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GEOMETRICAL DESCRIPTION OF HADRONIC COLLISIONS*

By H. I. Miettinen**

CERN, Geneva***

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We discuss the geometrical picture of hadronic collisions. ISR elastic scattering data are analyzed. Some features of multiparticle production are studied, emphasizing qualitative ways of discriminating between the geometrical and the multiperipheral models.

1. Introduction

In this talk I shall discuss a simple geometrical model for hadronic collisions[1]. In this model, the two incoming particles are pictured as two objects of finite spatial extension which propagate through each other. During the passage, their constituents may collide, resulting in the production and emission of secondary particles (see Fig. 1 [5]). Thus, the incoming hadron wave gets attenuated. This is often expressed by saying that it gets "absorbed" into the many open inelastic channels. The strength of the absorption, or the eikonal $\Omega(s, b)$, is usually assumed to be proportional to the amount of overlap of the two matter distributions. In a central collision, this overlap is large and, consequently, the probability that an inelastic interaction occurs is also large. In a peripheral collision, only the "tails" of the two hadrons go through each other. In this case, the chance that an interaction occurs is much smaller than in the previous case.

In the above picture, elastic scattering is regarded as being the shadow of absorption. The elastic amplitude is predominantly imaginary and is related to the eikonal by the formula

$$h_{el}(s, b) = 2(1-e^{-\Omega(s,b)}) = 2\Omega(s, b)-[\Omega(s, b)]^2+\dots \quad (1)$$

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** Herman Rosenberg Foundation Fellow. On leave of absence from the Research Institute for Theoretical Physics, University of Helsinki, Finland.

*** Address: CERN, CH 1211 Geneva 23, Switzerland.

[1] This model was first formulated in a clear-cut way by Yang and his collaborators [1]. Geometrical ideas of hadronic scattering have been studied earlier by many authors, of course. See, for instance, the works of Heisenberg [2], Bhabha [3] and Krisch [4].

From this equation we immediately see several important results:

(*i*) the elastic amplitude at a certain impact parameter is generated by the absorption into the inelastic channels at the same impact parameter; this very important property follows from angular momentum conservation[2];

(*ii*) since the eikonal is assumed to be given by the matter overlap, it is a non-negative quantity; this implies that unitarity is automatically satisfied;

(*iii*) the first and the second terms of the expansion on the right-hand side of Eq. (1) have opposite signs; thus, they interfere destructively. In momentum space, this may easily give dips in $d\sigma/dt$.

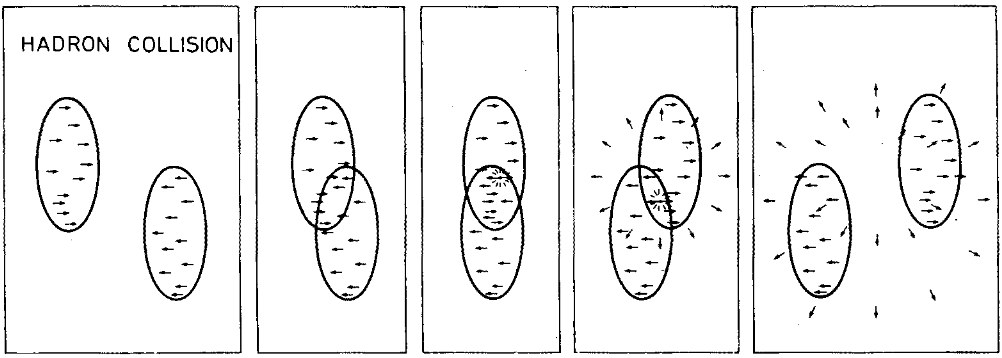


Fig. 1. The evolution of a hadronic collision. In the third and fourth picture one has a "hit". The hadrons move in the directions indicated by the arrows. From Ref. [5]

The above Chou-Yang model works very well phenomenologically. Several authors have shown that it provides a good description for the ISR elastic scattering data, including the "break" at small t values and the diffraction minimum around $t = -1.4 \text{ GeV}^2$ [7]. It should be remembered that the existence of this minimum was *predicted* by Chou and Yang already back in 1968. Thus, that it was experimentally discovered at the ISR in 1972 should certainly be regarded as a success for Chou and Yang.

The geometrical picture is simple and has much intuitive appeal. This, together with its phenomenological successes, suggests that it might be useful to develop it further. What I have in my mind here is, in particular, the application of geometrical ideas to the study of multiparticle processes. There are many interesting problems to be analyzed. To mention a few of them: what is the average multiplicity of particles produced in a collision at a given impact parameter? Are the longitudinal momenta of the secondary particles correlated with the impact parameter positions at which these particles are born and with the over-all impact parameter of the collision? Are the leading protons correlated with each other? I am almost certain that the answer to this last question is affirmative. In a peripheral collision where the incoming protons only slightly touch each other, they should emerge from the collision retaining a large fraction of their longitudinal momenta. In a central collision, on the other hand, the two protons "go through" each other. In this

[2] For a useful discussion, see Ref. [6].

case, one would expect them *both* to lose a sizeable fraction of their longitudinal momenta. In other words, there should exist a positive long-range correlation between the two leading protons, around the axis $y_c = -y_d$.

A further, very interesting, problem to be analyzed is that of the production of heavy particles. The production of baryon-antibaryon pairs, say, obviously requires a lot of energy. In a central collision, the energy available for particle production is probably much larger than in a peripheral one. Thus, at the present energies, one may expect that heavy particles are produced predominantly in central collisions. If true, this would mean that triggering on antiprotons should provide us with a practical way for selecting small impact parameter events!

