ON THE DEPENDENCE OF THE MULTIPLICITY OF PARTICLES PRODUCED IN PROTON-NUCLEUS INTERACTIONS AT HIGH ENERGIES ON THE MASS NUMBER OF TARGET

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The mean values $\langle N_h \rangle$ and the mean multiplicities $\langle n_s \rangle$ in the interactions of high-energy protons with two groups of nucle (CNO and AgBr) were evaluated from the experimental emulsion data at different energies. The experimental data of $\langle n_s \rangle$ are well fitted by the dependence: $\langle n_s \rangle = C + DA^{1/3}$ (A — the mass number of target), which seems to be universal in a very wide interval of primary energies from 6.2 to thousands of GeV. A comparison of the experimental results with the predictions of theoretical calculations was made.

1. Introduction

During recent years some theoretical works of Dar and Vary [1], Fishbane and Trefil [2] and Gottfried [3] were published from which it follows that the investigations of proton-nucleus interactions at high energies make it possible to differentiate between two classes of models of particle production in elementary interactions. The first of these classes contains models of the cascade type which assume that the secondary particles are produced in proton-nucleus interaction immediately in several successive interactions of the primary particle and high energy secondary pions with nucleons of the nucleus. In models of the second class it is assumed that in the collision of the primary proton with the nucleon of the target nucleus, excited intermediate states are generated (fireball [4], nova [5] etc.) which produce the secondary particles. The idea of the production of particles in the proton-nucleus interactions through the intermediate states was proposed some years ago by Mięsowicz on the basis of cosmic ray results ([6], [7]). Both classes of models of particle production were discussed in Ref. [1].

Fishbane and Trefil suggested that in the case of particle production through the intermediate states the relation: $R = \langle n \rangle_A / \langle n \rangle_H$ (where $\langle n \rangle_A$ — average multiplicity in the interaction p-nucleus with mass number A, $\langle n \rangle_H$ — average multiplicity in pp interaction at the same energy) ought to be the linear function of $A^{1/3}$ independently of the

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primary energy. From the analysis of Dar and Vary (see Fig. 2 of the work [1]) one can also conclude the linear dependence R vs $A^{1/3}$ (for $10 \lesssim A \lesssim 100$) independently of the primary energy.

On the other hand, the calculations of Dar and Vary lead to the conclusion that in the case of particle production through the internuclear cascade the dependence of R on the mass number A for higher energies should be stronger than for lower energies (see Fig. 4 of this work).

It is interesting to investigate whether the experimental data of average multiplicity (and thereby of R) in p-nucleus interactions at different energies are well fitted by the dependence: $C+DA^{1/3}$. From the more general point of view it would be interesting to examine also other similar types of dependence $\langle n_s \rangle$ vs A, for instance $\langle n_s \rangle = C_\gamma + D_\gamma A^\gamma$ at different values of γ or $\langle n_s \rangle = yA^x$.

2. *Experimental data*

Nuclear emulsion consists mainly of two groups of nuclei: C, N, O and Ag, Br. The mass numbers of nuclei within each of these groups are similar, whereas the difference in the average mass numbers of the two groups ($\langle A \rangle = 14$ for CNO, $\langle A \rangle = 95$ for AgBr) is large. It appears that it is possible to make a fairly good estimate of mean multiplici-

