Vol. B6 (1975) ACTA PHYSICA POLONICA No 4

GEOMETRICAL SCALING, QUARKS AND THE POMERON

By J. Dias de Deus

The Niels Bohr Institute, University of Copenhagen

and

Theory Division, Rutherford Laboratory, Chilton*

(*Received December 10, 1974*)

From quark model additivity applied in the impact parameter plane to the inelastic overlap function we obtain a kind of scale invariant factorizable Pomeron. Quarks themselves are seen as behaving asymptotically like extended objects (quark pancakes), cross-sections and multiplicities being related to their overlap in a high energy collision. Predictions, which can be tested soon at NAL, are given for overlap functions, cross-sections, the ratio σ^{el}/σ^{tot} in the case of various reactions. Universality features of multiplicity distributions are explained in a natural way and an attempt is made to compute the modifications coming from the leading particle effect.

1. *Introduction*

High energy experiments at the CERN-ISR and at NAL will, hopefully, provide useful information concerning the mechanism responsible for the high energy behaviour of strong interactions, leading to some understanding of the nature of the Pomeron. The ISR experiments have already shown that, at least in the 10^2-10^3 GeV/c region, the Pomeron seems to be a scale invariant Pomeron in the sense that it satisfies geometrical scaling [1, 2]. The NAL experiments using various different beam particles will now allow the study of the internal symmetry properties of this Pomeron. In particular the predictions of the (most) naive quark model [3] may perhaps be tested at NAL.

It is far from being obvious that geometrical scaling is not just an accident happening in pp scattering only and only in a restricted energy region. At higher energies no one knows if the scaling will remain valid. For other elastic processes, Kp and πp scattering for instance, its validity was not yet checked. However very soon new data from NAL probably will allow such tests to be carried out and it would then be interesting to see if scaling is really a general feature of strong interactions at high energy and, if so, how the various distributions (in the impact parameter and in t) for various processes are related.

*Address: Theory Division, Rutherford Laboratory, Chilton, Didcot, Berkshire, OX11 OQX England.

In this paper we take the point of view of considering geometrical scaling as a stable property of the Pomeron. If this is the case it must be present not only in pp but in all the other elastic processes and our main purpose is then to relate the scale invariant distributions of different processes. We thus try to answer questions like: how are the shapes of differential cross-sections or overlap functions related; where is the diffractive dip, which occurs in pp, to be expected in πp and Kp scattering; what is the value of the inelastic overlap function $G^{i}(b)$ at $b = 0$ in πp and Kp scattering (this value is 0.94 in pp).

In this paper we make an attempt to incorporate quarks in the framework of geometrical models in order to obtain some desirable universality properties of hadron distributions. Our arguments will be mostly intuitive and phenomenologically oriented and no claims of rigour are made. An interpretation of the results is given in terms of the scattering of what we would like to call quark-pancakes. This interpretation is however somewhat independent of the testable predictions we make.

The paper is organised as follows. In Section 2 we briefly review the geometrical models and the connection to KNO scaling. In Section 3 we discuss the impact parameter structures of the Pomeron and the non-Pomeron (Reggeon) exchanges and relate them to the way hadrons interact. In Section 4 we introduce additivity for extended quarks (quark pancakes) and obtain predictions for overlap functions, elastic differential cross--sections and cross-section ratios for πp and Kp scattering using the pp data as input. Applications of quark additivity to multiplicity distributions including the leading particle effect and the modified improved KNO scaling are presented in Section 5. We finally draw some conclusions in Section 6.

2. *Geometrical models and multiparticle production*

In geometrical models the emphasis is put on the impact parameter plane and statements are made about matter distributions. The best known model of this kind is the Chou-Yang model [4]. Their rather intuitive ideas about hadrons as extended objects (pancakes) interacting in a semi-classical way, the impact parameter cross-sections being related to the matter overlap are usually accepted as basic ingredients in constructing geometrical models. It is a model of this type that we have in mind in the present paper where quarks are supposed to interact as extended objects (pancakes) in Chou-Yang sense.

Let us, however, first review the geometrical scaling ideas. Geometrical scaling is essentially a statement of scale invariance of the transverse (impact parameter) distribution of hadronic matter in hadron collisions. More precisely, it states that in a AB collision the inelastic overlap function $G^{i}_{AB}(b^2, s)$,

$$\frac{-d\sigma^{i}_{AB}}{db^2} \equiv \pi G^{i}_{AB}(b^2, s) \tag{1}$$

is asymptotically only a function of the scaling variable β [1]:

$$G^{i}_{AB}(b^2, s) \xrightarrow[s\to\infty]{} G^{i}_{AB}\left(\beta \equiv \frac{b^2}{R^2(s)}\right), \tag{2}$$

