NON-EQUIVALENT HAMILTONIANS AND RELATIVISTIC SYMMETRY OF MECHANICS

By Z. Chyliński

Institute of Nuclear Physics, Cracow*

(*Received November 4, 1974*)

The analysis of relativistic free-particle systems inclines us to define a set of non-equivalent Hamiltonians which imply the factorization of all degrees of freedom into groups corresponding to different "clustering" of the constituents of these systems. Within the quantum description all these Hamiltonians lead to selfconsistent solutions, although only one of them — call it H_1 — respects the full relativistic symmetry of the laws of motion. In the classical framework the only consistent description of motion follows from H_1, and thus the relativistic covariance is here unavoidable.

1. *Introduction*

The relativistic symmetry excludes instantaneous action at a distance, and hence it excludes the usual single-time canonical formalism of the classical, as well as quantum mechanics. The adjustment of both, the relativistic (R) and the canonical symmetries call for field theory, i. e. a theory of systems with infinite degrees of freedom. Within the framework of the perturbation theory of quantum field theory one regains the possibility of treating systems with a finite number of degrees of freedom, but difficulties connected particularly with the R-bound state problem clearly show that the problem cannot be regarded as completely solved, although the literature concerning it is immense.

In paper [1] it is shown that any mechanical system (with finite number of degrees of freedom) which fulfils: 1° the canonical symmetry, 2° the R-symmetry, and 3° respects the geometrical character of the classical trajectory must be a system of free particles. On the other hand, if one replaces the R-symmetry by the non-relativistic (NR) symmetry, one regains well-known freedom of dynamics resulting from arbitrary potentials depending on the distances between the particles (action at a distance). Thus there is a discontinuity between both symmetries reflecting the fundamental discontinuity in the number of invariants of these theories. The unique invariant $(\Delta s)^2$ of the R group goes, when $c \to \infty$, into two invariants, r and Δt ($(\Delta s)^2 = r^2 - c^2(\Delta t)^2$) of the Galilean (G) group. We emphasize this discontinuity opposing it to the continuous transition of all

* Address: Instytut Fizyki Jądrowej, Kawiory 26a, 30-055 Kraków, Poland.

kinematic relations; e. g. the kinetic energy of a particle of mass m, $E^{\mathrm{R}}_{\mathrm{kin}} = (m^2c^4+c^2\boldsymbol{p}^2)^{1/2} - mc^2 \to \boldsymbol{p}^2/2m = E^{\mathrm{G}}_{\mathrm{kin}}$, if $p/mc \to 0$. The point is that all kinematic relations are spanned on the momentum space, where the c-number parameter p/mc exists a priori. In the x-space the numerical values of momenta (or velocities) depend on the initial conditions which, as such, do not enter the structure of equations of motion formulated in the x-space. In consequence, these equations encounter the aforementioned discontinuity, being inherently connected with the space-time metric.

An exceptional situation takes place for one-body systems in an external field of forces. The R-symmetry does not exclude the "action at a distance" of an infinitely heavy centre which is the source of the external field. From the viewpoint of the relativity this follows from the fact that an infinitely heavy source does not suffer from the velocity-recoil (although it suffers from the momentum-recoil) due to the motion of the particle, and hence it does not reveal the way of propagation of this field. However, the very concept of an external field is limited, and when the mass m_1 of the source becomes finite — independently how large! — and if one includes this source-particle into the corresponding two-body system, then the situation changes discontinuously because of finite velocity-recoil of the source. Then the R-symmetry eliminates the external field together with the instantaneous action at a distance. This discontinuity is also reflected in the Bethe-Salpeter equation which in the limit $m_1 \to \infty$ does not reproduce the one-body equation in the external field [2]. This strange discontinuity inclines us to speculate in the following direction.

