ODD NEUTRON HOLE $N = 81$ NUCLEI IN THE INTERMEDIATE COUPLING UNIFIED MODEL

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The intermediate coupling unified model is applied to the odd neutron hole $N = 81$ nuclei ^{145}Gd, ^{143}Sm, ^{141}Nd, ^{139}Ce and ^{137}Ba. In the calculations all the single-neutron hole states of the major shell 50–82 are coupled to the $N = 82$ core excitations up to three phonons. The spectra of the levels with the spins up to 19/2 are in agreement with the experiment if the first 4^+ states in the neighbouring $N = 82$ nuclei are approximated by two-phonon states.

1. *Introduction*

In the unified model the single particle or the single hole motion of extra-core nucleons (holes) are coupled to the collective motion of the even-even core. When the interaction is not too strong we are dealing with the intermediate coupling. The intermediate coupling model is applicable to the nuclei in the region not far from closed shells. In this model the individual particle motion is coupled to the vibrational motion of the spherical core.

The first discussion of this model was given by Bohr and Mottelson in 1953 [1]. From this moment the intermediate coupling scheme has been applied by many people to different nuclei. This method is still interesting because more and more experimental material is available.

In the present paper the intermediate coupling unified model is applied to the odd neutron hole nuclei $^{145}_{81}$Gd$_{64}$, $^{143}_{81}$Sm$_{62}$, $^{141}_{81}$Nd$_{60}$, $^{139}_{81}$Ce$_{58}$ and $^{137}_{81}$Ba$_{56}$. The energies of the one-phonon and two-phonon states are taken to be equal to the excitation energies of the lowest 2^+ and 4^+ levels, respectively, in the neighbouring doubly even $N = 82$ nuclei.

The low-spin states of the $N = 81$ nuclei in the harmonic model have been treated in the paper [2]. The energies and spectroscopic factors of the low spin states calculated by us are similar to those of the paper [2]. The position of the high spin odd-parity states strongly depends on the two-phonon state energy, and is improved if the anharmonic correction is introduced.

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2. *Short review of the formalism of the intermediate coupling model*

In the unified model the shell structure and nuclear deformation interplay with each other. The motion of nucleons in a nucleus is described in terms of independent particle and collective degrees of freedom. The Hamiltonian describing this motion has the form

$$H = H_{\mathrm{p}}+H_{\mathrm{s}}(\dot{\alpha}_{2\mu})+H_{\mathrm{int}}(\alpha_{2\mu}, x), \tag{1}$$

where H_{p} is the shell-model Hamiltonian of the individual particle, H_{s} describes the nuclear surface. For the quadrupole oscillations H_{s} is given by

$$H(\alpha_{2\mu}) = \sum_{\mu} \{\tfrac{1}{2} B_2|\dot{\alpha}_{2\mu}|^2+\tfrac{1}{2} C_2|\alpha_{2\mu}|^2\}. \tag{2}$$

The coordinates $\alpha_{2\mu}$ correspond to the quadrupole deformation of the nuclear surface, B_2 is the mass parameter and C_2 denotes the nuclear rigidity. The frequency of the surface oscillations is given by

$$\omega = \sqrt{\frac{C_2}{B_2}}. \tag{3}$$

The coupling term H_{int} represents the interaction of the particle (hole) with the nuclear deformation and has the form

$$H_{\mathrm{int}} = \mp k(r) \sum_{\mu} \alpha_{2\mu} Y_{2\mu}(\theta, \varphi) \tag{4}$$

to the first order in $\alpha_{2\mu}$. Here r, θ and φ are the polar coordinates of the particle. The coefficient $k(r)$ is connected with the change of the potential at the boundary. It is given by [3]

$$k(r) = r\frac{\partial V(r)}{\partial r}. \tag{5}$$

For a sharp nuclear boundary the expectation value of $k(r)$ is

$$\langle nl|k(r)|n'l'\rangle = V_0 R_0^3 \mathscr{R}_{nl}(R_0)\mathscr{R}_{n'l'}(R_0). \tag{6}$$

For binding energies in the region 5–10 MeV it is of the order of 40 MeV [1].

In the intermediate coupling treatment, the representation of the uncoupled motion is usually employed. The wave function may be written as

$$\psi = \sum_{jNR} |j, NR; IM\rangle \langle j, NR; IM|\psi\rangle, \tag{7}$$

where N denotes the number of phonons, R is the angular momentum of the phonons I is the total angular momentum, and M is its z component.