$R(s)$ being the effective radius in the collision, $R^2 \sim \langle b^2 \rangle$. From (2), neglecting spin and the real part of the amplitude, τ-scaling [2] is derived:

$$\Phi_{AB}(t, s) \equiv \frac{1}{\sigma^2(s)} \frac{d\sigma(s)}{dt} \xrightarrow[s\to\infty]{} \Phi_{AB}(\tau \equiv -tR^2(s)). \tag{3}$$

Consequences of τ-scaling are the following:

(*i*) The ratio of the elastic cross-section $\sigma^e(s)$ over the total cross-section $\sigma^t(s)$ becomes constant

$$\frac{\sigma^e_{AB}(s)}{\sigma^t_{AB}(s)} \to r_{AB}. \tag{4}$$

(*ii*) The slope parameter becomes proportional to $\sigma^t(s)$:

$$B_{AB}(s) \equiv \frac{d\sigma}{dt}\left(\ln \frac{d\sigma}{dt}\right)_{t=0} \sim \sigma^t_{AB}(s). \tag{5}$$

(*iii*) Dips on the differential cross-section occur at fixed τ_{AB} (not at fixed t).

Concerning particle production, in geometrical models the produced particle multiplicity is assumed to be, at a given energy, a sharp distribution, function of the matter overlap (see, for instance, Ref. [5]). As, from (2), the matter overlap is a function of β, the number n of produced particles in a collision at impact parameter b is also a function of β. The available centre of mass energy $\sqrt{s}$ is further supposed to appear via an overall multiplicative factor $\eta(s)$ in such a way that we can write

$$n(b^2, s) = \eta(s)\phi(\beta). \tag{6}$$

We note that a relation with the structure of Eq. (6) also occurs in Mueller's Regge approach. The meaning of the functions $R(s)$ and $\eta(s)$ is the following:

$$\sigma^t(s) \sim R^2(s)$$

and

$$\langle n(s) \rangle \sim \eta(s).$$

Using (1), (2) and (6) KNO multiplicity scaling [6] follows:

$$\Psi_{AB}\left(z \equiv \frac{n}{\langle n \rangle}, s\right) \equiv \langle n \rangle \cdot \frac{\sigma_n(s)}{\sigma^i(s)} \xrightarrow[s\to\infty]{} \Psi_{AB}(z). \tag{7}$$

Experimentally KNO scaling for charged produced particles is well satisfied — see however the discussions in Section 5 — the functions $\Psi(z)$ and $\langle n(s) \rangle$ being, to a good approximation, universal (for a recent analysis see [7]). The τ-scaling (geometrical scaling) is also well satisfied in pp scattering at NAL-ISR energies (see Refs [2] and [8]).

3. Impact parameter structures of the Pomeron and non-Pomeron contributions to inelastic scattering

Phenomenological analysis of intermediate and high energy data has led to the idea that the imaginary part of the Pomeron amplitude in the impact parameter plane is central (grey disc of radius around 0.9 fermi at present energies) while the non-Pomeron (Reggeon) amplitudes are peripheral (say a δ-function at $b = 1$ fermi) [9]. The fact that these two distributions are non-overlapping has the consequence that the elastic cross-section does not receive an appreciable contribution from the secondary trajectories with the result that particle and anti-particle elastic cross-sections are practically equal even at relatively low energies [9].

To make this point clear let us write the unitarity equation for the overlap functions (neglecting imaginary parts):

$$2G^{e}(b^2, s) = G^{e^2}(b^2, s) + G^{i}(b^2, s), \tag{8}$$

where $G^{e}(b^2, s)$ is the impact parameter elastic amplitude,

$$\sigma^{e}(s) = \pi \int G^{e^2}(b^2, s) db^2. \tag{9}$$

The normalisation in (8) is such that $0 \leqslant G^{e,i} \leqslant 1$. We consider now the two contributions in the inelastic overlap function: the dominant one $P(b^2, s)$ (central Pomeron) and secondary one $R(b^2, s)$ (peripheral Reggeon).

$$G^{i}(b^2, s) = P(b^2, s) + R(b^2, s). \tag{10}$$

Assuming now that $G_i^2(b^2, s) \ll G_i(b^2, s)$ (which is not a very good approximation at $b \simeq 0$) and that $R^2(b^2, s) \ll P^2(b^2, s)$ we obtain

$$G^{e^2}(b^2, s) = \tfrac{1}{4} P(b^2, s) \{P(b^2, s) + 2R(b^2, s)\}. \tag{11}$$

However the second term, $P \times R$ in (11), because P and R do not overlap in the b plane approximately vanishes. So in a first approximation we have:

$$G^{e^2}(b^2, s) \simeq \tfrac{1}{4} P^2(b^2, s). \tag{12}$$

Comparing (12) to (10) we observe that σ^{e} is less sensitive to the secondary contributions than σ^{i} or σ^{t}. In other words, the Pomeranchuk theorem should be satisfied earlier for the elastic cross-sections ($\sigma^{e}_{AB} = \sigma^{e}_{A\bar{B}}$).