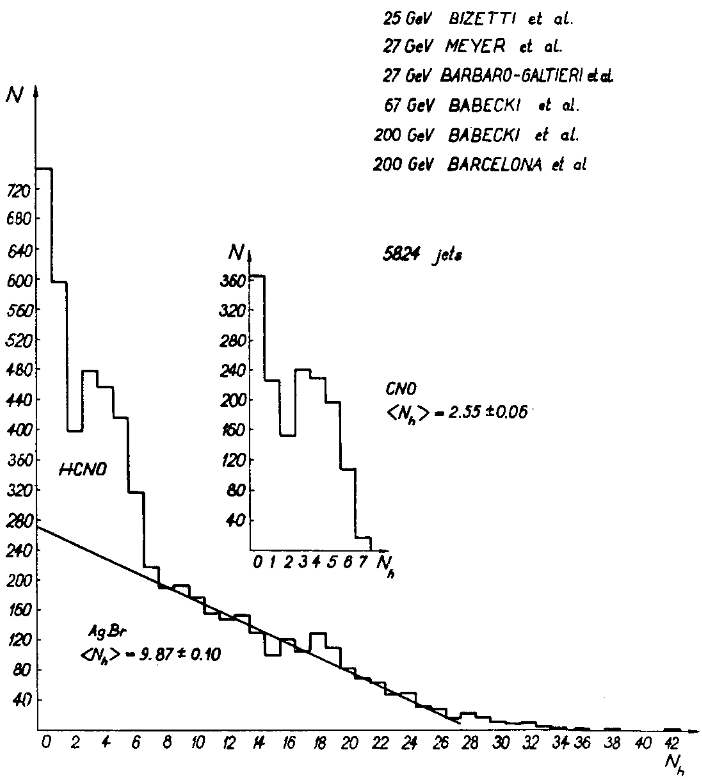

Fig. 1. Composite distributions of N_h for energies from 25 to 200 GeV

ties of particles produced in the interactions with the nuclei of each of these groups and thus to obtain the information on the dependence of average multiplicity on A without the use of experiments with pure targets.

Comparing the distributions of the numbers of heavily ionizing particles N_h in p-Em interactions at different primary energies (Bizetti et al. [8], Meyer et al. [9], Barbaro-Galtieri et al. [10], Barcelona and other Laboratories [29] and Cracow results at 67 and 200 GeV) it may be stated that the N_h-distribution does not depend on the primary energy (for $E_0 \gtrsim 25$ GeV). In Fig. 1 the composite distribution of N_h for the energies from 25 to 200 GeV is presented (at 67 and 200 GeV coherent events were rejected). The straight line which fits this distribution well in its central part ($8 \leqslant N_h \leqslant 25$) representing only the interactions with AgBr, divides the whole N_h-distribution into two parts, the areas of which stand in the same relation to one another as the expected numbers of interactions with HCNO-nuclei (~28%) and with AgBr-nuclei (~72%) (see Powell et al. [11]). One must be aware however that the extension of this straight line to the region of small N_h-values introduces some uncertainties which are difficult to estimate at the present time. Of course "the tail" of the N_h-distribution ($N_h > 25$) is included in the part of interactions with AgBr. For the interactions with HCNO the values of $N_h > 8$ are impossible because the largest number of protons in the nucleus of oxygen is 8. Hence we have two distributions of N_h: for HCNO and for AgBr divided with good approximation and we can evaluate average $\langle N_h \rangle$ for AgBr (Fig. 1). Now if we want to have the N_h-distribution for p-CNO interactions we must eliminate from the distribution of N_h for HCNO the cases belonging to the collisions of the primary particles with free protons in emulsion (~4% of all interactions), assuming that among the recoil protons in collisions with hydrogen there are ~50% of black and grey tracks ($\beta < 0.7$).[1] Having N_h-distribution for the group CNO we can calculate $\langle N_h \rangle$ for the interactions with the nuclei of this group (Fig. 1). In such a manner we obtain: $\langle N_h \rangle = 9.87 \pm 0.10$ for p-AgBr interactions and $\langle N_h \rangle = 2.55 \pm 0.06$ for p-CNO interactions independently of the primary energy (for $E_0 \gtrsim 25$ GeV). The only statistical errors of $\langle N_h \rangle$ are given.

We know the experimental dependence of the multiplicity n_s on the number of slow particles N_h in emulsion for different primary energies.[2] Hence we can evaluate for these energies $\langle n_s \rangle$ for interactions with CNO and with AgBr knowing $\langle N_h \rangle$ for these groups of nuclei. Such a method of evaluating $\langle n_s \rangle$ for interactions with CNO and AgBr may be applied only on the assumption that the dependence n_s vs N_h for interactions with CNO and with AgBr is similar (of course for $N_h < 8$). But our results of $\langle n_s \rangle$ for interactions with CNO and AgBr are in good agreement with those obtained by other authors using other methods. Our results at the energy 67 GeV are in very good agreement with those obtained by Abdo et al. [13] by a quite different but also only statistical method. Our cal-

[1] Such a composition of recoil protons was calculated by Calucci et al. [12] on the basis of the experimental data from a bubble chamber for pp collisions at 200 GeV.

[2] The linear dependence $\langle n_s \rangle = a + bN_h$ at different energies were presented e. g. in Refs [10] and [29]. The coefficients a and b increase with primary energy. In this work we fitted the experimental data at all available energies by the dependence $\langle n_s \rangle = a + bN_h$ (or $\langle n_s \rangle = a + bN_h + cN_h^2$ which is better for low energies). The errors of the coefficients a, b, c are taken into account in our later calculations.

culations of $\langle n_s \rangle$ at the energies ~21 GeV and ~3000 GeV are in good agreement with the experimental results obtained in the works of Lohrmann et al. ([9] and [14]) where special criteria were used for the individual jets to classify them into the groups of interactions with CNO or with AgBr (see Table I).