Let us suppose that the force binding two particles is an attribute of both particles, not of each of them separately, as it must be within the relativity theory. Such a "field" could generate the interaction between remote particles, as it does not depend on their motion. Of course, the realization of this hypothesis conflicts with the R-symmetry, much like any static interaction, and thus it results in partial breaking of this symmetry. The question arises, whether this is logically possible, and if it is, whether it does not conflict with well-known facts confirming the R-symmetry. We try to show that a positive answer is due to quantum physics, or more precisely, to the fact that not all quantities are directly accessible to measurement. In particular, internal forces responsible for binding of the particles belong to this class of quantities. The R-covariance is certainly required in parametrizing directly measurable quantities, such as cross-sections, invariant masses, etc., but these quantities can always be parametrized by directly measurable momentum invariants [3].

2. *Kinematics*

Within the R-kinematics we deal with two expressions of the energy of two free-particle system:

$$E = (m_1^2+\boldsymbol{p}_1^2)^{1/2}+(m_2^2+\boldsymbol{p}_2^2)^{1/2}, \quad \text{(a)}$$

$$E = [(p_1+p_2)^2+(\boldsymbol{p}_1+\boldsymbol{p}_2)^2]^{1/2} \equiv (M^2+\boldsymbol{P}^2)^{1/2}, \quad \text{(b)} \qquad (2.1)$$

where $M = [(p_1+p_2)^2]^{1/2}$ is the invariant mass of the system, and $\boldsymbol{P} = \boldsymbol{p}_1+\boldsymbol{p}_2$ is its total momentum. According to this we introduce two Hamiltonians (operators) of the system. The first Hamiltonian

$$\hat{H}_1 = (m_1^2+\hat{\boldsymbol{p}}_1^2)^{1/2}+(m_2^2+\hat{\boldsymbol{p}}_2^2)^{1/2} \tag{2.2}$$

is the typical one, and $\hat{\boldsymbol{p}}_{1,2}$ denote the momenta canonically conjugate to the Lorentz coordinates $\hat{\boldsymbol{x}}_{1,2}$, hence ($\hbar = 1$)[1]

$$[\hat{x}_{Aj}, \hat{p}_{Bk}] = i\delta_{AB}\delta_{jk}, \quad [\hat{x}_{Aj}, \hat{x}_{Bk}] = [\hat{p}_{Aj}, \hat{p}_{Bk}] = 0,$$
$$(j, k = 1, 2, 3; \quad A, B = 1, 2). \tag{2.3}$$

The second Hamiltonian $\hat{H}_2$ is equal to

$$\hat{H}_2 = (\hat{h}^2+\hat{\boldsymbol{P}}^2)^{1/2}, \quad \hat{h} = (m_1^2+\hat{\boldsymbol{q}}^2)^{1/2}+(m_2^2+\hat{\boldsymbol{q}}^2)^{1/2}, \tag{2.4}$$

and it defines a new set of variables, regarded henceforth as the canonical variables. The total momentum $\hat{\boldsymbol{P}}$ determines the canonically conjugate overall coordinate $\hat{\boldsymbol{X}}$ of the system, while the absolute, relative momentum $\hat{\boldsymbol{q}}$ determines the canonically conjugate absolute, relative position $\hat{\boldsymbol{y}}$ of the constituent particles. Of first importance is that the absolute — i. e. independent of the reference frame — character of the $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{q}}$ coordinates is given a priori on the level of equations of motion, that is apart from any boundary condition. In consequence, $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{q}}$ provide the parameters which describe the internal (absolute) laws of motion, and thus the internal (absolute) structure of the system. It is assumed that

$$[\hat{X}_j, \hat{X}_k] = [\hat{X}_j, \hat{y}_k] = [\hat{X}_j, \hat{q}_k] = [\hat{P}_j, \hat{P}_k] = [\hat{P}_j, \hat{y}_k] = [\hat{P}_j, \hat{q}_k]$$
$$= [\hat{q}_j, \hat{q}_k] = [\hat{y}_j, \hat{y}_k] = 0, \quad [\hat{X}_j, \hat{P}_k] = [\hat{y}_j, \hat{q}_k] = i\delta_{jk}. \tag{2.5}$$

Let us assume that both, $\hat{H}_1$ and $\hat{H}_2$ are the time translation generators, hence for $\hat{H}_2$ one gets the Schroedinger equation

$$i\,\partial\,\Psi(\boldsymbol{X}, \boldsymbol{y}, t)/\partial t = \hat{H}_2\Psi(\boldsymbol{X}, \boldsymbol{y}, t). \tag{2.6}$$

