ON $\Lambda\Sigma$ CONVERSION IN $^4_\Lambda$H AND $^4_\Lambda$He

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The effect of $\Lambda\Sigma$ conversion on the binding energy, B_Λ, of $^4_\Lambda$H and $^4_\Lambda$He is calculated in the "hyperon + rigid $3N$ core" model. For a ΛN potential derived from $^4_\Lambda$He, we obtain 1.6 MeV for the excitation energy of the $J = 1$ state, and a very small negative contribution to $B_\Lambda(^4_\Lambda\text{He}) - B_\Lambda(^4_\Lambda\text{H})$ due to Coulomb interaction of Σ^+ and Σ^-.

1. *Introduction*

The threshold energy for $\Lambda\Sigma$ conversion in the ΛN system, $\Lambda\text{N} \to \Sigma\text{N}$, is only about 78 MeV. Consequently, one expects $\Lambda\Sigma$ conversion to be of importance in hypernuclei, as was suggested originally by Bodmer [1]. Although no detailed calculations of binding energies of hypernuclei, which would include $\Lambda\Sigma$ conversion, have been performed so far, all the existing approximate calculations and estimates show indeed that by taking into account $\Lambda\Sigma$ conversion one may expect to resolve the difficulties encountered in attempting to correlate the measured hypernuclear binding energies and the hyperon-nucleon scattering data (see, e. g., [2, 3] where further references are given).

In the present paper, we discuss the effects of $\Lambda\Sigma$ conversion in the isodoublet pair of hypernuclei $^4_\Lambda$H$-^4_\Lambda$He. From the qualitative discussion of [3], based on a perturbative treatment of the $\Lambda\Sigma$ coupling, we expect the $\Lambda\Sigma$ coupling to increase the excitation energy in the $J = 1$ state compared to the $J = 0$ ground state. So far, the only quantitative discussion of $\Lambda\Sigma$ coupling in $^4_\Lambda$H$-^4_\Lambda$He is that by Gibson, Goldberg, and Weiss [4], who have assumed both hypernuclei to consist of a rigid $A = 3$ nucleus plus the hyperon. The simplicity of this "rigid nuclear core + hyperon" model applied in [4] allows one to find very easily the energies of the $J = 0$ and $J = 1$ states in $^4_\Lambda$H$-^4_\Lambda$He, and also the contribution to $\Delta B = B_\Lambda(^4_\Lambda\text{He}) - B_\Lambda(^4_\Lambda\text{H})$ which results from $\Lambda\Sigma$ coupling. Both quantities are important; the difference in the energies of the $J = 0$ and $J = 1$ states may be compared directly with the energy of the observed hypernuclear γ-transition [5], and the magnitude of the positive difference ΔB in the Λ binding energies in $^4_\Lambda$He and $^4_\Lambda$H is crucial in determining the charge-symmetry-breaking (CSB) component of the ΛN interaction. Unfortunately,

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the spin-isospin average values of hyperon-nucleon interactions have been calculated incorrectly in [4] (see [6, 7]), and consequently, we cannot draw any conclusions concerning the effects of $\Lambda\Sigma$ conversion in ${}^4_\Lambda\text{H}-{}^4_\Lambda\text{He}$ from the results obtained in [4].

In the present paper, we calculate the energies of the $J = 0$ and $J = 1$ states in ${}^4_\Lambda\text{H}-{}^4_\Lambda\text{He}$ and the effect of $\Lambda\Sigma$ conversion on ΔB in the frame of the "rigid nuclear core + hyperon" model of [4]. In Sect. 2, we present the Schroedinger equation for the motion of the hyperon in the field of the rigid nuclear core. The expression for ΔB, which follows from the model applied, is presented in Sect. 3. The input parameters of our calculations and the results are presented and discussed in Sect. 4. The spin-isospin functions of ${}^4_\Lambda\text{H}-{}^4_\Lambda\text{He}$ and the spin-isospin average values of the hyperon-nucleon interaction are presented in Appendix 1. Details of our numerical procedure are given in Appendix 2.

2. *The rigid nuclear core model of* ${}^4_\Lambda H-{}^4_\Lambda He$

According to the rigid nuclear core model described in [4], the ${}^4_\Lambda\text{H}$ hypernucleus (with charge symmetric interactions and with Coulomb interaction being neglected, the treatment of ${}^4_\Lambda\text{He}$ is analogous) is assumed to consist of a Λ particle interacting with the inert ${}^3\text{H}$ core. The $\Lambda\Sigma$ coupling introduces into the ${}^4_\Lambda\text{H}$ state a Σ channel admixture which again is assumed to consist of a Σ^0 (Σ^-) particle interacting with an inert ${}^3\text{H}$ (${}^3\text{He}$) core. Both core nuclei, ${}^3\text{H}$ and ${}^3\text{He}$, are assumed to have the same structure (they differ only by the third component of the isospin). These assumptions allow us to write the Λ and Σ channel components of the ${}^4_\Lambda\text{H}$ system in a state with total spin J (and total isospin $T = 1/2$), and with the hyperon in the S state, in the form