Many of the predictions of the above "naive" geometrical picture are in striking disagreement with those of the multiperipheral model. An example of such a prediction is provided by that of the correlation between the leading protons. In the geometrical picture, one expects these protons to feel a positive long-range correlation, as discussed above. In the multiperipheral model, on the other hand, the leading protons should be essentially uncorrelated. We shall discuss later also some other quantities, whose measurement should be a particularly good way of distinguishing between the geometrical and the multiperipheral pictures.

2. *Elastic scattering*

As already mentioned, the Chou-Yang model provides a very adequate description for the proton-proton elastic scattering data. The ISR data, together with the model calculation, are shown in Fig. 2 [8]. As seen, their agreement is very good.

It is easy to understand how the two interesting features of the data, the small t "break" (not visible in the figure) and the diffraction minimum, are produced in the model. In the model the eikonal is calculated by assuming the distribution of the hadronic matter inside the proton to be similar to that of the charge. From this, it follows that the eikonal is proportional to the square of the electric form factor

$$\Omega(s, t) \sim G^2(t) = \left[\frac{\mu^2}{\mu^2 - t}\right]^4, \quad \mu^2 = 0.71 \text{ GeV}^2. \tag{2}$$

The contributions of the two leading terms of the eikonal expansion are sketched in Fig. 3. The eikonal, given by Eq. (2), shows continuous curvature. In the forward direction, the contribution of the second term is much smaller than that of the first one. Its slope, however, is only half of that of the first term. Thus the contribution of the second term becomes increasingly important when the momentum transfer increases. The sum of the two terms shows continuous curvature at small $|t|$ values. Moving out in $|t|$, this behaviour changes around $|t| = 0.1 - 0.2 \text{ GeV}^2$ into an approximately exponential one; this gives the appearance of a "break" in this $|t|$ range. At much larger $|t|$ values, the eikonal and the absorption correction cancel each other out, producing a zero in Im $h_{el}(s, t)$ and a dip in the differential cross-section.

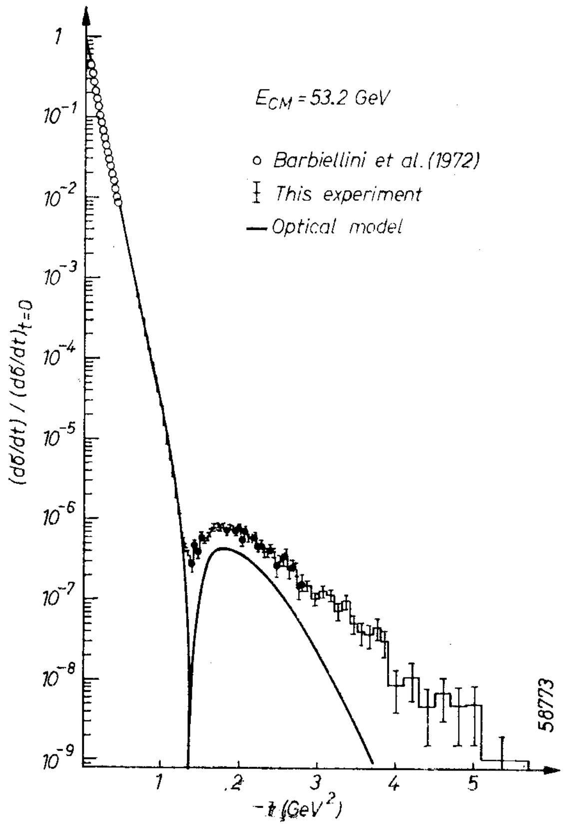


Fig. 2

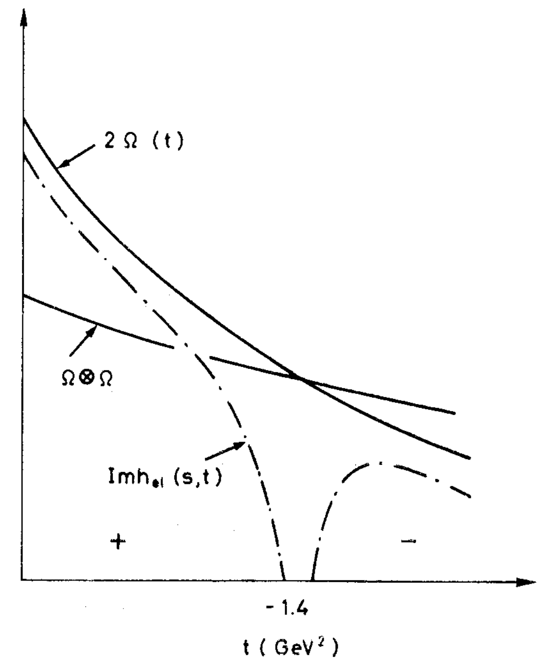


Fig. 3

Fig. 2. Differential cross-section of proton-proton elastic scattering at centre-of-mass energy $\sqrt{s}$ = 53 GeV (Aachen—CERN—Harvard—Genova—Torino collaboration, Ref. [8]). The solid curve is calculated using the Chou-Yang model

Fig. 3. Illustration of the negative interference between the two leading terms of the eikonal expansion

The above discussion is obviously a very rough one, but it illustrates the main point well. In the Chou-Yang approach, both the small $|t|$ curvature and the $t = -1.4\ \text{GeV}^2$ dip are caused by a destructive interference between a strongly curving eikonal and the negative absorption corrections.