Eq. (2.6) is separable in the external $\boldsymbol{X}$, and the internal $\boldsymbol{y}$ variables, which fact reflects naturally the independence of the internal from the external laws of motion. Note that this is alien to the covariant parametrization. By putting $\Psi = \Psi(\boldsymbol{X}, t)\,\psi(\boldsymbol{y})$ one obtains the relativistic Klein-Gordon equation in $\boldsymbol{X}$, t

$$(\partial_t^2-\nabla_X^2+M^2)\Psi(\boldsymbol{X}, t) = 0, \tag{2.7}$$

where the separation constant M means the invariant mass of the system, being the eigenvalue of the internal Hamiltonian $\hat{h}$

$$\hat{h}\,\psi(\boldsymbol{y}) = M\,\psi(\boldsymbol{y}). \tag{2.8}$$

As we see, the price for this separation is the violation of the R-symmetry by Eq. (2.8). This equation is known in the literature as the "semi-relativistic" equation [4].

---

[1] By $\hat{a}$ we denote the q-number, and by a its c-number eigenvalue.

3. Non-equivalent Hamiltonians

The commutation relations (2.3), (2.5) imply that between the variables $\hat{x}_{1,2}$, $\hat{p}_{1,2}$ and $\hat{X}, \hat{P}, \hat{y}, \hat{q}$ must stand a canonical transformation. The most general one is of the form

$$\hat{P} = \hat{p}_1 + \hat{p}_2, \quad \hat{q} = [a\hat{p}_2 - (1-a)\hat{p}_1]L^{-1},$$
$$\hat{X} = a\hat{x}_1 + (1-a)\hat{x}_2, \quad \hat{y} = L(\hat{x}_2 - \hat{x}_1), \tag{3.1}$$

where a, and the three-dimensional matrix L must be independent of the dynamical variables (operators), but so far quite arbitrary. On the other hand, the structure of $\hat{H}_2$ implies that the eigenvalue q of $\hat{q}$ coincides with the eigenvalue p_2 of $\hat{p}_2$ represented in the c.m. system S^*, i.e.: $q = p_2^* = -p_1^*$. If $M_{1,2} = (m_{1,2}^2 + q^2)^{1/2}$, and taking into account that $p_{1,2} = (E_{1,2}, p_{1,2})$ are four-momenta we have

$$(p_{1,2})_{\|} = \gamma(\mp q_{\|} + vM_{1,2}), \quad (p_{1,2})_{\perp} = (\mp q_{\perp}), \quad E_{1,2} = \gamma(M_{1,2} \mp vq), \tag{3.2}$$

where v is the velocity of an arbitrary reference frame S in S^*, and $\|$, $\perp$ denote the directions parallel and perpendicular to v. If one takes the third axis parallel to v, then from (3.2) and (3.1) one obtains

$$L = L(v) = \begin{pmatrix} 1, & 0, & 0 \\ 0, & 1, & 0 \\ 0, & 0, & \gamma \end{pmatrix}, \quad a = M_1/M = \tfrac{1}{2}\,[1 + (m_1^2 - m_2^2)/M^2],$$

$$\gamma = E/M, \quad M = M_1 + M_2, \quad E = (M^2 + P^2)^{1/2}. \tag{3.3}$$

As the parameters a and L depend on quantum numbers of a definite state, there is no canonical transformation which should transform the Hamiltonian $\hat{H}_1$ into $\hat{H}_2$, and vice-versa. We shall say that these Hamiltonians are *non-equivalent*,

$$\hat{H}_1(\hat{x}_{1,2}, \hat{p}_{1,2}) \not\equiv \hat{H}_2(\hat{X}, \hat{P}, \hat{y}, \hat{q}). \tag{3.4}$$

The transformation (3.1) with a, L as in (3.3) enables one only to change the parametrization of a state with given quantum numbers E, P. Thus it is given a posteriori, not a priori which requires the equivalence (identity) of $\hat{H}_1$ and $\hat{H}_2$.