This simple situation for the overlap functions in the impact parameter, with the Pomeron central region well separated from the Reggeon peripheral one, is lost when going to the t-plane. In fact at each value of t, the elastic amplitude $T(s, t)$ receives contributions from all values of b (Fourier-Bessel transform) and thus the Pomeron and Reggeon contribute together. The Reggeon contributions are particularly strong at small t, due to their impact parameter peripheral nature, causing the observed appreciable differences in the $t \simeq 0$ slopes of the particle and anti-particle differential cross-sections.

If we try to give a realistic interpretation to these impact parameter distributions and to treat the hadrons as extended objects we see that to produce the Pomeron term the hadron must act like some kind of disc of diameter around 0.9 fermi (actually this diameter grows with the energy) and to produce the Reggeon term the hadron must act like a ring of radius r (no overlap except at $b \simeq 2r$). We note that in the string model we have this second kind of situation: "matter" concentrated at the ends and peripheral interaction. Our ring could be generated by a rotating string.

In the present paper we concentrate mostly in the dominant Pomeron term, i.e. in the central interactions.

4. The additive quark model in the impact parameter plane and factorization

We use now the basic ideas of the naive quark model regarding high energy strong interactions in the near forward direction [3] and try to incorporate them in the framework of geometrical models. Quark additivity is formulated for the inelastic overlap function (impact parameter cross-section) in the following way:

$$G^{i}_{AB}(b^2, s) = \sum_{a}\sum_{b} g^{i}_{P}(b^2, s) + \sum_{a}\sum_{b} g^{i}_{R}(b^2, s)\delta_{a\bar{b}}, \quad (13)$$

where the sums are over the quarks of A and B, the first term giving the Pomeron and the second one (involving quark-antiquark $a\bar{a}$ annihilations as in the usual quark duality diagrams) gives the Reggeons, and $g^{i}_{P,R}(b^2, s)$ are SU(3) invariant quark-quark overlap functions.

A way of interpreting (13) is by considering the hadron as behaving not just like one pancake surrounded by a ring (as discussed before) but as a collection of independent

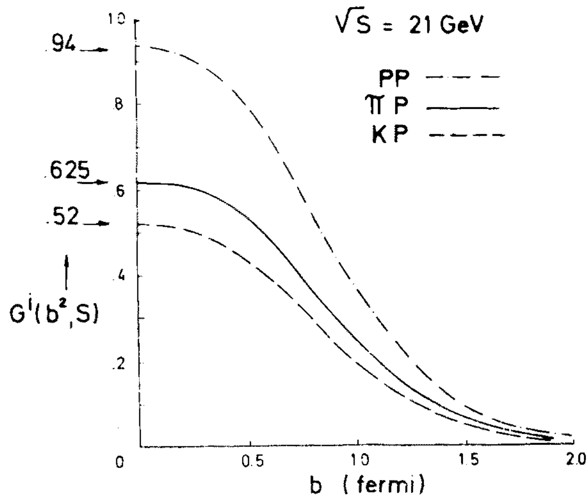


Fig. 1. Inelastic overlap functions at $\sqrt{s} = 21$ GeV. The pp overlap function is from Ref. [10]. The πp and Kp ones are predictions. Indicated in the figure are also the values of $G^{i}(\beta = 0)$

quark pancakes each one surrounded by a quark ring carrying the appropriate quark quantum numbers (charge, strangeness, spin etc.). From now on we just keep the first term in (13) and write the asymptotic additivity relation (dropping the index P) as:

$$G^{i}_{AB}(b^2, s) \underset{s\to\infty}{\simeq} n_A n_B g^{i}(\beta \equiv b^2/R^2(s)), \quad (14)$$

where n_A (n_B) is the number of quarks in A (B) and $g^{i}(\beta)$ the basic scale invariant quark-quark overlap function. The quantity $R(s)$ is assumed to be the only relevant length in high energy hadronic interactions and can be interpreted as a universal parameter measuring the basic quark-quark radius of interaction.