The expression for the matrix elements of the interaction Hamiltonian H_{int} in the representation of the uncoupled motion has been derived in detail in [4]. It may be written in the form

$$\langle lj, NR; IM|H_{\text{int}}|l'j', N'R'; IM\rangle$$

$$= \tfrac{1}{2}\,\xi\hbar\omega(-1)^{J+R'-l-1/2}\sqrt{(2j+1)(2j'+1)}\sqrt{(2R'+1)(2l'+1)}\,(l'200|l0)$$

$$\times \begin{Bmatrix} R' & R & 2 \\ j & j' & I \end{Bmatrix} \begin{Bmatrix} j' & l' & \frac{1}{2} \\ l & j & 2 \end{Bmatrix} (\langle NR\|b\|N'R'\rangle + \langle N'R'\|b\|NR\rangle), \tag{8}$$

where b is the annihilation operator of phonons. The dimensionless parameter

$$\xi = k\sqrt{5/2\pi\hbar\omega C_2} \tag{9}$$

characterizes the strength of the interaction. The intermediate coupling corresponds to the value of ξ of the order of 1–4. The approximate value of the coefficient ξ can be determined from the reduced transition probability $B(\text{E2}, 2^+ \to 0^+)$, providing the rigidity C_2 for harmonic vibrations is given by

$$C_2 = \frac{\hbar\omega}{2B(E2, 2^+ \to 0^+)}\left(\frac{3}{4\pi} Ze\mathscr{R}_0^2\right)^2. \tag{10}$$

Usually ξ is treated as parameter. The other parameters are: individual particle energies ε_j and phonon energies $\hbar\omega$. The energies and wave functions of a nucleus are obtained by diagonalization of the total Hamiltonian (1) in the representation (7). If the wave functions are known the spectroscopic factors defined as

$$S_l = |\langle lj, 00; j|\Psi\rangle|^2(2j+1) \tag{11}$$

can be calculated and compared with the experimental values obtained in pick-up reactions. The spectroscopic factor is a measure of the admixture of the single particle state j coupled to the zero phonon state in the total wave function.

3. *Calculations and results*

In the calculations all the single-hole states in the major shell 50–82 are taken into account. These states are: $2d_{3/2}$, $3s_{1/2}$, $1h_{11/2}$ $2d_{5/2}$, and $1g_{7/2}$. The single neutron hole energy spacing $\Delta s_{1/2} = E(s_{1/2}) - E(d_{3/2})$, $\Delta h_{11/2} = E(h_{11/2}) - E(d_{3/2})$ and coupling strength ξ are treated as parameters. The energy spacings $\Delta d_{5/2} = E(d_{5/2}) - E(d_{3/2})$ and $\Delta g_{7/2} = E(g_{7/2}) - E(d_{3/2})$ are taken from the experiment [5]. The one-phonon and two-phonon energies are assumed to be equal to the 2^+ and 4^+ energies, respectively, in the neighbouring $N = 82$ nuclei. The values of the single neutron hole energies, phonon energies and coupling strength used in our calculations are shown in Table I. The best fit is obtained for the same values of ξ and $\Delta h_{11/2}$ as in paper [2], also the values of the parameter $\Delta s_{1/2}$ are close to those in [2].

The results of the calculations are shown in Figs 1–5. The spectra of the even-parity low spin states have similar features as those for the harmonic vibrations [2]. In compar-

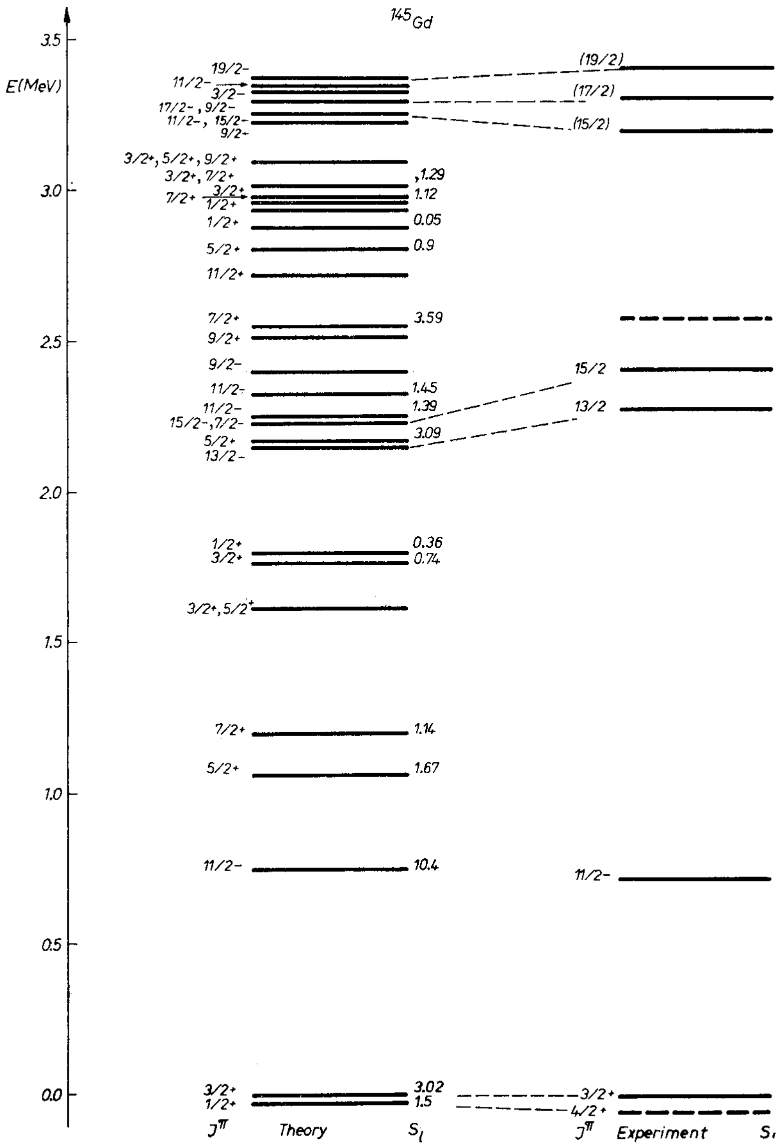
TABLE I

Single neutron hole energies, one-, two-, three-phonon energies and ξ used in the calculations