An exceptional situation takes place in the NR limit ($c \to \infty$). Then $a \to m_1/(m_1+m_2)$, and $L \to 1$ (unit matrix), which a priori are c-numbers. It turns out that the corresponding NR Hamiltonians $\hat{H}_1^G$, $\hat{H}_2^G$ become identical (equivalent).

The second limiting case to the classical mechanics ($\hbar \to 0$) shows that $H_2 = \hat{H}_2$ leads to an inconsistent description of motion, and therefore it must be refuted. Indeed, the Hamiltonian H_2 gives the following trajectory:

$$X = (\overset{0}{P}/\overset{0}{E})t + \overset{0}{X}, \quad y = (\overset{0}{M}/E)\,(1/M_1 + 1/M_2)\overset{0}{q} \cdot t + \overset{0}{y}, \tag{3.5}$$

where the quantities with the superscript "0" denote the initial values. The trajectory (3.5) translated — with the help of (3.1) and (3.3) — into the $x_{1,2}$, $p_{1,2}$ variables should result in the trajectory

$$x_{1,2} = (\overset{0}{p}_{1,2}/\overset{0}{E}_{1,2}) \cdot t + \overset{0}{x}_{1,2}, \tag{3.6}$$

which follows from the Hamiltonian $H_1 = \hat{H}_1$. One easily finds that this is not so. Since the trajectories (3.6) behave correctly under the Lorentz transformations, H_2 cannot be accepted as the Hamiltonian of the system. Thus within the framework of the classical mechanics there is no place for other Hamiltonians than H_1 with its Lorentz-covariant parametrization. In other words, the concept of an a priori absolute y-space is inconsistent.

The situation is quite different in quantum description, and the above dilemma disappears together with the classical trajectory itself. Let us consider the plane wave solution of Eq. (2.6) which is quite general as it provides the complete basis for the general solution of this equation. One finds that

$$\Psi(X, y, t) = \exp\left[i(\overset{0}{P}X + \overset{0}{q}y - Et)\right], \tag{3.7}$$

where $\overset{0}{E} = \overset{0}{E}_1 + \overset{0}{E}_2$ is the total energy. The transformation (3.1) leaves invariant the following expression

$$PX + qy \equiv p_1 x_1 + p_2 x_2, \tag{3.8}$$

even for arbitrary a and L. Thus

$$\Psi = \exp\left[i(\overset{0}{p}_1 x_1 + \overset{0}{p}_2 x_2 - (\overset{0}{E}_1 + \overset{0}{E}_2)t)\right] \tag{3.9}$$

which coincides with the equal-time two-body plane wave solution of the Schroedinger equation (2.6) with $\hat{H}_2 \to \hat{H}_1$. We see then that both quantum "trajectories" coincide.

It is remarkable that the expression (3.8), representing the space part of the classical action, is identical in both parametrizations. The inconsistency comes with the determination of the classical trajectory from general solution of the Hamilton-Jacobi equation. Then the identity (3.8) and the equality $E = E_1 + E_2$ between the eigenvalues of $\hat{H}_1$ and $\hat{H}_2$ are not sufficient for the classical trajectories in both parametrizations to coincide. One needs the equality of the Hamiltonians as functions of their dynamical variables which does not take place — cf. (3.4).

4. *Interaction and Lorentz-Poincaré group*

The heretofore kinematical considerations can be extended onto interaction by modifying the internal Hamiltonian $\hat{h}$. We can put

$$\hat{h} = (m_1^2 + \hat{q}^2)^{1/2} + (m_2^2 + \hat{q}^2)^{1/2} + \hat{U}(\hat{y}), \tag{4.1}$$

where $\hat{U}$ is called the absolute potential, and as was stated in Section 1, it is regarded as an attribute of both particles. Thus $\hat{U}$ does account for instantaneous action at a distance, but only in the absolute space spanned on the y variable which variable a priori cannot be identified with any Lorentz relative coordinate. We explain below this difference and the compatibility of the "static" action with the relativity theory. Note that the modification of $\hat{h}$ introduced with $\hat{U}$ retains Eq. (2.6) separable in the $\boldsymbol{X}$ and $\boldsymbol{y}$ variables, and Eq. (2.8) determines now the absolute masses M of interacting particles.