When detailed NAL data will be available Eq. (14) may be directly tested. In Fig. 1, using as input the pp overlap function [10], we show our predictions for the πp and Kp overlap functions at $\sqrt{s} = 21$ GeV. For Kp we used the usual λ-quark coupling reduced factor such that $n_K \simeq 1.7$. Clearly our overlap functions satisfy factorisation:

$$G^{i}_{AB}(b^2, s)^2 = G^{i}_{AA}(b^2, s) \cdot G^{i}_{BB}(b^2, s). \tag{15}$$

In the shadow scattering limit and in the same approximation as before (see Eq. (12)) we derive:

$$\frac{-d\sigma^{e}_{AB}}{db^2} \equiv \pi G^{e}_{AB}(b^2, s)^2 \simeq \frac{\pi}{4}(n_A n_B)^2 g^{i}(\beta)^2. \tag{16}$$

In these limits all cross-sections (elastic, inelastic, total) for all reactions will have asymptotically the same s dependence:

$$\sigma^{i}_{AB}(s), \sigma^{t}_{AB}(s) \sim n_A n_B R^2(s), \tag{17}$$

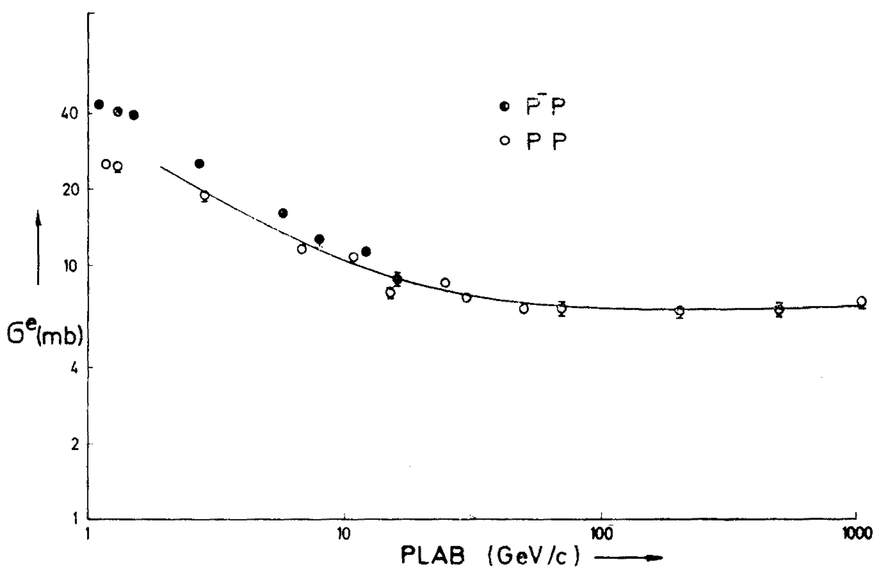
$$\sigma^{e}_{AB}(s) \sim (n_A n_B)^2 R^2(s). \tag{18}$$

For the τ-scaling asymptotic predictions indicated in the Introduction we obtain:

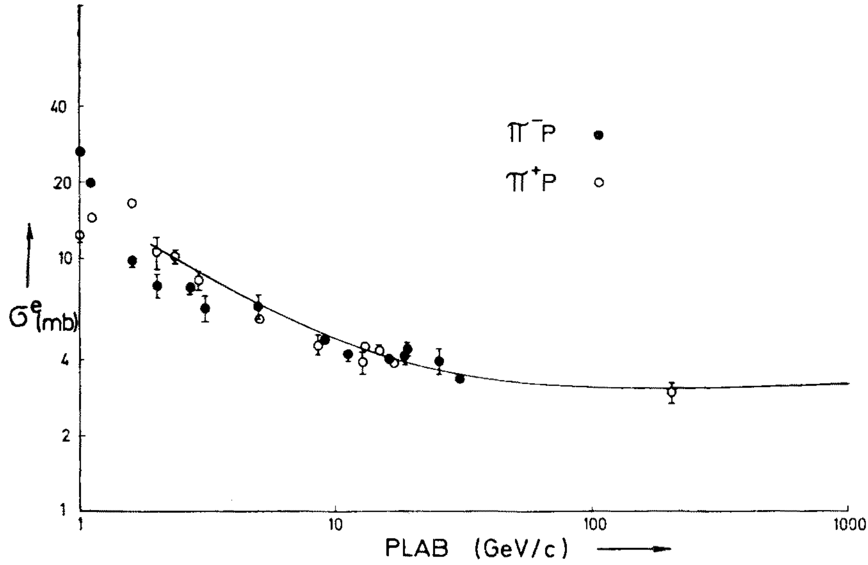
(*i*) $$r_{AB} \equiv \frac{\sigma^{e}_{AB}(s)}{\sigma^{i}_{AB}(s)} \sim n_A n_B, \tag{19}$$

(*ii*) a universal slope parameter: $B_{AB}(s) \sim R^2(s)$,
(*iii*) universal dip positions τ_{dip}.