The eikonal expansion of Eq. (1) has a simple physical interpretation. The first term, $2\Omega(s, b)$, represents the sum of the elementary probabilities for the constituents of the

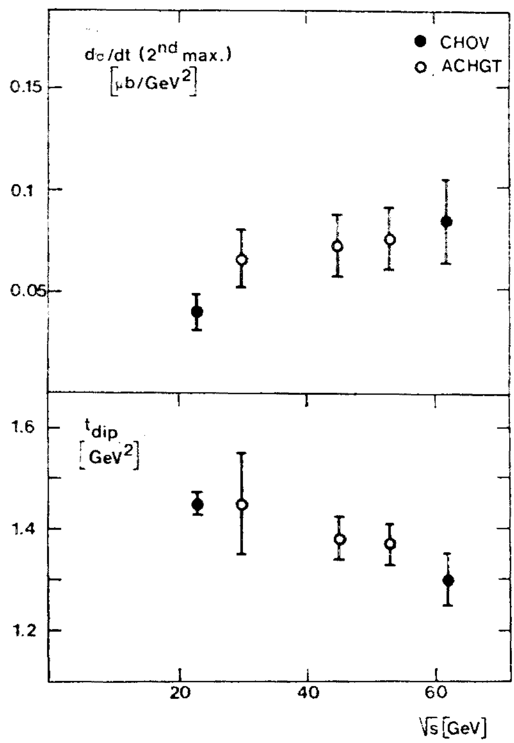


Fig. 4 a) Differential cross-section at the second maximum as a function of energy. b) Position of the minimum near $t = -1.4\ \text{GeV}^2$ as a function of energy (from Ref. [9])

left-moving proton to hit those of the right-moving one. If many hits occur in each collision, this term may clearly exceed the unitarity limit. Unitarity is then restored by the rest of the expansion. The physical interpretation of the correction terms is that they represent the effect of the shielding of the back part of the proton by its front part. This shielding effects is completely analogous to that of the Glauber model description of proton-nucleus scattering.

Although the Chou-Yang model provides a good description for the t dependence of elastic scattering, it does not make any natural predictions for its s dependence. Experimentally, the proton-proton data exhibit clear energy dependence. With increasing energy, 1) the optical point (i.e., the total cross-section) increases, 2) the differential cross--section shrinks, 3) the diffraction minimum moves slowly inwards and 4) the secondary maximum rises (see Fig. 4) [9]. Such an energy dependence may well be incorporated into the model but it is certainly not predicted by it.

There are (at least) two ways in which one may introduce an s dependence into the model [10]. Firstly, one may assume that the way in which the constituents of the two hadrons interact depends on the incoming energy. Thus, one may write

$$\Omega(s, b) = \int \varrho_A(\bar{x}_1)\varrho_B(\bar{x}_2)I(s; \bar{b}-\bar{x}_1+\bar{x}_2)d\bar{x}_1 d\bar{x}_2. \tag{3}$$

Here, $\varrho(\bar{x})$ is the matter distribution and $I(s; \bar{y})$ a function describing the interaction of the constituents of the colliding particles. At very high energies, the interaction is expected to become a local and contact one, so that $I(s; \bar{y})$ should approach a delta function. At finite energies, however, the interaction may be non-local and this may make the eikonal s dependent. The second way of making the eikonal s dependent is much more straightforward: one simply assumes the matter distribution to be s dependent, i.e., that $\varrho(\bar{x}) \to \varrho(s; \bar{x})$. It is clear that in this way one may fit an arbitrary s dependence. The problem of understanding the physical origin of this s dependence, however, still remains.

Let us now look at the b space structure and the s dependence of the ISR elastic scattering data in more detail. For this purpose, we write the s channel unitarity equation as follows [11]

$$\mathrm{Im}\, h_{el}(s, b) = \tfrac{1}{4}\, |h_{el}(s, b)|^2 + G_{inel}(s, b). \tag{4}$$

Here, the first term on the right-hand side corresponds to the shadow of elastic scattering and the second one to that of the inelastic channels. They are called the elastic and inelastic overlap functions, respectively [12]. The unitarity equation is illustrated in Fig. 5.

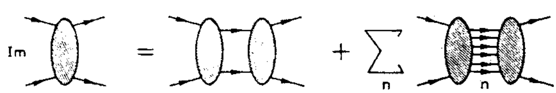


Fig. 5. Illustration of the s channel unitarity equation

The three terms of Eq. (4) have a simple physical interpretation. They tell us how the total, elastic and inelastic cross-sections, respectively, are distributed as functions of the impact parameter. For example,

$$G_{inel}(s, b) \equiv \sum_n |T_{ni}(s, b)|^2 \equiv \frac{1}{\pi}\frac{d\sigma_{inel}}{db^2}. \tag{5}$$

Here, $T_{ni}(s, b)$ is the amplitude for producing the inelastic state n at the impact parameter b.

The inelastic overlap function $G_{inel}(s, b)$ may be obtained from the elastic scattering data. The results I shall show are from a recent analysis by Pirilä and myself [13, 14]. Similar analyses have been performed also by several other authors [11, 15]. Our results are in a reasonably good qualitative agreement with those of most of the other analyses, although important quantitative differences occur.

The results of our analysis are shown in Figs 6 — 8. They may be summarized as follows [14]:

a) Im $h_{el}(s, b)$ is very nearly a Gaussian over the b^2 range from 0 to 2 (Fermi)^2; at larger impact parameters it levels off; this large b tail is directly related to the sharp break of the differential cross-section at $t = -0.15\ \mathrm{GeV}^2$;

b) $G_{inel}(s, b)$ bends down near $b = 0$; in t space, this corresponds to a zero of $G_{inel}(s, t)$ around $t = -0.6\ \mathrm{GeV}^2$;

c) at $b = 0$, the value of $G_{inel}(s, b)$ is $(94 \pm 1)\%$ of the maximum value allowed by unitarity ("the black disk limit"); it stays constant through the ISR energy range;

d) the rise of the total cross-section comes from a relatively narrow region around 1 Fermi.