Let us consider the structure of the Lorentz-Poincaré (L–P) group of our system. Let $\hat{J}_k$, $\hat{K}_k$ denote six generators of the homogeneous, and $\hat{H}$, $\hat{P}_k$ four translation generators of the L–P group. Let $\hat{J}_k$ be of the dimension of an angular momentum, $\hat{K}_k$ – of (g. cm), $\hat{P}_k$ — of the dimension of a momentum, and $\hat{H}$ — of energy. Then the ten generators fulfil the commutation relations

$$\begin{aligned} &[\hat{J}_j, \hat{J}_k] = i\hbar e_{jks}\hat{J}_s, \quad && [\hat{K}_j, \hat{H}] = i\hbar\hat{P}_j, \\ &[\hat{J}_j, \hat{K}_k] = i\hbar e_{jks}\hat{K}_s, \quad && [\hat{P}_j, \hat{P}_k] = 0, \\ &[\hat{J}_j, \hat{P}_k] = i\hbar e_{jks}\hat{P}_s; \quad && [\hat{P}_j, \hat{H}] = 0, \\ &[\hat{J}_j, \hat{H}] = 0, && \end{aligned} \tag{4.2a}$$

and

$$[\hat{K}_j, \hat{K}_k] = -\frac{i\hbar}{c^2} e_{jks}\hat{J}_s, \quad [\hat{K}_j, \hat{P}_k] = \frac{i\hbar}{c^2}\delta_{jk}\hat{H}. \tag{4.2b}$$

In these units one easily performs the limiting procedure to the NR physics ($c \to \infty$) and the Galilean (G) group structure. The commutation relations (4.2a) remain unmodified, while instead of (4.2b) we have

$$[\hat{K}^G_j, \hat{K}^G_k] = 0, \quad [\hat{K}^G_j, \hat{P}^G_k] = i\hbar\delta_{jk}m, \tag{4.2b'}$$

where m is the neutral element of the G group, of the dimension of mass. The lack of energy-mass relation forces us to postulate the mass of the system.

Let us start with the NR case. For our two-body system the ten generators can be made equal to:

$$\begin{aligned} \hat{J}^G_j = e_{jks}(\hat{X}_k\hat{P}_s + \hat{y}_k\hat{q}_s), \quad \hat{P}^G_j = \hat{P}_j, \quad \hat{K}^G_j = m\hat{X}_j, \\ \hat{H}^G = \hat{\boldsymbol{P}}^2/2m + (\hat{\boldsymbol{q}}^2/2\mu + \hat{U}(\hat{\boldsymbol{y}})), \end{aligned} \tag{4.3}$$

where $m = m_1 + m_2$ (neutral element), $\mu = m_1 m_2/m$ — the reduced mass, and $\hat{\boldsymbol{X}}, \hat{\boldsymbol{P}}$; $\hat{\boldsymbol{y}}, \hat{\boldsymbol{q}}$ are determined by (3.1) with $a = m_1/m$, $L = 1$. Thus $\hat{\boldsymbol{X}}$ is the NR centre of gravity, and $\boldsymbol{y}$ the relative (absolute) coordinate between the constituents. As $\hat{J}^G_k$ commute with $\hat{H}^G$, the absolute potential $\hat{U}$ must be rotation-invariant, i. e. $\hat{U} = \hat{U}(\hat{\boldsymbol{y}}^2)$. Beside this the structure of the G group does not impose any other restriction on the NR dynamics, which is well-known from mechanics.