TABLE I

Comparison of our results of $\langle n_s \rangle$ with those of other authors

E_0 (GeV)	$\langle n_s \rangle$ for CNO	$\langle n_s \rangle$ for AgBr
~21	4.33 ± 0.17	5.92 ± 0.26
~24 from [9]	4.80 ± 0.20	6.30 ± 0.10
67	7.23 ± 0.30	10.55 ± 0.50
67 from [13]	7.53 ± 0.27	10.53 ± 0.48
~3000	20.1 ± 1.2	27.9 ± 1.9
~3000 from [14]	17.1 ± 2.0	26.8 ± 2.0

A method almost identical with our was used for evaluating $\langle n_s \rangle$ for the interactions with CNO and AgBr in the work of Florian et al. [15] at 200 GeV.

The dependences $\langle n_s \rangle$ (estimated in the above described manner) vs $A^{1/3}$ are shown in Fig. 2 for primary energies from 6.2 GeV up to ~3000 GeV. At 6.2 GeV and ~21 GeV the separate N_h-distributions valid for these energies were used. Data for 6.2, ~21 (20.5 and 22.5 together) and 27 GeV were taken from the works of Winzeler [16], Meyer et al. [9] and Barbaro-Galtieri et al. [10]. Data for 67, 200, ~1000 and ~3000 GeV were mostly obtained and collected in the Cracow Emulsion Laboratory (Babecki et al. ([17]), [28]), Gierula and Wolter [18]). Points for ~1000 GeV refer to the interactions of cosmic particles with emulsion [18] and to the pp interactions in ISR [19]. Data for ~3000 GeV come from the large emulsion stacks irradiated by cosmic rays in balloon flights (Texas-stack [20], ICEF-stack [21], Brawley-stack [22], Sydney-stack [23] and Cracow-sandwich-stack [24]) and refer to jets found in these stacks by tracing back along electromagnetic cascades of large energies. We also used the data at 200 GeV published in the work of Barcelona et al. [29].

The values of $\langle n_s \rangle_H$ for $A = 1$ were obtained by subtraction of 0.5 from $\langle n_{ch} \rangle$-values from bubble chambers (Barish et al. [25], Ammosov et al. [26], Czyżewski and Rybicki [27]). Such a correction should be made for identical treatment of the data from emulsions and those from bubble chambers, because the average multiplicities $\langle n_s \rangle$ for p-Em interactions are always given with slow ($\beta < 0.7$) protons subtracted. 0.5 is the number of slow protons in one pp collision as calculated by Calucci et al. [12].[3]

The method of estimating of primary energy for cosmic jets from large stacks (~3000 GeV) will be published separately [30]. We extrapolated to higher energies the

[3] The significance of this correction at large energies is not too large (~6% at 200 GeV).

curve $\langle n_{ch} \rangle$ vs log E_0 for pp interactions which was estimated on the basis of $\langle n_{ch} \rangle$-data at lower energies (Czyżewski and Rybicki [27], Antinucci et al. [19]) and in such a way we found $\langle n_s \rangle_H$ for pp interactions at the energy ~3000 GeV.

In Fig. 2 it is seen that in the wide interval of primary energies from 6.2 up to ~3000 GeV the experimental data are well fitted by the dependence: $\langle n_s \rangle = C + DA^{1/3}$.

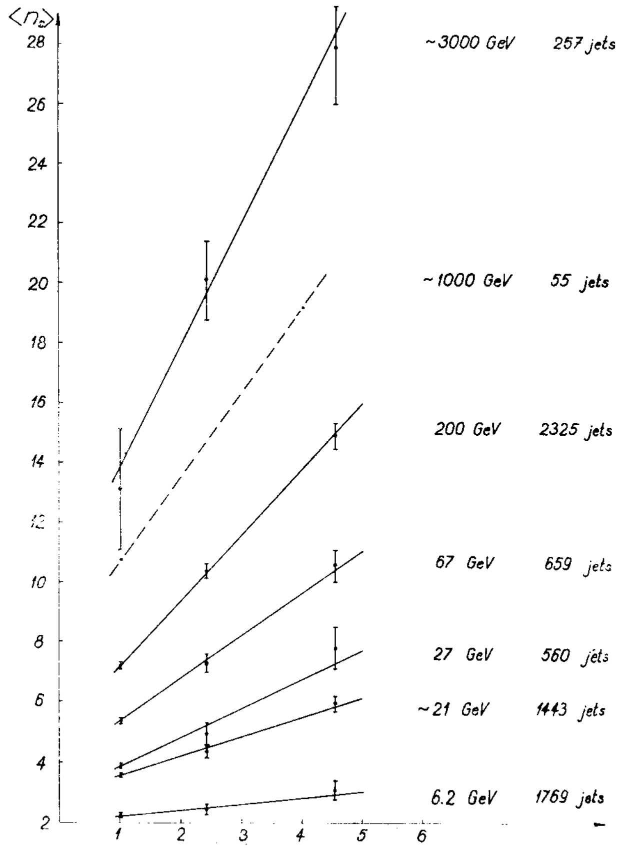

Fig. 2. Dependence of $\langle n_s \rangle$ vs $A^{1/3}$ at different primary energies