These results obviously provide strong constraints for theoretical models of multiple production. The observation that the amount of S wave absorption stays constant at a level of 94% of the black disk limit is particularly intriguing. Why does it stay constant? And why at the 94% level? One may speculate in (at least) two different directions[14]:

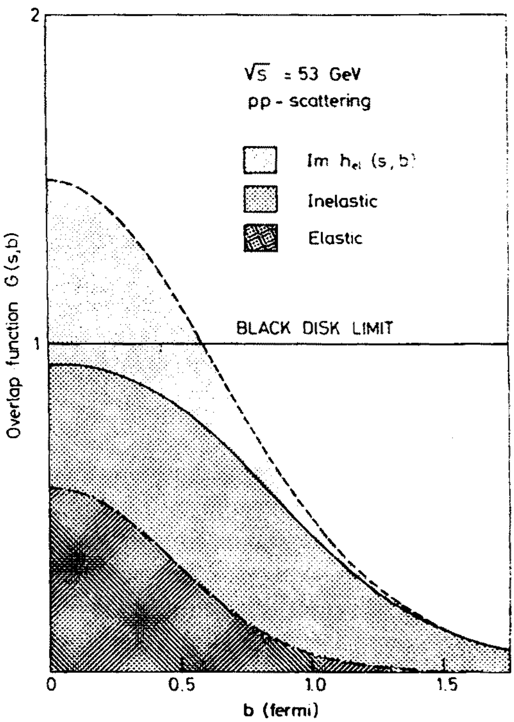


Fig. 6

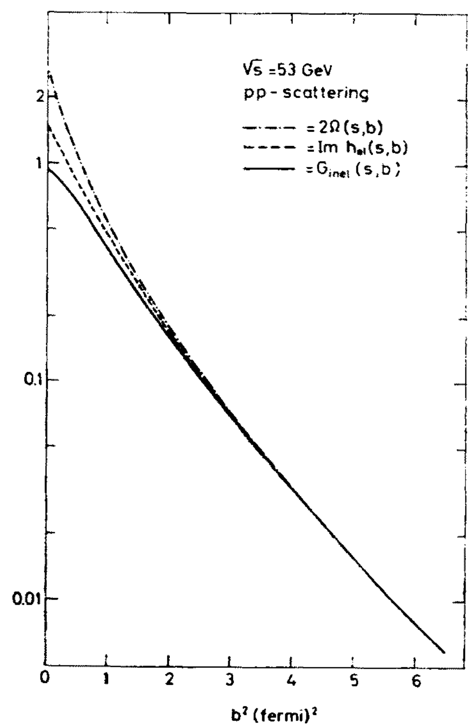


Fig. 7

Fig. 6. Impact structure of proton-proton scattering at $\sqrt{s} = 53$ GeV. Shown are Im $h_{el}(s, b)$ and the inelastic and elastic overlap functions extracted from experimental data. The "black disk limit" indicates the maximum value of the inelastic overlap function allowed by unitarity (100% absorption) (from Ref. [13])

Fig. 7. The amplitude, inelastic overlap function and eikonal extracted from experimental data at $\sqrt{s} = 53$ GeV. Notice the large b tail and the flattening of $G_{inel}(s, b)$ near $b = 0$ (from Ref. [13])

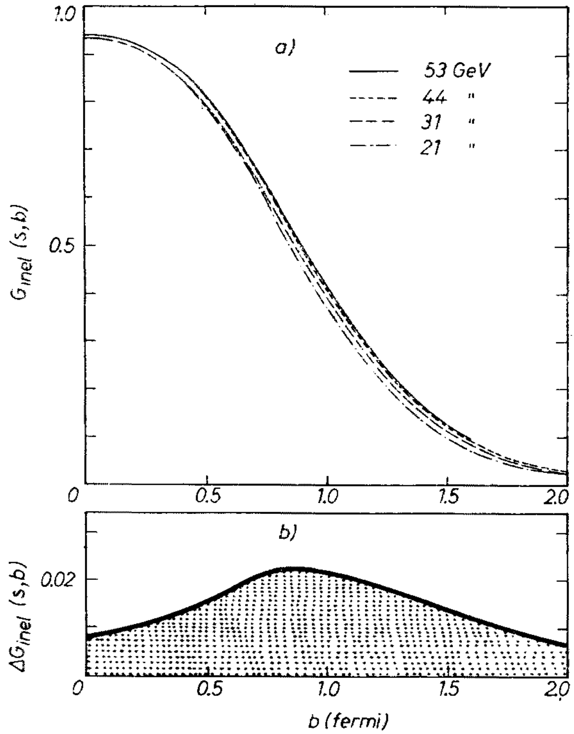


Fig. 8 a) Inelastic overlap functions calculated from the $\sqrt{s}$ = 21, 31, 44 and 53 GeV ISR data. b) Difference of the $\sqrt{s}$ = 53 and 31 GeV inelastic overlap functions, $G_{inel}(s, b)$, showing that the cross--section increase comes from a rather narrow region around 1 Fermi (from Ref. [13])

(*i*) there exists an "effective" unitarity limit at a level of 94% of the real black disk limit, caused by some unknown mechanism; the overlap function saturates this limit;

(*ii*) the constancy of $G_{inel}(s, b=0)$ at the ISR energy range is a "fake-effect", caused by this quantity passing through a broad minimum.

It will be very interesting to study the behaviour of $G_{inel}(s, b)$ in kaon-nucleon and pion-nucleon scattering in the FNAL energy range. This is so since both in KN and πN scattering, $G_{inel}(s, b)$ is much below the unitarity limit [the main reason why $\sigma_{tot}(K^+p)$ is much smaller than $\sigma_{tot}(pp)$ is *not* that the radius of the kaon would be much smaller than that of the proton but that the *opaqueness* of the kaon is less than that of the proton]. Thus, unitarity effects are expected to be much less important in KN and πN scattering than in proton-proton scattering.