In the L–P group the situation becomes different. The time translation generator $\hat{H} = \hat{H}_2$ implies the factorization of the internal — Eq. (2.8) — from the external — Eq. (2.7) — laws of motion, but the latter deals already with the invariant mass M which means that the internal state (eigenstate of $\hat{h}$) must be already determined. The only invariant group of Eq. (2.8) is the three-dimensional rotation group in the y-space, and $\hat{j}_k = e_{kjs}\hat{y}_j\hat{q}_s$ are three generators of this group. Since $\hat{h}$ must be rotation-invariant; $[\hat{h}, \hat{j}_k] = 0$, the absolute potential $\hat{U}$, much like in the NR case, must depend on $\hat{y}^2$. For the sake of simplicity, let us assume that the solution $\psi(y)$ is the eigenstate not only of $\hat{h}$, but also of $\hat{j}^2$, i.e.

$$\hat{h}\psi_{Ml}(y) = M\psi_{Ml}, \qquad \hat{j}^2\psi_{Ml} = \hbar^2 l(l+1)\psi_{Ml}. \tag{4.4}$$

Now, i.e. a posteriori, we introduce three $(2l+1)\cdot(2l+1)$-dimensional matrices $S_k^{(l)}$ which are the rotation generators of the $(2l+1)$-dimensional representation ψ_{Ml}. On having this, ten generators of the L–P group can be taken in the following form

$$\hat{J}_k = e_{kjs}\hat{X}_j\hat{P}_s + \hbar S_k^{(l)}, \qquad \hat{P}_k = \hat{P}_k,$$

$$\hat{K}_k = \frac{1}{c^2}\hat{X}_k\hat{H} + \frac{i\hbar}{c}S_k^{(l)}, \qquad \hat{H} = \hat{H}_2 = (\hat{h}^2 + c^2\hat{\boldsymbol{P}}^2)^{1/2}, \tag{4.5}$$

where

$$[S_k^{(l)}, S_j^{(l)}] = ie_{kjs}S_s^{(l)}.$$

The Hamiltonian $\hat{H} = \hat{H}_2$, as the only one generator retains the dependence on the internal coordinates $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{q}}$, which exhibits the distinguished position of the time variable. Since the boost generators $\hat{K}_j$ do not commute with themselves (Thomas precession), they and the rotation generators $\hat{J}_k$ must be independent of $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{q}}$, and so they must be determined a posteriori. Here is the fundamental difference between the G and the L–P groups consisting in the fact that for finite c the y-space ceases to be isomorphic with the space spanned on the relative (Lorentz) coordinates. In the NR limit ($c \to \infty$) $\hat{K}_j^G$ commute and both spaces become a priori isomorphic.

Within the proposed scheme one gets no constraints for the internal dynamics; only the potential $\hat{U}(\hat{y}^2)$ must vanish for $\hat{y}^2 \to \infty$. Indeed, the mass M of the system, as the eigenvalues of $\hat{h}$, normalizes $\hat{U}$, unlike in the NR case.

5. *Relativization*

The eigenvalue of any scalar operator in the phase-space of $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{q}}$ is a priori an absolute quantity. This absolute c-number can be a posteriori identified with an invariant of the Lorentz geometry. Thus the relativization should work a posteriori. Of course, this conflicts with full covariance which requires all quantities to be covariant a priori. In this sense the Lorentz contraction effect is also an a posteriori effect. In the first stage the internal laws of motion — such as Eq. (2.8) — determine from point particles an absolute structure described by $\psi(y)$, and in the second stage (a posteriori) this c-number shape can be projected onto the Lorentz space-time. As $y = L(x_2 - x_1)$, the same shape in the Lorentz parametrization takes the form $\psi[L(x_2 - x_1)]$ and it exhibits the Lorentz contraction effect.

The Hamiltonian $\hat{H} = \hat{H}_2$ results also in the relativistic time dilatation effect. Let us consider the meta-stable state

$$\Psi = \Psi(X)\psi(y)\exp(-i\,Et) \tag{5.1}$$

with $E = (P^2 + M^2)^{1/2}$, where

$$M = M_0 - i\Delta M \quad (\Delta M/M_0 \ll 1). \tag{5.2}$$

This means that the internal structure is unstable and its life-time is given by squaring;

$$|\Psi|^2 = \Psi|(X)\psi(y)|^2 \exp(-2\Delta M/\gamma t),$$

where $\gamma = E_0/M_0$, $E_0 = (P^2 + M_0^2)^{1/2}$. Thus the life-times, Δt in S, and Δt^* in S^* are connected through the relation

$$\Delta t = \gamma \Delta t^*. \tag{5.3}$$

This expresses the time dilatation of any internal process in a moving reference frame S. By comparing (5.3) with the Lorentz transformation

$$\Delta t = \gamma(\Delta t^* - v\Delta x^*),$$

we see that in the proposed picture the internal motion (in the y-space) does not affect the time flow. This again exhibits the difference between the $\hat{y}$ and the relative Lorentz coordinate $\Delta\hat{x}^*$.