$$\Psi^J_\Lambda = (\Phi^J_\Lambda(r)/\sqrt{4\pi}\, r)R\chi^J_\Lambda, \qquad \Psi^J_\Sigma = (\Phi^J_\Sigma(r)/\sqrt{4\pi}\, r)R\chi^J_\Sigma, \tag{2.1}$$

where $\Phi^J_{\Lambda(\Sigma)}(r)$ describes the relative motion of the $\Lambda(\Sigma)$ particle and the core nucleus; R is the normalized intrinsic spatial function of the core nucleus, symmetric in the coordiates of the three nucleons, and $\chi^J_{\Lambda(\Sigma)}$ are spin-isospin functions of all four particles, which nare chosen to yield a (J, T) values of (0, 1/2) and (1, 1/2).

Ansatz (2.1) leads to two coupled Schroedinger equations for Φ^J_Λ and Φ^J_Σ:

$$\begin{aligned} -(\hbar^2/2\mu_\Lambda)d^2\Phi^J_\Lambda(r)/dr^2 + V^J_{\Lambda\Lambda}(r)\Phi^J_\Lambda(r) + V^J_{\Lambda\Sigma}(r)\Phi^J_\Sigma(r) &= E^J\Phi^J_\Lambda(r), \\ -(\hbar^2/2\mu_\Sigma)d^2\Phi^J_\Sigma(r)/dr^2 + V^J_{\Sigma\Sigma}(r)\Phi^J_\Sigma(r) + V^J_{\Sigma\Lambda}(r)\Phi^J_\Lambda(r) &= (E^J-\Delta)\Phi^J_\Sigma(r), \end{aligned} \tag{2.2}$$

where $\Delta = M_\Sigma - M_\Lambda$, $\mu_Y = 3M_N M_Y/(3M_N + M_Y)$ (Y stands for the hyperon which may be Λ or Σ). The single particle elastic $\Lambda(\Sigma)$ potentials $V^J_{\Lambda\Lambda}(V^J_{\Sigma\Sigma})$ and the $\Lambda\Sigma$ coupling potentials $V^J_{\Lambda\Sigma} = V^J_{\Sigma\Lambda}$ are obtained from the two-body YN interaction potentials by folding the two-body potentials into the nucleon density distribution $\varrho(r')$ of the core nucleus:

$$V^J_{Y'Y}(r) = \int d\boldsymbol{r}'\varrho(r')V^J_{Y'Y\text{N}}(x), \qquad \boldsymbol{x} = \boldsymbol{r}-\boldsymbol{r}', \tag{2.3}$$

$$V^J_{Y'Y\text{N}}(x) = \langle\chi^J_{Y'}|\hat{V}_{Y'Y\text{N}}(x)|\chi^J_Y\rangle, \tag{2.4}$$

where $\hat{V}_{Y'YN}$ is the two-body $YN \to Y'$N interaction (which is an operator in the nucleon and hyperon spin and isospin space) between any one of the three core nucleons and the hyperon, and ϱ is normalized according to $\int dr' \ \varrho(r') = 3$.

The explicit form of the spin-isospin functions χ^J_Y, and expressions for $V^J_{Y'YN}$ are given in Appendix 1.

3. *The difference between* $B_\Lambda({}^4_\Lambda He)$ *and* $B_\Lambda({}^4_\Lambda H)$

With charge symmetric YN interaction and without Coulomb forces, the energies E are the same for both hypernuclei: ${}^4_\Lambda$He and ${}^4_\Lambda$H (here, for the sake of simplicity, we do not indicate explicitly the value of J). However, the measured Λ binding energies in the two hypernuclei (i. e., the separation energies B_Λ in the $J = 0$ ground state) differ by the positive quantity

$$\{\Delta B\}_{\rm exp} = B_\Lambda({}^4_\Lambda{\rm He}) - B_\Lambda({}^4_\Lambda{\rm H}) \cong 0.3 \text{ MeV}. \tag{3.1}$$

As is well known [8–10], consideration of Coulomb effects in the one-channel approach (i. e., consideration of differences in the rms radii of the nucleon distributions in the core nuclei due to Coulomb repulsion in ^{3}He, and consideration of additional Coulomb energy associated with compression of the nucleon core in ${}^4_\Lambda$He) leads to a negative value of $\{\Delta B\}_{\rm Coul} = \Delta B_{\rm C}$. To account for the total difference $\{\Delta B\}_{\rm exp} - \Delta B_{\rm C}$, one introduces a CSB component into the ΛN interaction with the strength adjusted as to reproduce the observed value of ΔB, Eq. (3.1).

In this connection, it is important to estimate the contribution to $\Delta B_{\rm C}$, which arises from the presence of the Σ component in the wave function of ${}^4_\Lambda$He and ${}^4_\Lambda$H. Such an estimate has been made in [4]. We shall briefly outline this estimate which simply consists of calculating $\Delta B_{\rm C}$ in the rigid nuclear core model. Namely, in this model, the whole effect of Coulomb interaction on B_Λ arises from the presence of the charged Σ hyperons.