3. *Multiparticle production*

Let us now proceed to the analysis of multiparticle production. Since there exist no quantitative geometrical models for particle production, we shall be satisfied with trying to find qualitative ways to discriminate between the geometrical picture and other approaches.

Average multiplicity versus impact parameter

To measure experimentally the multiplicity of the particles produced in a collision at a given impact parameter is a very hard task. In order to do it, one should determine all the n particle amplitudes, including their *phases*, and Fourier-Bessel transform them into the impact parameter representation. With our present knowledge of the many-body amplitudes, such an analysis is clearly impossible to carry out. Still, something may be said about the b dependence of the multiplicity. In a recent paper, Białas and Białas show that it is possible to derive *upper and lower limits* for the average multiplicity at a given impact parameter [16]. Although these limits are not very stringent, it turns out that they are stringent enough to rule out some popular models for $\bar{n}(b)$!

Let us write the over-all multiplicity distribution $P(n)$ as an integral over the fixed b multiplicity distributions $p(n, b)$

$$P(n) = \frac{1}{\sigma_{\rm inel}} \int d^2 b G_{\rm inel}(s, b) p(n, b). \tag{6}$$

This equation is completely general. To derive from it the properties of the function $p(n, b)$ further assumptions are needed. In their analysis, Białas and Białas assume that:

(*i*) $p(n, b)$ is a very narrow distribution: $p(n, b) \approx \delta(n - \bar{n}(b))$;

(*ii*) the average multiplicity $\bar{n}(b)$ is a monotonic function of the impact parameter.

With these assumptions, one may immediately perform the integral in Eq. (6):

$$P(n) = \frac{\pi}{\sigma_{\rm inel}} G_{\rm inel}(b) \frac{1}{\left| \frac{d\bar{n}(b)}{db^2} \right|_{b=b_n}}, \tag{7}$$

where b_n is the solution of the equation $\bar{n}(b_n) = n$.

In Eq. (7), only the absolute value of $d\bar{n}(b)/db^2$ appears. Thus, one has a sign ambiguity. From this it follows that there exist *two* solutions. Choosing the negative sign on the right-hand side of Eq. (7) gives $\bar{n}(b)$ decreasing with increasing b. In contrast to this, the positive sign solution increases with increasing b. The two solutions, as obtained by Białas and Białas, are shown in Fig. 9.

Looking back at the assumptions needed to derive the above results, one may obviously question the assumption that $p(n, b)$ is a very narrow distribution. How would the above results be modified in the case of $p(n, b)$ having a finite width? The answer to this question is that there would still exist two solutions, one decreasing and one increasing with increasing b, but these solutions would be less rapidly varying that the previous ones. Thus, varying the assumption about the width of $p(n, b)$ one may obtain an infinite set of solutions, all of which would be lying in the cone determined by the zero width ones. From this we conclude that the curves shown in Fig. 9 may be regarded as upper and lower limits for $\bar{n}(b)$, and that the experimental function $\bar{n}(b)$ should lie between them.

In the geometrical picture, $\bar{n}(b)$ is expected to be a decreasing function of b. A popular assumption is that $\bar{n}(b)$ is proportional to the eikonal $\Omega(s, b)$. Now, one may ask if such

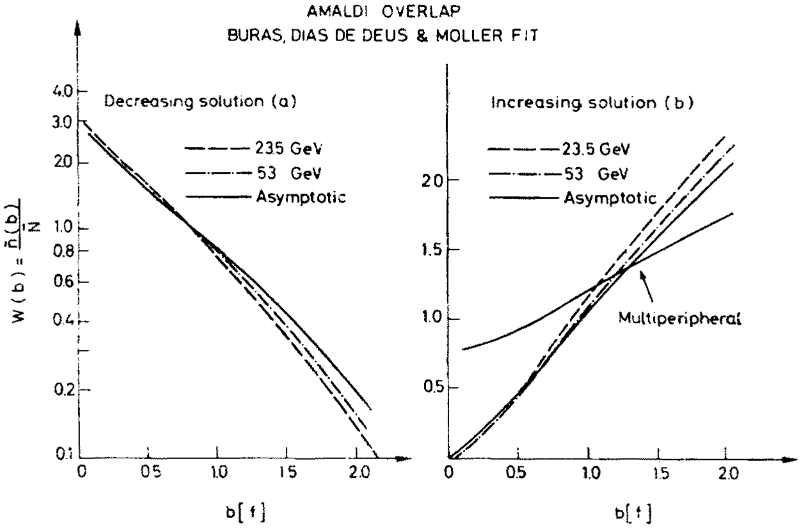
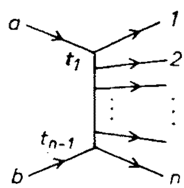


Fig. 9. Average multiplicity $\bar{n}(b)$ of negative particles produced in high energy proton-proton collisions as a function of the impact parameter: a) decreasing solution; b) increasing solution (from Ref. [16])

an assumption is consistent with the results of Białas and Białas. Comparing the experimental eikonal shown in Fig. 7, with the solutions of Białas and Białas, we see that the eikonal decreases faster than the limiting solution a). Thus, we conclude that the popular assumption of $\bar{n}(b)$ being proportional to the eikonal is indeed ruled out by the experimental data!