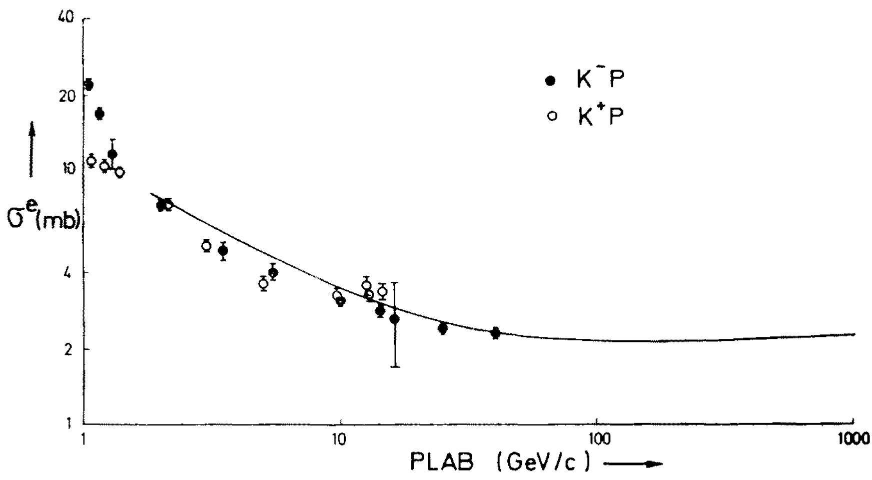
We turn now to comparison with experiment. First we want to compare the energy behaviour of cross-sections. Because of the difference in the impact parameter structures



a



b

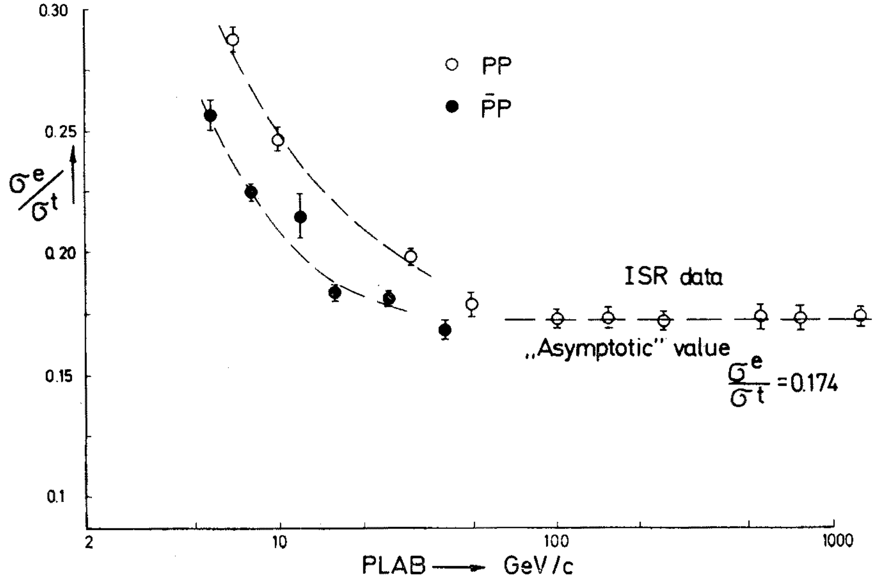


c

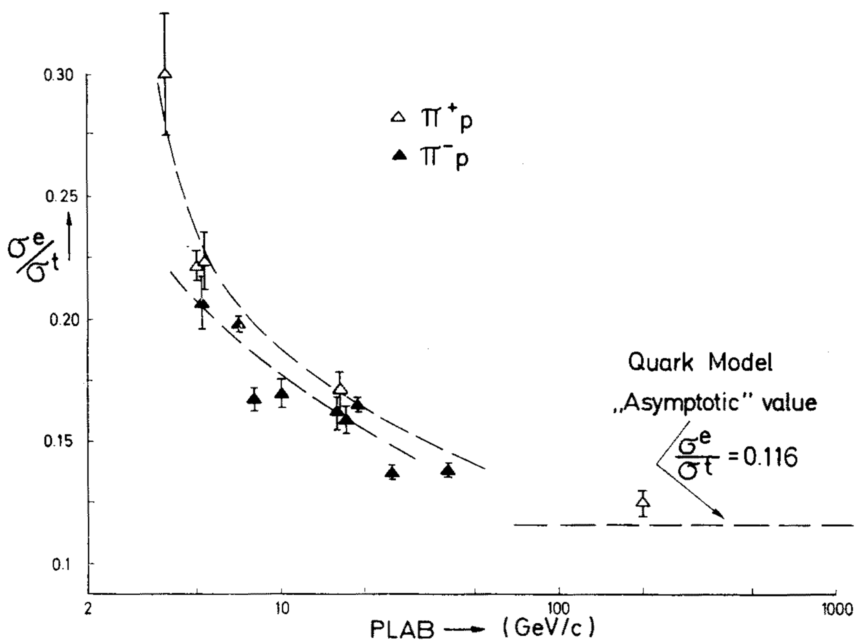
Fig. 2. Elastic cross-sections. The pp curve is used as input the πp and Kp ones are the same curve multiplied by quark model factors: $(6/9)^2$ and $(5/9)^2$ respectively. Data from Ref. [16]; a) pp, b) πp, c) Kp