Let us write Ansatz (2.1) in the form

$$|{}^4_\Lambda{\rm He}\rangle = |\Lambda\rangle|{}^3{\rm He}\rangle + \frac{1}{\sqrt{3}}|\Sigma^0\rangle|{}^3{\rm He}\rangle - \sqrt{\frac{2}{3}}|\Sigma^+\rangle|{}^3{\rm H}\rangle,$$

$$|{}^4_\Lambda{\rm H}\rangle = |\Lambda\rangle|{}^3{\rm H}\rangle - \frac{1}{\sqrt{3}}|\Sigma^0\rangle|{}^3{\rm H}\rangle + \sqrt{\frac{2}{3}}|\Sigma^-\rangle|{}^3{\rm He}\rangle; \tag{3.2}$$

which shows explicitly the isospin structure of the χ^J_Y functions (it is understood that the hyperon and nuclear core states in (3.2) are coupled to the desired J value). The probabilities of the Λ and Σ components are

$$P_\Lambda = |\langle\Lambda|\Lambda\rangle|^2 = \int dr \Phi^2_\Lambda(r) = 1 - P_\Sigma,$$

$$P_\Sigma = |\langle\Sigma^0|\Sigma^0\rangle|^2 = |\langle\Sigma^\pm|\Sigma^\pm\rangle|^2 = \int dr \Phi^2_\Sigma(r), \tag{3.3}$$

We treat Coulomb interaction as a perturbation whose effect may be calculated with the wave functions (3.2). Coulomb interaction has two effects:

(*i*) It changes the energy of the ^{3}He core nucleus by $E_C(^3\text{He}) = 0.764$ MeV.

(*ii*) It introduces Coulomb energy of Σ^- hyperon present in $^4_\Lambda$H,

$$E_C(\Sigma^-) = -e^2 \int \frac{d\boldsymbol{r}d\boldsymbol{r}'}{4\pi r^2} \Phi_\Sigma^2(r)\varrho_C(^3\text{He}, r')/|\boldsymbol{r}-\boldsymbol{r}'|, \tag{3.4}$$

and of the Σ^+ hyperon present in $^4_\Lambda$He,

$$E_C(\Sigma^+) = e^2 \int \frac{d\boldsymbol{r}d\boldsymbol{r}'}{4\pi r^2} \Phi_\Sigma^2(r)\varrho_C(^3\text{H}, r')/|\boldsymbol{r}-\boldsymbol{r}'|, \tag{3.5}$$

where ϱ_C is the charge density of ^{3}He and ^{3}H, respectively, normalized according to $\int d\boldsymbol{r}' \varrho_C(r') = Z$.

The total contributions of Coulomb interaction to the energies of $^4_\Lambda$H and $^4_\Lambda$He are then:

$$E_C(^4_\Lambda\text{H}) = \tfrac{2}{3} P_\Sigma E_C(^3\text{He}) + \tfrac{2}{3} E_C(\Sigma^-),$$

$$E_C(^4_\Lambda\text{He}) = (P_\Lambda + \tfrac{1}{3} P_\Sigma)E_C(^3\text{He}) + \tfrac{2}{3} E_C(\Sigma^+). \tag{3.6}$$

Since

$$B_\Lambda(^4_\Lambda\text{H}) = -[E(^4_\Lambda\text{H}) - E(^3\text{H})], \quad B_\Lambda(^4_\Lambda\text{He}) = -[E(^4_\Lambda\text{He}) - E(^3\text{He})], \tag{3.7}$$

we get for the Coulomb contribution to ΔB,

$$\Delta B_C = \frac{4}{3} P_\Sigma E_C(^3\text{He}) - \frac{2}{3} e^2 \int \frac{d\boldsymbol{r}d\boldsymbol{r}'}{4\pi r^2} \Phi_\Sigma^2(r) \{\varrho_C(^3\text{H}, r') + \varrho_C(^3\text{He}, r')\}/|\boldsymbol{r}-\boldsymbol{r}'|. \tag{3.8}$$

Notice that our expression for ΔB_C differs in sign from the expression given in [4]. Because of this difference, we found it necessary to present here the steps leading to Eq. (3.8).

4. *Results and discussion*

In our calculations, we have used the simple, pure attractive, spin independent two-body ΛN potential of a Gaussian form, applied in [4]:

$$\hat{V}_{\Lambda\Lambda N}(x) = V^J_{\Lambda\Lambda N}(x) = V(x) = -V_0 \exp[-(x/b)^2], \tag{4.1}$$

with $b = 1.05$ fm, $V_0 = 38.2$ MeV. The value of b corresponds to the exchange of two pions between Λ and N. To get the above value of V_0 one proceeds in the following way. One assumes for $^5_\Lambda$He the model in which Λ is bound to a rigid α particle core. This model implies a complete suppression of Σ channel due to isospin conservation. Within this model, one adjusts V_0 so as to reproduce the experimental Λ separation energy for $^5_\Lambda$He, 3.1 MeV. Details of this procedure are given in [4].