The question arises whether the proposed partial breaking of the R-symmetry does not conflict with the experiment. We know already that quantum "trajectory" remains consistent. Moreover, the whole kinematics based on $\hat{H}_2$ (momentum space relations) remains ex-definitione unmodified. On the other hand, any internal structure in the y-space, connected already with dynamics — e. g. the potential $\hat{U}$ — is measurable only indirectly, namely through its Fourier transform in the momentum space. The measurement of $U(y)$ has nothing to do with the determination of the space-time coincidences determining directly the x-shape of U, as it takes place in the classical measurement of a force. One always determines momenta, while an exact determination of a momentum eliminates the space-time localization. Here is the fundamental difference between the quantum and classical frameworks which — in principle — opens the possibility of partial breaking of the R-symmetry. Since the absolute momentum transfer $(\Delta q)^2$ can be translated into the Lorentz momentum invariants, they provide us with the unique variables which parametrize the directly measurable structures. Therefore the momentum-invariant language of the S-matrix theory does not imply that this theory follows from fully covariant laws of motion formulated in space-time.

Let us now consider two limiting cases of Eq. (2.6), a) the NR, and b) the infinitely heavy centre limit. a) In the NR limit — as we know — both Hamiltonians $\hat{H}_1^G$ and $\hat{H}_2^G$ become equivalent and the y-space becomes isomorphic with the (absolute) space of the relative Galilean coordinates. b) Let now the mass m_1 of the system go to infinity. If $W = M - m_1$, Eq. (2.8), in the limit $m_1 \to \infty$ takes the following form

$$[(m_2^2 + \hat{q}^2)^{1/2} + \hat{U}(\hat{y})]\psi = W\psi. \tag{5.4}$$

If $\hat{q} \to -i\partial/\partial y$, and $W \to i\partial/\partial\tau$ one obtains from (5.4) the Klein-Gordon equation which regains the L–P group of covariance. We see then that the subtraction of infinitely heavy constituent m_1 results in the relativistic one-body problem in the external field $U(y)$ acceptable by the R-symmetry. The coordinates (τ, y) can be then identified with the Lorentz coordinates of the particle m_2 in the rest-frame of infinitely heavy particle m_1.

These two limits show us that the proposed scheme rules out the aforementioned discontinuity between the R and NR symmetries, as they both are the particular (limiting) symmetries of the same symmetry.

Note finally that for many-body systems one can define many different non-equivalent Hamiltonians according to different groupings of the constituents of this system. Only one of them — called $\hat{H}_1$ — which does not perform any grouping, is admissible within the framework of the classical mechanics, and only this one remains consistent with full R-covariance of the corresponding equations of motion. One easily shows that all these Hamiltonians lead to the same kinematics, while modifications come with the interaction. Then the choice of the suitable Hamiltonian depends on the initial asymptotic state in which some of the constituents can already be "clustered" in bund states.

REFERENCES

[1] D. G. Currie, T. P. Jordan, E. C. G. Sudarshan, *Rev. Mod. Phys.* **35**, 350 (1963).

[2] G. C. Wick, *Phys. Rev.* **96**, 1124 (1954); R. E. Cutkosky, *Phys. Rev.* **96**, 1135 (1954).

[3] Z. Chyliński, *Acta Phys. Pol.* **30**, 293 (1966); **31**, 3 (1967).

[4] G. Breit, *Phys. Rev.* **34**, 553 (1929); L. L. Foldy, S. A. Wouthuysen, *Phys. Rev.* **78**, 29 (1950); A. A. Logunov, A. N. Tavkhelidze, *Nuovo Cimento* **29**, 380 (1963).