of the Pomeron and the other Reggeons, discussed before in Section 3, the convenient cross-section to check is the elastic cross-section, as there the Pomeron alone, $(\mathrm{Im}P)^2+(\mathrm{Re}P)^2$, and perhaps some residual $P\times f_0$ interference, dominates even at low energies. In Fig. 2 we check relation (18). The pp curve is used as input and the πp and Kp curves are just the pp one multiplied by the quark model factors $(6/9)^2$, and $\sim(5/9)^2$, respectively. The agreement is good, starting from $P_{\mathrm{LAB}}\sim 2$ GeV/c and it should be interesting to check it at much higher energies. Similar checks for total cross-sections, Eq. (17), can only be carried out at higher energies, where approximate equality of particle and antiparticle cross-sections may be observed.

In Fig. 3 we use Eq. (19) and the pp ISR data as "asymptotic" input to predict the "asymptotic" values of the ratio σ^e/σ^t: 0.116 in πp and 0.099 in Kp scattering. The low



a



b

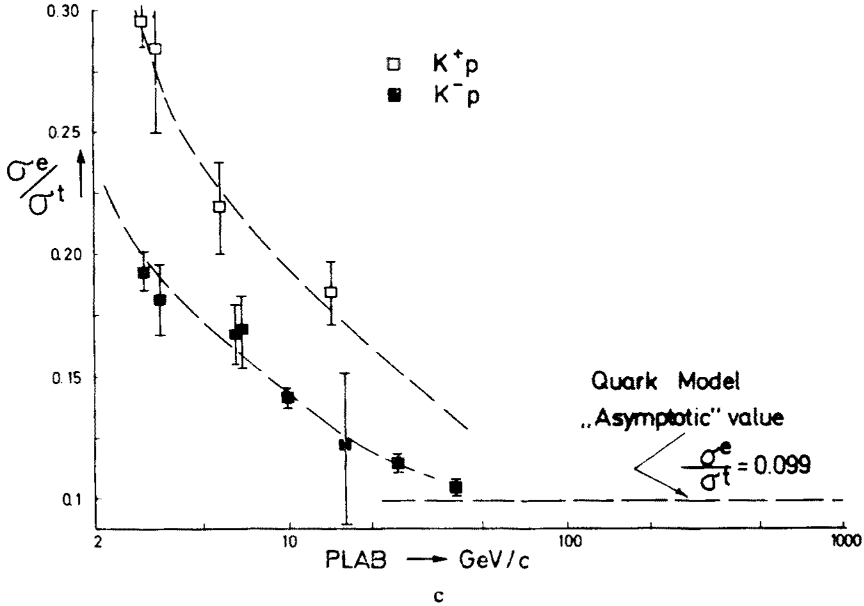


Fig. 3. Ratios σ^e/σ^t. The asymptotic (constant) value for pp is used as input to derive the asymptotic values for πp and Kp. The low energy curves are just guides to the eye. Data From Ref. [16]; a) pp, b) πp, c) Kp

energy strong s dependence of these ratios and their variation in going from particle to antiparticle reflects the non asymptotic character, in general, of σ^t at such energies.

For the predictions concerning the slope parameter and the dip position no detailed high energy data are at the moment available in the πp and Kp cases. The tests are here more delicate because, as mentioned before, the slope and the position of the dip may be affected appreciably by the secondary trajectories. Also small changes in the impact parameter overlap functions may strongly affect the slope and the dip position. Even if Eqs (14) and (16) are approximately valid the universality of $\Phi(\tau \equiv -tR^2(s))$ may be appreciably violated. We note that the asymptotic universality of the slope parameter is equivalent to the universality of the Pomeron trajectory slope. Such universality is not observed experimentally at least at present energies [8].

We come now back to Eqs (17) and (18). They suggest a Pomeron behaving as a renormalised, single Pomeron exchange where factorisation is preserved. A "good" asymptotic forward amplitude, from a theoretical and experimental point of view, could be:

$$T_{AB}(E, t) \simeq C_1 n_A n_B (iE)\left(\ln E - i\frac{\pi}{2}\right)^{\nu} \exp\left\{C_2\left(\ln E - i\frac{\pi}{2}\right)^{\mu} t\right\}, \tag{20}$$

where $E \simeq s/2m_p$, $0 < \nu \leqslant 2$, $\nu \leqslant \mu \leqslant 2$, and C_1 and C_2 are constants. Geometrical scaling requires $\nu = \mu$. The Reggeon field theoretical approach using renormalisation group techniques [11] produces a renormalised Pomeron of the form (20) but not satisfying, so far, the geometrical scaling condition. The dipole model of Phillips [12] has $\nu = \mu = 1$ and so scaling is there satisfied.