The nucleon density distribution $\varrho(r')$ of the $A = 3$ core nucleus in ${}^4_\Lambda\text{H}-{}^4_\Lambda\text{He}$ is assumed to be Gaussian,

$$\varrho(r') = (3/\pi^{3/2}\beta^3)\exp[-(r'/\beta)^2], \tag{4.2}$$

where β, according to [4], is obtained by taking an average value of the mas radia of ^{3}He and ^{3}H:

$$\beta^2 = \tfrac{2}{3}\{\tfrac{2}{3}[r^2_{ch}({}^3\text{He})-r^2_p]+\tfrac{1}{3}[r^2_{ch}({}^3\text{H})-r^2_p]\}, \tag{4.3}$$

with the values:

$$r_{ch}({}^3\text{He}) = 1.84 \text{ fm}, \quad r_{ch}({}^3\text{H}) = 1.70 \text{ fm}, \quad r_p = 0.8 \text{ fm}, \tag{4.4}$$

for the charge radia of ^{3}He, ^{3}H, and proton, respectively.

With the Gaussian shape of V and ϱ, one gets for the single particle potential $V^J_{\Lambda\Lambda} = V_{\Lambda\Lambda}$, Eq. (2.3),

$$V_{\Lambda\Lambda}(r) = -v_0 \exp[-(r/\alpha)^2], \tag{4.5}$$

where

$$v_0 = 3V_0(b/\alpha)^3, \quad \alpha^2 = b^2+\beta^2. \tag{4.6}$$

We assume here that $\Lambda\Sigma$ coupling takes place only in the spin triplet state, as is suggested by analysis of the $\Sigma^- \text{p} \to \Lambda\text{n}$ reaction [11–13]. According to Eq. (A 1.8), we have then

$$V^J_{\Lambda\Sigma\text{N}}(x) = \begin{cases} -U_0 \exp[-(x/b_x)^2] & \text{for } J = 0, \\ -\tfrac{1}{3}U_0 \exp[-(x/b_x)^2] & \text{for } J = 1, \end{cases} \tag{4.7}$$

where the two-body $\Lambda\Sigma$ coupling potential is assumed to have Gaussian shape and the depth U_t, connected with U_0 by:

$$U_0 = \tfrac{1}{2}U_t. \tag{4.8}$$

Notice that the sign of U_t is irrelevant for determining E^J, because Eqs (2.2) are invariant under the transformation $V^J_{\Sigma\Lambda} \to -V^J_{\Sigma\Lambda}$, $\Phi^J_\Sigma \to -\Phi^J_\Sigma$.

With ϱ given by Eq. (4.2), one gets for the single particle coupling potential

$$V^J_{\Lambda\Sigma}(r) = -u^J_0 \exp[-(r/\alpha_x)^2], \quad \alpha^2_x = b^2_x+\beta^2, \tag{4.9}$$

where

$$u^J_0 = \begin{cases} 3U_0(b_x/\alpha_x)^3 & \text{for } J = 0, \\ U_0(b_x/\alpha_x)^3 & \text{for } J = 1. \end{cases} \tag{4.10}$$

For b_x, we take the value $b_x = 2b = 2.1$ fm which corresponds to one pion exchange. For comparison, the value $b_x = b$ is also considered. Since the effect of $\Lambda\Sigma$ coupling on E^J turns out to be more sensitive to the strength of the coupling potential $V^J_{\Lambda\Sigma\text{N}}$ than to the strength of the elastic ΣN potential $V^J_{\Sigma\Sigma N}$, in most of the considered cases we put $V^J_{\Sigma\Sigma N} = 0$, and adjust U_0 (by solving Eqs (2.2)) so as to reproduce the experimental Λ separation energy for ${}^4_\Lambda$H, 2.03 MeV. For comparison, we consider also the case of a non-vanishing spin and isospin independent ΣN potential:

$$\hat{V}_{\Sigma\Sigma N}(x) = V^J_{\Sigma\Sigma N}(x) = W(x) = -W_0 \exp[-(x/b)^2], \tag{4.11}$$

with the same range as the ΛN potential (4.1). For the single particle potential $V_{\Sigma\Sigma}$ we have then

$$V_{\Sigma\Sigma}(r) = -w_0 \exp[-(r/\alpha)^2], \quad w_0 = 3W_0(b/\alpha)^3. \tag{4.12}$$

For a given value of U_0, the value of W_0 is adjusted (by solving Eqs (2.2)) so as to reproduce the experimental Λ separation energy for ${}^4_\Lambda$H.