The impact parameter structure of the multiperipheral model is easy to analyze in the weak coupling limit. In simple versions of the MPM, the amplitude is assumed to factorize into a product of functions of the momentum transfers t_j (short-range order):

$T(ab \to 1 \ldots n) =$



$= \prod_i e^{a t_i}$

In the weak coupling limit, the variables conjugate to the momentum transfers are the impact parameter steps Δb_j, i.e., the differences between the impact parameters of the particles produced next to each other in the chain. The assumption that the t_j's factorize implies that the impact parameter steps are independent. This means that *the MPM is equivalent to a random walk in impact parameter* [17].

A basic property of a random walk is that the total length of the walk is proportional to the square root of the number of steps: $b = \text{const.} \sqrt{\bar{n}(b)}$. Inverting this, we find that $\bar{n}(b) \sim b^2$. Thus, $\bar{n}(b)$ is a rapidly *increasing* function of b.

The above discussion is clearly a very simplified one. Its weakest side is that it assumes the coupling constant to be small. Experimentally, however, the coupling constant (i.e., the particle density) is very large. For high particle densities, the above simple relation between the t_j's and the Δb_j's is no longer valid, and the analysis becomes much more complicated. Also other effects, such as the Reggeization of the particles exchanged, should be considered. In a recent paper, Jadach and Turnau pointed out that for these more complicated cases $\bar{n}(b)$ may even be qualitatively different from the above result and *decrease* with increasing b [18]. For a further discussion, we refer to their paper.

Factorization of the two hemispheres

A simple qualitative way to discriminate between the geometrical and the multiperipheral pictures is provided by the study of the correlations. Basically, the multiperipheral model is a short-range correlation model, while the geometrical picture predicts that all kinds of interesting long-range correlations should be present. Here, we shall discuss two simple examples of such correlations.

As a first example, let us study the multiplicity correlation between the two hemispheres. Let n_R (n_L) be the multiplicity of particles in the right (left) hemisphere, $p(n)$ the multi-

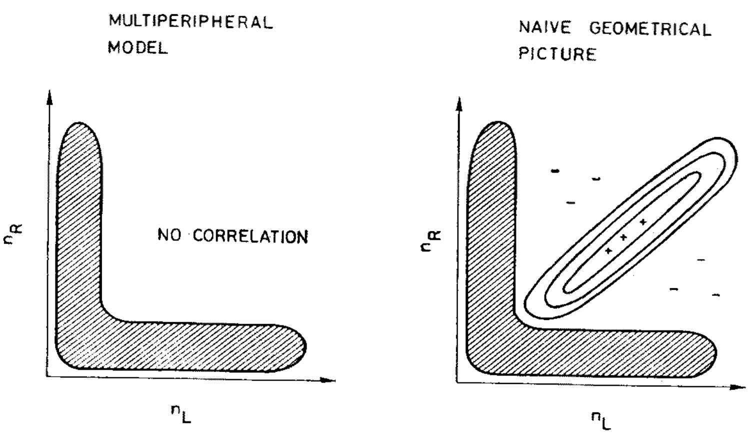


Fig. 10. Illustration of the left-right multiplicity correlation expected in the multiperipheral and the geometrical models. In the shadowed area, correlations due to inelastic diffraction may be present

plicity distribution in one hemisphere and $p(n_R, n_L)$ the probability distribution for there being n_R particles in the right hemisphere *and* n_L particles in the left one. Define the multiplicity correlation function $C(n_R, n_L)$ as follows:

$$C(n_R, n_L) = p(n_R, n_L) - p(n_R)p(n_L). \tag{8}$$

This correlation function is normalized as follows:

$$\sum_{n_R} C(n_R, n_L) = \sum_{n_L} C(n_R, n_L) = 0. \tag{9}$$

The multiperipheral model is a short-range correlation model. In this model, the multiplicities of the two hemispheres are essentially independent, i.e., $C(n_R, n_L) \approx 0$. In the geometrical model, on the other hand, one expects to find a high (low) multiplicity in *both* hemispheres, if the collision has been a central (peripheral) one. Thus, according to this picture, the correlation should be positive in the neighbourhood of the diagonal $n_R = n_L$. Due to the normalization condition Eq. (9), it should be negative in the other regions of the $n_R - n_L$ plot.

The above predictions are illustrated in Fig. 10. To test them, it is best to study relatively large multiplicities. This is to avoid the complications present at low multiplicities due to inelastic diffraction.

Our second example is provided by the correlation between the leading protons. In the multiperipheral model, such a correlation should be very small or zero. What is the prediction of the geometrical picture? In a "head on" collision, one expects, clearly, that *both* protons should lose a considerable fraction of their longitudinal momenta. In a peripheral collision, they both are expected to retain most of their momenta. Thus, one expects a positive correlation along the axis $y_c = -y_d$. Such a correlation would clearly be of long-range nature, since it would extend over the whole phase space.

The above proton-proton longitudinal correlation is expected to be a particularly useful quantity to be studied, since it is only little influenced by the "phase space effects".

Heavy particle production [19]

Experimentally, the antiproton spectrum is still rising fast at the ISR energies. Since the available centre-of-mass energy at the ISR is $\sqrt{s} = 20 - 60$ GeV, and since the minimum mass of a $N\bar{N}$ pair is less than 2 GeV, one may safely conclude that some dynamical mechanism must be delaying the $N\bar{N}$ production threshold.

In the multiperipheral framework such a delayed threshold for $N\bar{N}$ production is caused by a kinematic "t-min effect" [20]. It is fairly easy to see that the strength of such a t-min effect depends on the masses of the produced particles in such a way that the spectra of the light particles approach their scaling limits fast, while those of the heavy particles will scale very slowly.