5. *Quarks and multiplicity distributions*

In the spirit of the quark model quarks are supposed to scatter independently. The probability of an inelastic collision of pancakes $g^{i}(\beta)$ is so small that simultaneous double scattering and rescattering are negligible. When two hadrons are observed to collide inelastically what takes place is an inelastic collision of two quark pancakes. The produced particles are originated in such single collision of two quark pancakes. The Eq. (6) in Section 2 relating the number of produced particles to the impact parameter distribution thus applies to such collision. The functions $g^{i}(\beta)$ and $\eta(s)$ are now universal quark functions. From (2), (6) and (14) then naturally follows the universality of $\Psi(z)$ and $\langle n(s)\rangle$:

$$\Psi_{AB}(s) \to \Psi(s), \tag{21}$$

$$\langle n(s)\rangle_{AB} \to \langle n(s)\rangle. \tag{22}$$

The universality expressed by relations (21) and (22) being in a first approximation correct does not take however into account the presence of leading particles (as opposed to really produced ones). The improved versions of (21) and (22) include the leading particle effects (diffractive dissociation, non-diffractive dissociation, leading particle associated with pionization) through the parameter α measuring the average number of leading particles. The improved versions of (21) and (22) read:

$$\Psi_{AB}(z') \to \Psi(z'), \tag{23}$$

$$\langle n(s)\rangle_{AB}^{\mathrm{Prod}} \equiv \langle n(s)\rangle_{AB} - \alpha_{AB} \to \langle n(s)\rangle^{\mathrm{Prod}}, \tag{24}$$

where $z' \equiv (n-\alpha)/(\langle n\rangle - \alpha)$. Both Eqs (23) and (24) are in good agreement with experiment.

In order to derive expressions (23) and (24) we follow N–P Chang's idea of independent, factorisable fragmentation [13] applied to the quark pancakes. In an inelastic collision either one or both pancakes fragment. In the first case while one of them gets excited emitting particles the other pancake joins again its original hadron set to form a leading particle in the final state. Let us then rewrite (14) in the form:

$$\frac{d\sigma^{i}}{db^{2}} \simeq n_{A}n_{B}\left(2\frac{d\sigma^{\mathrm{s.e}}}{db^{2}} + \frac{d\sigma^{\mathrm{d.e}}}{db^{2}}\right), \tag{25}$$

where $d\sigma^{\mathrm{s.e}}/db^{2}$ stands for the single excitation cross-section and $d\sigma^{\mathrm{d.e}}/db^{2}$ for the double excitation one. Essentially repeating the calculation of Ref. [13] we get

$$\frac{d\sigma_{AB}^{\mathrm{s.e}}}{db^{2}} \simeq \pi n_{A}n_{B}\,\tfrac{1}{2}\,g^{i}(\beta) \tag{26}$$

and

$$\sigma_{AB}^{\mathrm{s.e}}(s) \simeq \tfrac{1}{2}\,\sigma_{AB}^{i}(s). \tag{27}$$

The number α of leading particles, given by the relation:

$$\alpha \equiv \frac{2\sigma^{\mathrm{s.e}}(s)}{\sigma^{i}(s)} \tag{28}$$

is then, from (27)

$$\alpha \simeq 1. \tag{29}$$

As (29) is true also at each b, Eq. (6) for the bulk of the produced particles should then be modified to

$$n(b^2, s) - \alpha = \eta(s) \cdot \phi(\beta). \tag{30}$$

From (30), and (1) and (2) again, now follows (23) and (24). Independent estimates of α [14, 15, 7], in the case of various reactions also give values close to 1.

6. *Conclusions*

The main purpose of this paper was to include quark additivity in the framework of geometrical models. Applying additivity to the impact parameter inelastic cross-sections and geometrical scaling to the basic quark-quark overlap function, Eq. (14), we obtained the following predictions:

(*i*) Inelastic overlap function, scaling variable $\beta \equiv b^2/R^2(s)$,

$$G^{i}_{AB}(b^2, s) \underset{s\to\infty}{\to} G^{i}_{AB}(\beta) \simeq n_A n_B g^{i}(\beta),$$

with $g^{i}(\beta)$ universal;

(*ii*) Inelastic, Total and Elastic cross-sections,

$$\sigma^{i}_{AB}(s), \sigma^{t}_{AB}(s) \sim n_A n_B R^2(s), \qquad \sigma^{e}_{AB}(s) \sim (n_A n_B)^2 R^2(s)$$

with $R(s)$ universal and in particular

$$\frac{\sigma^{e}_{AB}(s)}{\sigma^{t}_{AB}(s)} = n_A n_B \cdot \text{const.};$$

(*iii*) Differential elastic cross-section, scaling variable $\tau \equiv -tR^2(s)$,

$$\Phi_{AB}(t, s) \equiv \frac{1}{\sigma^2}\frac{d\sigma^{e}}{dt} \to \Phi(\tau)$$

implying asymptotically the same shrinkage and the same dip position for all elastic processes.