In calculating ΔB_C, Eq. (3.8), the charge densities of ^{3}He and ^{3}H are assumed to be Gaussian,

$$\varrho_C({}^3\text{He}, r) = (2/\pi^{3/2}\beta^3_{\text{He}}) \exp[-(r/\beta_{\text{He}})^2],$$
$$\varrho_C({}^3\text{H}, r) = (1/\pi^{3/2}\beta^3_{\text{H}}) \exp[-(r/\beta_{\text{H}})^2], \tag{4.13}$$

where β_{He} and β_{H} are determined by the charge radia of ^{3}He and ^{3}H given in (4.4),

$$\beta_{\text{He}} = \sqrt{\tfrac{2}{3}}\, r_{\text{ch}}({}^3\text{He}), \quad \beta_{\text{H}} = \sqrt{\tfrac{2}{3}}\, r_{\text{ch}}({}^3\text{H}). \tag{4.14}$$

The results of the present calculations are shown in Table I. They differ from the results of [4] in two respects. First, our values of U_0 (and W_0) are much bigger than the values obtained in [4], although in both calculations they have been adjusted to $B_\Lambda(\text{H}^4_\Lambda) = 2$ MeV. To have a direct comparison with [4], we have used the values $U_0 = 15$ MeV,

TABLE I

Results of the present calculations for $V_0 = 38.2$ MeV and for the indicated values of U_0 and W_0 which give $E^{J=0} = -2.03$ MeV. E^* is the excitation energy of the $J = 1$ state. All energies are in MeV

b_x	U_0	W_0	J	$P_\Sigma(\%)$	E^*	B_C	B_m
$2b$	16.5	0	0	2.3	—	−0.01	0.13
			1	0.2	1.63	-0.00_1	0.01
b	60.9	0	0	2.3	—	−0.02	0.12
			1	0.1	1.64	-0.00_1	0.01
	50.0	82.7	0	3.3	—	−0.03	0.18
			1	0.2	1.64	-0.00_1	0.01

$W_0 = 77$ MeV, and $U_0 = 25$ MeV, $W_0 = 48$ MeV, and have obtained (with $b_x = b$) for B_Λ the values 0.4 and 0.5 MeV, and not the value 2 MeV quoted in [4]. Also, without $\Lambda\Sigma$ coupling, we get $B_\Lambda \cong 0.3$ MeV, whereas in [4] no bound state was found for this case. To help to clarify the matter, details of our numerical procedure are given in Appendix 2. The second difference is that our calculated values of ΔB_C are negative whereas those calculated in [4] are positive. We believe that this difference is connected with the difference in sign between our expression (3.8) and the corresponding expression (4) of Ref. [4]. (The remaining obvious differences, connected with taking the proper spin--isospin average values of YN interactions, have been mentioned already in Sect. 1.)

As is seen from Table I, the results are very similar for the two ranges b_x of the coupling potential, and are not sensitive to the values of U_0, W_0 as long as they are adjusted to the same value of $E^{J=0}$. Within our simplified model, we find:

(*i*) The probability of the admixed Σ in the $J = 0$ ground state of ${}^4_\Lambda\text{H}-{}^4_\Lambda\text{He}$ is about 2–3%, and in the $J = 1$ excited state is very small (0.1–0.2%).

(*ii*) The excitation energy E^* of the $J = 1$ state is 1.6 MeV. In contradistinction to estimates made within the one-channel approach (see, e. g., [14]), our simplified treatment of the Σ channel gives an excitation energy which is bigger than the experimentally observed value of 1.09 MeV [5].

(*iii*) The contribution ΔB_C of the Coulomb interaction to $\Delta B = B_\Lambda({}^4_\Lambda\text{He})-B_\Lambda({}^4_\Lambda\text{H})$, which arises from the admixture of Σ^+ and Σ^-, is negative (i. e., it increases the amount of the CSB component in ΛN interaction, required to reproduce the observed positive value of ΔB). However, the whole effect is very small for the $J = 0$ ground state ($\Delta B_C \leqslant -0.03$ MeV), and completely negligible for the $J = 1$ excited state.

Within the present model, one may easily estimate the effect of a part of the breaking of charge-symmetry, which arises from the Σ mass differences. As indicated in [4], by taking into account the mass differences of the Σ triplet, one finds that the appropriate value of Δ in Eqs (2.2) is given by:

$$\Delta = \begin{cases} \frac{2}{3}\Delta^+ + \frac{1}{3}\Delta^0 = 74.8 \text{ MeV} & \text{for } {}^4_\Lambda\text{He}, \\ \frac{2}{3}\Delta^- + \frac{1}{3}\Delta^0 = 80.1 \text{ MeV} & \text{for } {}^4_\Lambda\text{H}, \end{cases} \tag{4.15}$$

where $\Delta^\pm$ and Δ^0 are the $\Sigma^\pm - \Lambda$ and $\Sigma^0 - \Lambda$ mass differences. By solving Eqs (2.2) with the values of Δ given in (4.15), one obtains different values of $E^J = -B^J_\Lambda$ for ${}^4_\Lambda\text{He}$ and ${}^4_\Lambda\text{H}$. The resulting differences $\Delta B_m = B_\Lambda({}^4_\Lambda\text{He})-B_\Lambda({}^4_\Lambda\text{H})$ for $J = 0, 1$ are shown in the last column of Table I. For the $J = 0$ ground state our calculated, positive values of ΔB_m account for about half of the experimental value of ΔB, Eq. (3.1), which is in qualitative agreement with the early estimate of Ref. [8]. For the $J = 1$ excited state the calculated values of ΔB_m are negligibly small.

