 2 are neglected, i.e., we assume these momenta to vanish outside the range of $v_{NN,S}$ and $v_{\Lambda N,S}$. Obviously, the momentum of the Λ-particle vanishes outside the range of $v_{\Lambda N,S}$ in the ground state of the Λ+nuclear matter system.

It is readily verified that with the form (2.3) of ψ, $\varepsilon_{\Lambda NN}$ is given by

$$\varepsilon_{\Lambda NN} = \left(\frac{\hbar^2}{2m_N} I_1 + \frac{\hbar^2}{4m_\Lambda} I_2\right)\Big/\Omega^2, \tag{2.7}$$

where

$$I_1 = 4\pi^2 \int_0^\infty dx \frac{df_{NN}^2(x)}{dx} \int_0^\infty dy \frac{df_{\Lambda N}^2(y)}{dy} \int_{|x-y|}^{x+y} dzz(z^2-x^2-y^2)f_{\Lambda N}^2(z), \tag{2.8}$$

$$I_2 = 4\pi^2 \int_0^\infty dx \frac{df_{\Lambda N}^2(x)}{dx} \int_0^\infty dy \frac{df_{\Lambda N}^2(y)}{dy} \int_{|x-y|}^{x+y} dzz(z^2-x^2-y^2)f_{NN}^2(z). \tag{2.9}$$

The quantity Ω denotes the normalization volume, and we put

$$\langle\psi|\psi\rangle \simeq \Omega^3. \tag{2.10}$$

To get the total ΛNN correlation enegy, $E_{\Lambda 3} = -B_{\Lambda 3}$, we have to multiply $\varepsilon_{\Lambda NN}$ by $\frac{1}{2}A^2\varphi$, where $\frac{1}{2}A^2$ is the number of interacting NN pairs, and $\varphi = \frac{3}{4}$ is the fraction of NN pairs in which the two nucleons have different σ_z and τ_z quantum numbers. We obtain

$$B_{\Lambda 3} = -E_{\Lambda 3} = -\frac{1}{2}A^2\varphi\varepsilon_{\Lambda NN} = -\frac{3}{16}\varrho^2\left(\frac{\hbar^2}{m_N}I_1 + \frac{\hbar^2}{2m_\Lambda}I_2\right). \tag{2.11}$$

To compare this expression with the result of Moszkowski [4] for the NNN correlation energy in nuclear matter, let us write Eq. (2.11) in the form:

$$E_{\Lambda 3} = \frac{9}{32}\frac{\hbar^2}{m_N}\varrho^2\bar{I}, \tag{2.12}$$

where

$$\bar{I} = \frac{2}{3}I_1 + \frac{1}{3}\frac{m_N}{m_\Lambda}I_2. \tag{2.13}$$

If we disregard the difference between the nucleon and Λ-particle masses, and between the NN and ΛN interaction, then Eq. (2.12) for $E_{\Lambda 3}$ differs only by an extra factor 6 from the analogical Eq. (22) of Ref. [4] for the NNN correlation energy per nucleon in nuclear matter, E^p/A. A factor 2 comes from our value of $\varphi = \frac{3}{4}$ which is twice the value which appears in Ref. [4]. In our case, we introduce the factor $\frac{1}{2}A^2$, equal to the number of NN pairs, whereas in Ref. [4] the analogical factor is the number of NNN triplets which is $\frac{1}{6}A^3$ (or $\frac{1}{6}A^2$ per nucleon). This accounts for the remaining factor 3.

As mentioned in Section 1, the short-range potentials $v_{NN,s}(r_{NN})$ and $v_{\Lambda N,s}(r_{\Lambda N})$ are equal to v_{NN} and $v_{\Lambda N}$ for $r_{NN} < d_{NN}$ and $r_{\Lambda N} < d_{\Lambda N}$, and vanish for $r_{NN} > d_{NN}$ and $r_{\Lambda N} > d_{\Lambda N}$ respectively. The separation distances d_{NN} and $d_{\Lambda N}$ are fixed by the requirement that $v_{NN,s}$ and $v_{\Lambda N,s}$ give zero scattering lengths. This means that $f_{NN}(r_{NN})$ and $f_{\Lambda N}(r_{\Lambda N})$, determined by Eqs (2.4) and (2.5), should have zero slope at $r_{NN} = d_{NN}$ and $r_{\Lambda N} = d_{\Lambda N}$ respectively [5]. Moszkowski approximates f_{NN} by

$$\begin{aligned} f_{NN}(r) &= 0, && r < c_{NN}, \\ &= (r - c_{NN})/(d_{NN} - c_{NN}), && c_{NN} < r < d_{NN}, \\ &= 1, && d_{NN} < r, \end{aligned} \tag{2.14}$$

where c_{NN} is the hard core radius of the NN interaction v_{NN}. In our calculation, we use the form (2.12) for f_{NN} with Moszkowski's choice of the parameters

$$c_{NN} = 0.3 \text{ fm}, \qquad d_{NN} = 0.9 \text{ fm}, \tag{2.14'}$$

adjusted to Wong's [6] potential v_{NN}.

For the ΛN correlation function $f_{\Lambda N}$, we also apply an analogical approximation, i.e., we put

$$\begin{aligned} f_{\Lambda N}(r) &= 0, && r < c_{\Lambda N}, \\ &= (r - c_{\Lambda N})/(d_{\Lambda N} - c_{\Lambda N}), && c_{\Lambda N} < r < d_{\Lambda N}, \\ &= 1, && d_{\Lambda N} < r, \end{aligned} \tag{2.15}$$

where $c_{\Lambda N}$ is the hard core radius of the ΛN interaction $v_{\Lambda N}$.