Making the usual geometrical models connections between the impact parameter and the number of produced particles, Eq. (6), we further obtained:

(*iv*) Multiplicity distributions, scaling variable $Z \equiv n/\langle n\rangle$,

$$\Psi_{AB}(Z, s) \equiv \langle n\rangle \frac{\sigma_n}{\sigma} \to \Psi(Z),$$

with $\Psi(Z)$ universal:

(*v*) Average multiplicity

$$\langle n(s)\rangle_{AB} \to \langle n(s)\rangle$$

with $\langle n(s)\rangle$ universal.

The results (i), (ii) and (iii) can probably be tested soon at NAL. The results (iv) and (v) are in good experimental agreement when the leading particle effect is taken into account [7].

Pictorically our results for the impact parameter distributions mean that at high energy all hadrons behave as if having the same interaction radius R but an overlap $G(0)$ proportional to the quark content. This is an extreme picture to be contrasted with the other extreme position of assuming the same overlap $G(0)$ but different radius R in different reactions [1]. The present picture, which is obviously an idealised limit, seems to be in better agreement with data than the one proposed in Ref. [1].

Finally we should like to mention that recently several other authors have tried to generalize and extend the observed pp geometrical scaling to other reactions [17]. In particular Barger, Luthe and Phillips emphasize the importance of the higher order terms in the expansion of G^{i} (or of the eikonal χ) which we completely neglected in our approach.

I would like to thank Andrzej Buras, Gerald Canning, Poul Olesen and Chia Tze from the Niels Bohr Institute and Roger Phillips from the Rutherford Laboratory for stimulating discussions and criticism.

REFERENCES

[1] J. Dias de Deus, *Nucl. Phys.* **B59**, 231 (1973).
[2] A. J. Buras, J. Dias de Deus, *Nucl. Phys.* **B71**, 481 (1974).
[3] E. M. Levin, L. L. Frankfurt, *Soviet Phys.-JETP Lett.* **2**, 65 (1965); H. J. Lipkin, F. Scheck, *Phys. Rev. Lett.* **16**, 336 (1966).
[4] T. T. Chou, C. N. Yang, *Phys. Rev. Lett.* **20**, 1213 (1968).
[5] A. J. Buras, Z. Koba, *Nuovo Cimento Lett.* **6**, 629 (1973); A. Białas, *CERN preprint* TH. 1788 (1973).
[6] Z. Koba, H. B. Nielsen, P. Olesen, *Nucl. Phys.* **B40**, 317 (1972).
[7] R. Møller, *Nucl. Phys.* **B74**, 145 (1974).
[8] V. Barger, Plenary Session talk at the *XVII International Conference on High Energy Physics*, London 1974.
[9] H. Harari, M. Davier, *Phys. Lett.* **35B**, 239 (1971); Y. Zarmi, *Phys. Lett.* **37B**, 393 (1971).
[10] H. I. Miettinen, *Impact structure of diffractive scattering*, talk presented at the *IXth Rencontre de Moriond* (1974).
[11] A. A. Migdal, A. M. Polyakov, K. A. Ter Martirosyan, *Phys. Lett.* **48B**, 239 (1974); H. D. I. Abarbanel, J. B. Bronzan, *Phys. Lett.* **48B**, 345 (1974).
[12] R. J. N. Phillips, *Rutherford Laboratory preprint* RL-74-034 (1974).
[13] N. P. Chang, *Impact parameter, a geometric picture of inelastic scattering, Max Planck Institute preprint* (1973).
[14] A. K. Wróblewski, *Warsaw preprint* IFD 5/73 (1973); A. J. Buras, J. Dias de Deus, R. Møller, *Phys. Lett.* **47B**, 251 (1973).
[15] A. Bassetto, J. Dias de Deus, *Nuovo Cimento Lett.* **9**, 525 (1974).
[16] CERN/HERA 72-1, 72-2, 73-1; Yu. M. Antipov et al., *Nucl. Phys.* **B57**, 333 (1973); S. P. Denisov et al., *Phys. Lett.* **36B**, 528 (1971); D. Bogert et al., *Phys. Rev. Lett.* **31**, 1271 (1973).
[17] M. Pennington, G. Ross (in preparation); V. Barger, J. Luthe, R. Phillips, to be published in *Nucl. Phys.* B.