The present estimate of the effects of $\Lambda\Sigma$ coupling in ${}^4_\Lambda\text{H}-{}^4_\Lambda\text{He}$ involves serious simplifications. To obtain more reliable, quantitative results, one should improve the YN potential matrix (by adjusting it to the known YN scattering data, as it has been done in [15] for a separable interaction model), and one should improve the model of ${}^4_\Lambda\text{H}-{}^4_\Lambda\text{He}$ by considering the distortion of the nuclear core.

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APPENDIX 1

Expressions for χ_Y^J and $V_{Y'YN}^J$

We start with the spin-isospin functions of ^{3}H and ^{3}He. We use the following order of coupling of spins and isospins of the three nucleons 1, 2, 3:

$$(\sigma_1+\sigma_2)+\sigma_3,\ (\tau_1+\tau_2)+\tau_3, \tag{A1.1}$$

and denote by

$$|J_{12}, M_N\rangle_3^\sigma, \qquad |T_{12}, \mu_N\rangle_3^\tau, \tag{A1.2}$$

the spin (isospin) states of the three nucleons with total spin 1/2 (isospin 1/2) and its third component M_N (μ_N). The spins (isospins) of nucleons 1 and 2 are coupled to J_{12} (T_{12}).

The totally antisymmetric spin-isospin state of the three nucleons, with total spin and isospin, and their third components equal (1/2, M_N), (1/2, μ_N) has the form [16]:

$$\chi^{M_N\mu_N}(123) = \frac{1}{\sqrt{2}}\{|0M_N\rangle_3^\sigma|1\mu_N\rangle_3^\tau - |1M_N\rangle_3^\sigma|0\mu_N\rangle_3^\tau\}. \tag{A1.3}$$

By coupling the total spin and isospin of the three nucleons with the spin and isospin of the hyperon Y (particle 4), we get for the spin-isospin function of the 3N+Y system, with total spin J and its third component M, and with total isospin 1/2 and its third component μ:

$$\chi_Y^{JM\mu}(1234) = \sum\{M_N m_Y \mu_N \mu_Y\}\ (\tfrac{1}{2}\,\tfrac{1}{2}\,M_N m_Y|JM)$$
$$\times(\tfrac{1}{2}\,t_Y \mu_N \mu_Y|\tfrac{1}{2}\,\mu)\chi^{M_N\mu_N}(123)\chi_Y^{m_Y\mu_Y}(4), \tag{A1.4}$$

where t_Y is the isospin of the hyperon ($t_\Lambda = 0$, $t_\Sigma = 1$), and $\chi^{m_Y\mu_Y}$ is the spin-isospin function of the hyperon with the third component of spin and isospin equal m_Y and μ_Y, respectively.

Most convenient for calculating $V_{Y'YN}^J$ is a form of χ_Y, in which spins and isospins of the four particles are recoupled in the order ((1+2)+(3+4)). Let us introduce the notation

$$|J_{12}J_{34};JM\rangle \equiv |J_{12}, J_{34}\rangle^\sigma = \sum\{M_{12}M_{34}\}$$
$$\times(J_{12}J_{34}M_{12}M_{34}|JM)\eta^{J_{12}M_{12}}(12)\eta^{J_{34}M_{34}}(34) \tag{A1.5}$$

for the spin state of the four particles, in which spins of particles 1, 2 and 3, 4 are first coupled to J_{12} and J_{34}, respectively (the corresponding spin functions are $\eta^{J_{12}M_{12}}$ and $\eta^{J_{34}M_{34}}$), and afterwards the spins J_{12} and J_{34} are coupled to the resulting spin J with the third component M. To simplify our notation, we suppress the index J, and also M which anyhow is irrelevant for calculating $V_{Y'YN}^J$. The analogical notation $|T_{12}, T_{34}\rangle_Y^\tau$ for the isospin state with total isospin 1/2 is selfexplanatory (we add here the subscript Y

to distinguish between the case when the isospin of particle 4 is equal 0 ($Y = \Lambda$) or 1 ($Y = \Sigma$).

With the known values of the Clebsch-Gordan coefficients, one obtains easily from (A1.4) the following expressions for $\chi_Y^{JM\mu}(1234) \equiv \chi_Y^J$:

$$\chi_\Lambda^{J=0} = \frac{1}{\sqrt{2}}|0, 0\rangle^\sigma \left|1, \frac{1}{2}\right\rangle_\Lambda^\tau - \frac{1}{\sqrt{2}}|1, 1\rangle^\sigma \left|0, \frac{1}{2}\right\rangle_\Lambda^\tau,$$

$$\chi_\Sigma^{J=0} = -\frac{1}{3\sqrt{2}}|0, 0\rangle^\sigma \left|1, \frac{1}{2}\right\rangle_\Sigma^\tau + \frac{2}{3}|0, 0\rangle^\sigma \left|1, \frac{3}{2}\right\rangle_\Sigma^\tau - \frac{1}{\sqrt{2}}|1, 1\rangle^\sigma \left|0, \frac{1}{2}\right\rangle_\Sigma^\tau,$$