In the geometrical picture, the delayed threshold of $N\bar{N}$ production may be understood as follows (see the illustration in Fig. 11) [19]. Consider a collision process at a fixed impact parameter $\bar{b}$. Assume that different parts of the production volume behave independently, i.e., production at a position $\bar{x}$ in the impact plane is controlled by the *local* energy density $\xi(s; \bar{b}, \bar{r})$[21]. Now, to produce a $N\bar{N}$ pair in a collision at an impact parameter $\bar{b}$ at the position $\bar{r}$, the local energy density $\xi(s; \bar{b}, \bar{r})$ must exceed some threshold value $\xi_{th}^{N\bar{N}}$. At low and medium energies ξ is generally smaller than $\xi_{th}^{N\bar{N}}$, and, consequently, very few $N\bar{N}$ pairs are produced. As the energy is increased, $\xi(s; \bar{b}, \bar{r})$ exceeds $\xi_{th}^{N\bar{N}}$. *This occurs first in small-impact-parameter collisions*, and $N\bar{N}$ production sets in. *In this transition energy region, heavy particles are expected to be produced in small-impact-parameter collisions only.* When the energy is further increased, the size of the part of the production volume "hot" enough to produce $N\bar{N}$ pairs grows and the difference between the impact structures of $N\bar{N}$ pair and pion production diminishes. The rapid growth of the ratio of the $N\bar{N}$ pair to the pion pair production at the NAL-ISR energies indicates that these energies still belong to the transition energy range.

According to the above arguments, triggering on antiprotons biases one towards central collisions. Thus, one expects the antiproton events to show many of the characteristics of the small b collisions previously discussed. Firstly, central collisions should have larger multiplicity than the peripheral ones. Our preliminary calculations within a simple

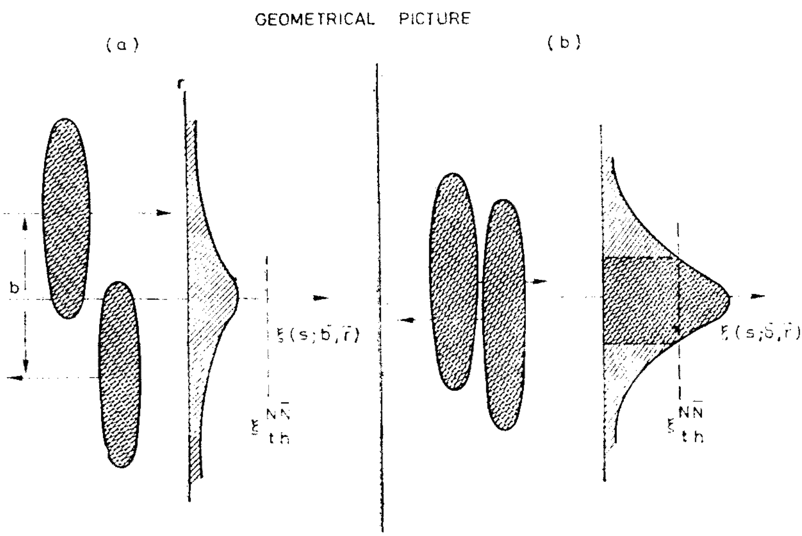


Fig. 11. a) Geometrical picture of a peripheral collision; no $N\bar{N}$ pairs are produced. b) Geometrical picture of a central collision; the energy density in the checkered region is large enough to allow $N\bar{N}$ production (from Ref. [19])

geometrical model indicate that this increase of the multiplicity should be large enough to overcome the decrease of the multiplicity due to the $N\bar{N}$ pair taking more energy than an average pion pair. Hence we expect the associated multiplicity $\langle n^- \rangle_{\bar{p}}$ to be larger than the over-all $\langle n^- \rangle$. Secondly, in a central collision, the fragments of incident protons are less likely to carry away a substantial fraction of the incident momentum; so we expect the associated single particle spectra $(\varrho_\pi)_{\bar{p}}$ and $(\varrho_p)_{\bar{p}}$ to be more peaked toward the central region than the normal single particle spectra ϱ_π and ϱ_p.

The above predictions are opposite to those of the multiperipheral model. The MPM predicts that events with heavy particles produced should look essentially similar to those with only pions produced[3].

Some predictions of the multiperipheral and the geometrical pictures are summarized below.

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[3] A small difference is to be expected, due to the fact that the $N\bar{N}$ replaces 3–4 pions in the chain.

Quantity	Multiperipheral model	Naive geometrical picture
left-right multiplicity correlation $C(n_R, n_L)$	no correlation	$C(n_R, n_L) > 0$ along the axis $n_R = n_L$
leading proton correlation	no correlation	$C_{pp}(y_c, y_d) > 0$ along the axis $y_c = -y_d$
associated multiplicity $\langle n_{ch}\rangle_{\bar{p}}$	$\langle n_{ch}\rangle_{\bar{p}} < \langle n_{ch}\rangle$	$\langle n_{ch}\rangle_{\bar{p}} > \langle n_{ch}\rangle$
associated pion spectra $(\varrho_\pi)_{\bar{p}}$	as usual or flatter	more central
associated leading proton spectra $(\varrho_p)_{\bar{p}}$ [4]	as usual	more central, may even peak at $x = 0$

4. *Conclusions*

The main points of this talk may be summarized as follows:

A) the geometrical model provides a good description for the t dependence of the ISR elastic scattering data, including the "break" at small t values and the diffraction minimum around $t = -1.4$ GeV^2;

B) the model does not make any natural predictions for the s dependence of the data, although an arbitrary s dependence may easily be incorporated into it;

C) the b and s dependence of the inelastic overlap function $G_{inel}(s, b)$ extracted from the ISR data were discussed; a particularly intriguing feature of this quantity is that at $b = 0$ it stays constant at a level of 94% of the black disk limit over the whole ISR energy range;

D) the study of $G_{inel}(s, b)$ in KN and πN scattering in the FNAL energy range should provide particularly useful information on the importance of the unitarity effects and on the origin of the rise of the total cross-sections;

E) upper and lower limits for the average multiplicity of the particles produced in a collision at a given impact parameter may be derived from the experimental data; these limits exclude the popular hypothesis that $\bar{n}(b) \sim \Omega(b)$;

F) neither the multiperipheral nor the geometrical model is ruled out by the existing data; the study of the left-right multiplicities, leading proton correlations and antiproton production should be a particularly good way of discriminating between these models; some predictions are given.