3. *Results and discussion*

The ΛN potentials applied in our estimate of $B_{\Lambda 3}$ are listed in Table I. All of them are of the form

$$v_{\Lambda N}(r) = \begin{cases} \infty, & r < c_{\Lambda N}, \\ -U_0 e^{-\lambda(r - c_{\Lambda N})}, & c_{\Lambda N} < r, \end{cases} \tag{3.1}$$

where U_0 has the value U_{0s} in the spin singlet state and U_{0t} in the spin triplet state. The HTS potential is an old potential fitted by Herndon, Tang, and Schmid [7] to the binding energies of the S-shell hypernuclei. Its intrinsic range b is equal to the intrinsic range of a purely attractive two-pion-exchange Yukawa potential. The new potentials H, HI, EII, and HII [8], [9] have been fitted to the new experimental values of the Λ binding in $^3_\Lambda$H, $^4_\Lambda$H, and $^4_\Lambda$He, and also to the Λp scattering data. These potentials have also charge-symmetry breaking components and suppression factors in odd angular momentum states which are not considered in our estimate of $B_{\Lambda 3}$.

TABLE I

Parameters of $v_{\Lambda N}$ and the calculated values of $B_{\Lambda 3}$ ($c_{\Lambda N}$, b, $d_{\Lambda N}$ in fm; λ in fm^{-1}; U_0, $B_{\Lambda 3}$ in MeV)

$v_{\Lambda N}$	Ref.	$c_{\Lambda N}$	b	λ	$\begin{cases} U_{0s} \\ U_{0t} \end{cases}$	$\begin{cases} d^s_{\Lambda N} \\ d^t_{\Lambda N} \end{cases}$	$B_{\Lambda 3}$
HTS	[7]	0.4	1.5	5.059	1221.1 954.1	0.81 0.92	−2.2
H	[8]	0.6	2.1	3.935	713.1 676.9	1.18 1.21	−4.8
HI	[9]	0.45	1.85	3.728	599.5 573.4	1.02 1.04	−3.2
EII	[9]	0.45	2.0	3.219	443.0 415.0	1.07 1.10	−3.6
HII	[9]	0.5	1.95	3.728	614.0 582.2	1.08 1.11	−3.8

In the simplified estimate of Section 2, with the approximation (2.15) for $f_{\Lambda N}$, the only parameters of $v_{\Lambda N}$ we need for calculating $B_{\Lambda 3}$ are the hard core radius $c_{\Lambda N}$ and the separation distance $d_{\Lambda N}$ (for zero relative momentum). The determination of $d_{\Lambda N}$ for the exponential potential $v_{\Lambda N}$, Eq. (3.1) is easy because the solution of Eq. (2.5) for this $v_{\Lambda N}$ may be expressed easily in terms of Bessel functions of zero order (see, e.g., [10]). The values of $d_{\Lambda N}$ obtained for all the ΛN potentials considered are shown in Table I.

Knowing the parameters $c_{\Lambda N}$ and $d_{\Lambda N}$ of the ΛN correlation function $f_{\Lambda N}$, Eq. (2.15), and using the Moszkowski values of the parameters c_{NN} and d_{NN}, Eq. (2.14′), we may compute the integrals I_1 and I_2, Eqs (2.8) and (2.9). For Fermi momentum $k_F = 1.35$ fm^{-1} we then obtain from Eq. (2.11) the values of $B_{\Lambda 3}$, listed in the last column of Table I.

Although the expression (2.11) has been derived under the assumption of spin independence of all the interactions, the ΛN potentials considered here depend on spin. For this reason we have performed the calculation with the spin singlet and triplet values of the separation distances, $d^s_{\Lambda N}$ and $d^t_{\Lambda N}$, for each potential $v_{\Lambda N}$. We then have taken the arithmetic average of the two values, $B^s_{\Lambda 3}$ and $B^t_{\Lambda 3}$. This is the value of $B_{\Lambda 3}$ shown in Table I for each ΛN potential. An alternative procedure of computing $B_{\Lambda 3}$ with an average value of the separation distances, $(d^s_{\Lambda N}+d^t_{\Lambda N})/2$, leads, within our accuracy, to the same results. For the new ΛN potentials H, HI, EII, HII, the spin dependence is so weak that the whole problem of spin dependence is of no significance within the accuracy of the present estimate of $B_{\Lambda 3}$. Even in the case of the HTS potential, which shows a stronger spin dependence, the whole effect is not very important ($B^s_{\Lambda 3} = 1.9$ MeV, $B^t_{\Lambda 3} = 2.5$ MeV).

Let us notice that according to the remarks made in Section 2 (after Eq. (2.13)), for ΛN potentials which are similar to the NN potential, we should expect values of $E_{\Lambda 3} = -B_{\Lambda 3}$ roughly about six times bigger than the value of E^p/A, estimated by Moszkowski [4] to be about 0.4 MeV. This, in fact, is revealed by our result for the HTS potential.

The results for $B_{\Lambda 3}$ listed in Table I show that the short-range ΛNN correlations decrease the Λ-particle binding in nuclear matter, B_Λ. The bigger the hard core radius $c_{\Lambda N}$ of the ΛN interaction, the bigger is this decrease. For the new ΛN potentials H, HI, EII, HII, we get for $B_{\Lambda 3}$ about -4 MeV which is a decrease of about 10% in the value of B_Λ calculated in the two-hole-lines approximation (equal about 40 MeV for these potentials [11]). This suggests that the ΛNN correlation energy should be considered as an important factor in the calculation of B_Λ and may help in obtaining an agreement with the empirical value of B_Λ.

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