$$\chi_\Lambda^{J=1} = \frac{1}{\sqrt{2}}|0, 1\rangle^\sigma \left|1, \frac{1}{2}\right\rangle_\Lambda^\tau - \frac{1}{\sqrt{3}}|1, 1\rangle^\sigma \left|0, \frac{1}{2}\right\rangle_\Lambda^\tau + \frac{1}{\sqrt{6}}|1, 0\rangle^\sigma \left|0, \frac{1}{2}\right\rangle_\Lambda^\tau,$$

$$\chi_\Sigma^{J=1} = -\frac{1}{3\sqrt{2}}|0, 1\rangle^\sigma \left|1, \frac{1}{2}\right\rangle_\Sigma^\tau + \frac{2}{3}|0, 1\rangle^\sigma \left|1, \frac{3}{2}\right\rangle_\Sigma^\tau$$
$$- \frac{1}{\sqrt{3}}|1, 1\rangle^\sigma \left|0, \frac{1}{2}\right\rangle_\Sigma^\tau + \frac{1}{\sqrt{6}}|1, 0\rangle^\sigma \left|0, \frac{1}{2}\right\rangle_\Sigma^\tau. \quad \text{(A1.6)}$$

Now, let us calculate the spin-isospin averages, $V_{Y'YN}^J(x)$, of the two-body interaction $\hat{V}_{Y'YN}$, Eq. (2.4). $\hat{V}_{Y'YN}(x)$ depends on spins and isospins of the two particles and on their separation x. We denote by $V^{\mathrm{T}}(x)$ and $V^{\mathrm{S}}(x)$ (and similarly by $U^{\mathrm{T}}(x)$ and $U^{\mathrm{S}}(x)$) the radial dependence of $\hat{V}_{\Lambda\Lambda N}$ (and $\hat{V}_{\Lambda\Sigma N}$) in the spin-triplet and singlet state of the hyperon-nucleon system, respectively. Notice that the ΛN system has isospin 1/2, and both potentials $\hat{V}_{\Lambda\Lambda N}$ and $\hat{V}_{\Lambda\Sigma N}$ act only in the isospin-doublet state. The ΣN system may exist in isospin-doublet ($T_{\Sigma N} = 1/2$) and quartet ($T_{\Sigma N} = 3/2$) states, and we have here four different spin-isospin states: spin-triplet-isospin-doublet (TD), spin-triplet-isospin-quartet (TQ), spin-singlet-isospin-doublet (SD), and spin-singlet-isospin-quartet (SQ). Consequently, we have four parts of $\hat{V}_{\Sigma\Sigma N}$: $W^{\mathrm{TD}}(x)$, $W^{\mathrm{TQ}}(x)$, $W^{\mathrm{SD}}(x)$, $W^{\mathrm{SQ}}(x)$.

In applying expressions (A1.6) in calculating $V_{Y'YN}^J$, one uses the two-body interaction $\hat{V}_{Y'YN}$ acting between particles 3 and 4, and obtains immediately the following results:

$$V_{\Lambda\Lambda N}^J(x) = \begin{cases} \frac{1}{2} V^{\mathrm{T}}(x) + \frac{1}{2} V^{\mathrm{S}}(x), & (J = 0), \\ \frac{5}{6} V^{\mathrm{T}}(x) + \frac{1}{6} V^{\mathrm{S}}(x), & (J = 1), \end{cases} \quad \text{(A1.7)}$$

$$V_{\Lambda\Sigma N}^J(x) = \begin{cases} \frac{1}{2} U^{\mathrm{T}}(x) - \frac{1}{6} U^{\mathrm{S}}(x), & (J = 0), \\ \frac{1}{6} U^{\mathrm{T}}(x) + \frac{1}{6} U^{\mathrm{S}}(x), & (J = 1), \end{cases} \quad \text{(A1.8)}$$

$$V_{\Sigma\Sigma N}^J(x) = \begin{cases} \frac{1}{2} W^{\mathrm{TD}}(x) + \frac{1}{2}\left[\frac{1}{9} W^{\mathrm{SD}}(x) + \frac{8}{9} W^{\mathrm{SQ}}(x)\right], & (J = 0), \\ \frac{5}{6}\left[\frac{7}{15} W^{\mathrm{TD}}(x) + \frac{8}{15} W^{\mathrm{TQ}}(x)\right] + \frac{1}{6} W^{\mathrm{SD}}(x), & (J = 1). \end{cases} \quad \text{(A1.9)}$$