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[4] "Leading proton spectra" = difference of the proton and antiproton spectra.

REFERENCES

[1] T. T. Wu, C. N. Yang, *Phys. Rev.* **137**, B708 (1965); N. Byers, C. N. Yang, *Phys. Rev.* **142**, 976 (1966); T. T. Chou, C. N. Yang, *High Energy Physics and Nuclear Structure*, G. Alexander, Ed., North-Holland Publishing Co., Amsterdam 1967, pp. 348–395; L. Durand III, R. Lipes, *Phys. Rev. Lett.* **20**, 637 (1968); T. T. Chou, C. N. Yang, *Phys. Rev.* **170**, 1591 (1968); *Phys. Rev. Lett.* **20**, 1213 (1968).

[2] W. Heisenberg, *Z. Phys.* **101**, 533 (1936); **113**, 61 (1939); **126**, 569 (1949); **133**, 65 (1952).

[3] H. J. Bhabha, *Proc. R. Soc.* **219A**, 293 (1963); see also: D. S. Narayan, *Nucl. Phys.* **B34**, 386 (1971).

[4] A. D. Krisch, *Phys. Rev.* **135**, B1456 (1964); *Phys. Lett.* **44B**, 71 (1973).

[5] H. B. Nielsen, P. Olesen, *Phys. Lett.* **B43**, 37 (1973).

[6] G. Calucci, R. Iengo, *Nuovo Cimento Lett.* **4**, 33 (1972).

[7] B. Carreras, J. N. J. White, *Nucl. Phys.* **B32**, 319 (1971); **B42**, 95; J. N. J. White, *Nucl. Phys.* **B51**, 23 (1973); M. Kac, *Nucl Phys.* **B62**, 402 (1973).

[8] A. Böhm et al., Aachen—CERN—Harvard—Genova—Torino collaboration, *Phys. Lett.* **49B**, 491 (1974).

[9] E. Nagy et al. CERN—Hamburg—Vienna collaboration, *Experimental results on large-angle elastic pp scattering at the CERN ISR*, paper submitted to the *XVII International Conference on High Energy Physics*, London 1974.

[10] For attempts to incorporate an s dependence into the Chou-Yang model, see Refs [7] and: F. Hayot, U. P. Sukhatme, *University of Michigan preprint UM HE* 73–31 (1973); R. Henzi, B. Margolis, P. Valin, *Phys. Rev. Lett.* **32**, 1077 (1974).

[11] For an elementary discussion of the s channel unitarity equation, see: E. H. de Groot, H. I. Miettinen, *Shadow approach to diffraction scattering*, *Proceedings of the VIII Rencontre de Moriond*, J. Tran Thanh Van, Ed., Méribel-les-Allues 1973.

[12] L. Van Hove, *Nuovo Cimento* **28**, 798 (1963); *Rev. Mod. Phys.* **36**, 655 (1964).

[13] P. Pirilä, H. I. Miettinen, in preparation. Some results of this analysis are given in Ref. [14].

[14] H. I. Miettinen, *Impact structure of diffraction scattering*, to be published in the *Proceedings of the IX Recontre de Moriond*, J. Tran Thanh Van, Ed., Méribel-les-Allues 1974; also *CERN preprint TH.* 1864 (1974).

[15] U. Amaldi, *Elastic and inelastic processes and the intersection storage rings*; the experiments and their impact parameter description, to be published in the *Proceedings of the Ettore Majorana International School of Subnuclear Physics*, Erice, July 1973; R. Henzi, P. Valin, *Phys. Lett.* **B48**, 119 (1974); F. S. Henyey, R. Hong Tuan, G. L. Kane, *Nucl. Phys.* **B70**, 445 (1974); A. W. Chao, C. N. Yang, *Phys. Rev.* **D8**, 2063 (1973); N. Sakai, J. N. J. White, *Nucl. Phys.* **B59**, 511 (1973).

[16] A. Białas, E. Białas, *Acta Phys. Pol.* **B5**, 373 (1974).

[17] That the weak coupling MPM corresponds to a random walk b space has been known for a long time among the experts. For a recent clear discussion, see: F. S. Henyey, *Phys. Lett.* **45B**, 469 (1973).

[18] S. Jadach, J. Turnau, *Phys. Lett.* **50B**, 369 (1974); See also: F. S. Henyey, *Longitudinal momentum transfers in the multiperipheral model, University of Michigan preprint UM HE* 74–8 (1974).

[19] T. K. Gaisser, H. I. Miettinen, Chung-I Tan, D. M. Tow, Is $\bar{p}$ production peripheral or central?, to be published in *Phys. Lett.*; also *Brown University preprint* COO-3130 TA-306 (1974).

[20] L. Caneschi, *Nucl. Phys.* **B68**, 77 (1974); S. Humble, *Phys. Lett.* **40B**, 373 (1972); R. Iengo, A. Krzywicki, B. Petersson, *Phys. Lett.* **43B**, 307 (1973); T. K. Gaisser, Chung-I Tan, *Phys. Rev.* **D8**, 3881 (1973).

[21] See, for example, Ref. [3].