These data are not so well fitted by the dependence $\langle n_s \rangle \sim A^x$ used by different authors (with the exception of the cosmic energies at which statistical errors are very large, and at the energy of 6.2 GeV). The confidence levels of fits are presented in Table II.

In Fig. 3 the dependence of normalized coefficients $c = C/\langle n_s \rangle_H$ and $d = D/\langle n_s \rangle_H$ on the primary energy and also the dependence of the average R for emulsion ($R_{Em} = \langle n_s \rangle_{Em}/\langle n_s \rangle_H$) on the primary energy are shown. The values c, d, and R_{Em} are listed in Table III. It seems that for $E_0 \gtrsim 50$ GeV these values do not depend on the primary energy.

Therefore on the basis of the composite data at 67 and 200 GeV we could obtain the dependence, common for these two energies of $R = \langle n_s \rangle_A/\langle n_s \rangle_H$ on the mass number

TABLE II

The confidence levels of fits for two types of dependence of $\langle n_s \rangle$ vs A*

E_0 (GeV)	$P(\chi^2)$ for $C+DA^{1/3}$	$P(\chi^2)$ for $\sim A^x$	x
6.2	0.45	0.25	0.05
~21	0.45	0.035	0.10
27	0.24	0.04	0.13
67	0.80	0.03	0.14
200	0.78	0.05	0.15
~3000	0.58	0.94	0.17

* To differentiate between these two types of dependence of $\langle n_s \rangle$ vs A we also used the experimental data at 67 GeV from the work of Abdo et al. [13].

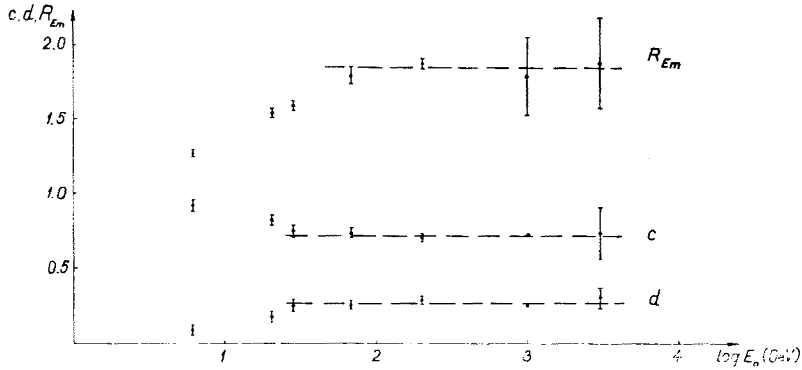

Fig. 3. Dependence of coefficients c, d and the value R_{Em} on the primary energy

TABLE III

E_0 (GeV)	c	d	R_{Em}	γ
6.2	0.91 ± 0.03	0.09 ± 0.03	1.27 ± 0.02	0.72 ± 0.48
~21	0.82 ± 0.03	0.18 ± 0.03	1.54 ± 0.03	0.46 ± 0.17
27	0.75 ± 0.04	0.25 ± 0.04	1.61 ± 0.03	0.60 ± 0.24
67	0.74 ± 0.03	0.26 ± 0.02	1.80 ± 0.05	0.35 ± 0.10
200	0.70 ± 0.02	0.30 ± 0.02	1.87 ± 0.03	0.31 ± 0.08
~1000	0.74	0.26	1.79 ± 0.26	
~3000	0.74 ± 0.17	0.31 ± 0.06	1.88 ± 0.30	0.18 ± 0.25

A of the target. Namely: $R = c+dA^{1/3}$, where: $c = (0.716 \pm 0.018)$ and $d = 0.283 \pm \pm 0.015)$.

To estimate the coefficients c and d we used only the data at 67 GeV (659 jets) and 200 GeV (2325 jets) which are the monoenergetic and unbiased samples of jets found in the along track scanning. A much smaller sample of 257 cosmic jets might have been biased in some manner from the point of view of n_s or N_h (for instance jets with very small multiplicities might be lost because of the special character of scanning).