APPENDIX 2

Numerical procedure

The system of two linear differential equations (2.2) for $\Phi_Y(r)$ ($Y = \Lambda, \Sigma$, the index J is dropped here) has been solved in the following way. For a given negative value of $\tilde{E} = -\tilde{B}$, we find two solutions of (2.2), Φ_{Y1} and Φ_{Y2}, which satisfy the initial conditions:

$$\Phi_{Yi}(0) = 0, \quad \Phi'_{Yi}(0) = d_{Yi}, \quad i = 1, 2, \tag{A2.1}$$

where $(d_{\Lambda 1}, d_{\Sigma 1})$ and $(d_{\Lambda 2}, d_{\Sigma 2})$ are two pairs of arbitrary, linearly independent constants. These two solutions have been found by applying the Runge-Kutta method with the step size 0.05 fm and with the upper bound $R = 5$ fm. For $r > R$ the solutions have already, to a very good approximation, their asymptotic forms,

$$\Phi_{Yi}(r) = A_{Yi} \exp(-\alpha_Y r) + B_{Yi} \exp(\alpha_Y r), \quad r > R, \tag{A2.2}$$

where $\alpha_\Lambda = \sqrt{2\mu_\Lambda \tilde{B}}/\hbar$, $\alpha_\Sigma = \sqrt{2\mu_\Sigma(\tilde{B}+\Delta)}/\hbar$. The constants A_{Yi}, B_{Yi} have been determined from the equations:

$$\left.\begin{matrix} A_{Yi} \\ \\ B_{Yi} \end{matrix}\right\} = \tfrac{1}{2} \exp(\pm \alpha_Y R) \left[\Phi_{Yi}(R) \mp \Phi'_{Yi}(R)/\alpha_Y\right]. \tag{A2.3}$$

The general solution Φ_Y of (2.2) may be written as a linear combination of two solutions,

$$\Phi_Y = C_1 \Phi_{Y1} + C_2 \Phi_{Y2}, \tag{A2.4}$$

and has the asymptotic form:

$$\Phi_Y(r) = \left(\sum_i C_i A_{Yi}\right) \exp(-\alpha_Y r) + \left(\sum_i C_i B_{Yi}\right) \exp(\alpha_Y r), \quad r > R. \tag{A2.5}$$

The coefficients C_i for a bound state are determined from the requirement:

$$\sum_i C_i B_{Yi} = 0, \quad Y = \Lambda, \Sigma. \tag{A2.6}$$

Equations (A2.6) for C_i have nonvanishing solution if the determinant

$$D = B_{\Lambda 1} B_{\Sigma 2} - B_{\Lambda 2} B_{\Sigma 1} = 0, \tag{A2.7}$$

which is the eigenvalue equation.

To find the bound state energy E, one has to solve Eqs (2.2) for a few values of $\tilde{E}$, to determine for each of them B_{Yi} from Eq. (A2.3), to calculate the corresponding value of D, till one finds such a value of $\tilde{E} = E$ for which $D(E) = 0$. For this value of E one has from Eq. (A2.6)

$$C_1 = -(B_{\Lambda 2}/B_{\Lambda 1}) C_2 = -(B_{\Sigma 2}/B_{\Sigma 1}) C_2, \tag{A2.8}$$

and for the eigenfunctions Φ_Y one gets

$$\Phi_Y = C_2\{-(B_{\Lambda 2}/B_{\Lambda 1})\Phi_{Y1}+\Phi_{Y2}\}, \tag{A2.9}$$

where C_2 is to be determined from the normalization condition

$$\int_0^R dr(\Phi_\Lambda^2+\Phi_\Sigma^2)+\Phi_\Lambda^2(R)/2\alpha_\Lambda+\Phi_\Sigma^2(R)/2\alpha_\Sigma = 1, \tag{A2.10}$$

in which, in the integration interval (R, ∞), the asymptotic form

$$\Phi_Y(r) = \Phi_Y(R)\exp[-\alpha_Y(r-R)], \qquad r > R \tag{A2.11}$$

has been used. The integration in the interval $(0, R)$ has been performed by means of the Simpson rule with step 0.05 fm.

The value of R has been adjusted so that a further increase in R would not change the results. The results should not depend on the particular choice of the constants d_{Yi}, as actually has been tested in one of the cases considered. Otherwise the values $d_{\Lambda i} = \delta_{1i}$, $d_{\Sigma i} = \delta_{2i}$ have been used.

With $\varrho_C(^3\text{He}, r)$, $\varrho_C(^3\text{H}, r)$ given by Eq. (4.13), we may reduce expression (3.8) for ΔB_C to

$$\Delta B_C = \tfrac{4}{3}P_\Sigma E_C(^3\text{He})-\tfrac{2}{3}e^2 I, \tag{A2.12}$$

where

$$I = \int_0^\infty dr\Phi_\Sigma^2(r)\{2\,\text{erf}\,(r/\beta_{\text{He}})+\text{erf}\,(r/\beta_{\text{H}})\}/r, \tag{A2.13}$$

where erf is the error function, defined as in [17]. The integral I has been computed numerically by means of the Simpson rule with step 0.05 fm in the interval $(0, R)$, and with step 0.1 fm in the interval between R and the cutoff radius 8 fm, where form (A2.11) of $\Phi_\Sigma(r)$ has been used.

Let us remark that erf $(r/\beta)/r$ is a slowly varying function in the interval, where Φ_Σ^2 has appreciable values, which implies that $I \sim P_\Sigma$. A rough estimate of I is thus possible, and it leads t negative values of ΔB_C which approximately agree with our calculated values.

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