We can also fit the experimental data of $\langle n_s \rangle$ for $A = 1$, 14 and 95 at different energies by the more general dependence: $\langle n_s \rangle = C_\gamma + D_\gamma A^\gamma$ changing the value of γ (we changed γ from 0.05 to 0.90) and finding $P(\chi^2)$ for each γ. Knowing three values of $\langle n_s \rangle$ (for $A = 1$, 14, 95), we obtained the best experimental values of γ at different primary energies and the statistical errors of these values of γ ($P(\chi^2) = 0.33$). They are listed in the last column of Table III. It is seen that at the energies from 6.2 to ~3000 GeV the value $\gamma = 1/3$ is in the limits of errors.

3. *Comparison of experimental data with theoretical predictions*

In Fig. 4 the experimental straight line $R = 0.716 + 0.283\, A^{1/3}$ and also theoretical curves R vs A from the work of Dar and Vary [1] are shown. These curves are the results of the assumptions of two different ways of particle production in p-nucleus interactions.

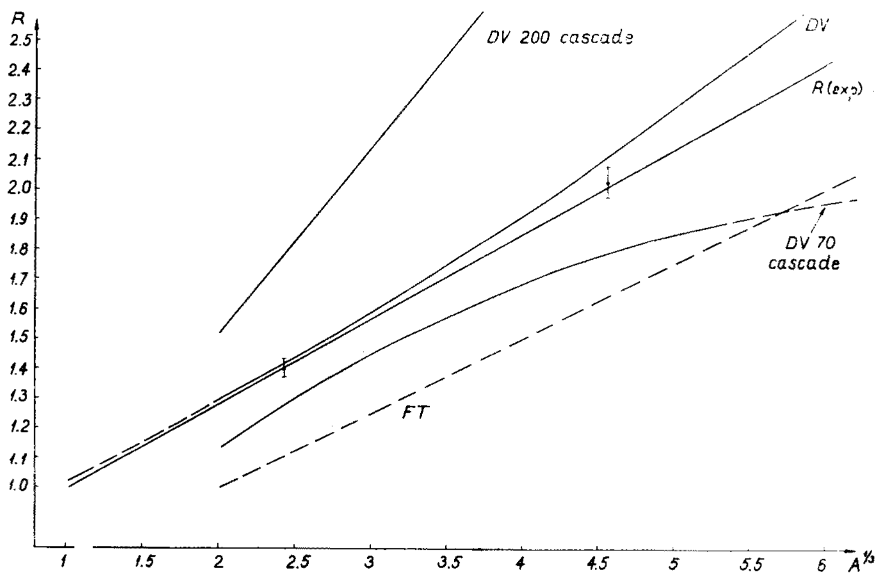

Fig. 4. Comparison of the dependence R vs $A^{1/3}$ with the predictions of Dar and Vary [1] and Fishbane and Trefil [2]

The curves were transferred from original drawings made in (log A, log R)-variables system to the ($A^{1/3}$, R)-variables system. It is seen that the curve DV evaluated assuming particle production through the intermediate states runs close to the experimental points.

On the other hand, in the work of Dar and Vary, the assumption of cascade particle production leads to different curves R vs A for different energies. In Fig. 4 such curves for two energies, 70 and 200 GeV, are drawn (DV 70 and DV 200). But the experimental dependence R vs A is the same for these two energies as was shown above. Hence our experimental results, in comparison with the predictions of Dar and Vary calculations, clearly favour the class of interaction models in which secondary particles are produced through the intermediate states and do not agree with the results of calculations based on the cascade models of particle production. The fact of the independence of R of the primary energy leads to the same conclusion ([1], [3]).

The line R vs $A^{1/3}$ (FT) proposed by Fishbane and Trefil [2] for the production through the intermediate states runs in Fig. 4 far from the experimental points.

It would be very interesting to obtain the experimental values of $\langle n_s \rangle$ with good statistics, for the interactions of high-energy protons with very heavy nuclei (e. g. Au, W or Pb) and to investigate whether the dependence R vs $A^{1/3}$ for very large A is still linear or perhaps parabolic.

4. *Conclusions*

a. The experimental data of $\langle n_s \rangle$ are sufficiently well fitted by the dependences: $\langle n_s \rangle = C+D\,A^{1/3}$ and $R = c+d\,A^{1/3}$ and it seems that these dependences are universal in a very wide interval of primary energies from 6.2 GeV up to $\sim$3000 GeV. The coefficients c and d are independent of the primary energy for $E_0 \gtrsim 50$ GeV.

b. Our experimental results, in comparison with theoretical predictions of Dar and Vary, clearly favour the production of secondary particles in the p-nucleus interactions via intermediate states and are in contradiction to the production of particles via internuclear cascade.

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