	^{145}Gd	^{143}Sm	^{141}Nd	^{139}Ce	^{137}Ba
$\Delta s_{1/2}$	0.00	0.22	0.35	0.48	0.38
$\Delta h_{11/2}$	0.39	0.39	0.41	0 44	0 54
$\Delta d_{5/2}$	1.52	1.52	1.47	1.72	1.71
$\Delta g_{7/2}$	2.12	2.12	2.20	2.90	2.93
E1	1.5795	1.6590	1.5760	1.5967	1.4350
E2	2.6580	2.1906	2.1010	2.0834	1.8990
E3	3.9870	3.2859	3.1520	3.1251	2.8480
ξ	1.5	1.5	1.5	1.5	1.0

ison with experiment [5], [6] the positions of the lowest $3/2^+$, $1/2^+$, $11/2^-$, $\overset{(1)}{5/2^+}$, $\overset{(1)}{7/2^+}$, $\overset{(2)}{5/2^+}$, $\overset{(3)}{5/2^+}$ and $\overset{(2)}{7/2^+}$ states are well reproduced. In all considered nuclei the ground state has spin and parity $3/2^+$ and is almost the pure $|2d_{3/2}^{-1}, 00,3/2^+\rangle$ state. The first excited state is the $1/2^+$ state with energy smaller then 0.5 MeV. It is mainly formed by the $|3s_{1/2}^{-1}, 00,1/2^+\rangle$ state. The second $1/2^+$ level appears in the energy interval 1.5–2.0 MeV and has, as the main component the $|2d_{3/2}^{-1}, 12,1/2^+\rangle$ single-hole state. The lowest three $5/2^+$ levels lying in the interval 1.0–2.5 MeV are built mainly from the uncoupled states $|2d_{5/2}^{-1}, 00,5/2^+\rangle$, $|2d_{3/2}^{-1}, 12,5/2^+\rangle$ and $|3s_{1/2}^{-1}, 12,5/2^+\rangle$. The lowest two $7/2^+$ levels lying in the intervals 1.0–1.5 MeV and 2.0–2.5 MeV are built mainly from the components $|2d_{3/2}^{-1}, 12,7/2^+\rangle$ and $|2d_{3/2}^{-1}, 22,7/2^+\rangle$, respectively. The first experimental $7/2^+$ state contains more of the state $|1g_{7/2}^{-1}, 00,7/2^+\rangle$ than the theoretical one. In the calculations the $7/2^+$ levels with a great admixture of the $|1g_{7/2}^{-1}, 00,7/2^+\rangle$ components appears at about 3.0 MeV for ^{139}Ce and ^{137}Ba.

The odd-parity high spin states $11/2^-$, $13/2^-$, $15/2^-$, $17/2^-$ and $19/2^-$ are formed by the single-neutron hole $1h_{11/2}$ state coupled to the core vibrations. In this case the core excitations up to three phonons are taken into account. The experimental data for these states are taken from papers [7] and [8]. The high spin odd parity states are built from the $1h_{11/2}$ single neutron hole state coupled mainly to the zero phonon state for $11/2^-$ levels, to one phonon for the $13/2^-$, $15/2^-$ doublets and to two-phonon state for the $15/2^-$, $17/2^-$, $19/2^-$ triplets. The parity of the levels with spins 13/2, 15/2, 17/2 and 19/2 has not been determined in the experiment. However, since their energies and spins correspond to the energies and spins of the theoretical levels formed by the $1h_{11/2}$ state coupled to the quadrupole vibrations, one can suppose that they really have the odd parity. In comparison with the experiment the order of the spin is correct. The calculated splitting of the levels in the doublets $13/2^-$, $15/2^-$ and in the triplets $15/2^-$, $17/2^-$, $19/2^-$ is smaller then the experimental one by the factor about 0.5. The lowering of the levels $13/2^-$, ... $19/2^-$ in the model with anharmonic correction is caused by the low energy of the 4^+ levels in the $N = 82$ nuclei. In the case of harmonic oscillations the theoretical triplets $15/2^-$, $17/2^-$, $19/2^-$ lie much higher then the experimental ones.



145Gd
E(MeV)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
19/2−
11/2−
3/2−
17/2−, 9/2−
11/2−, 15/2−
9/2−
3/2+, 5/2+, 9/2+
3/2+, 7/2+
3/2+
7/2+
1/2+
1/2+
5/2+
11/2+
7/2+
9/2+
9/2−
11/2−
11/2−
15/2−, 7/2−
5/2+
13/2−
1/2+
3/2+
3/2+, 5/2+
7/2+
5/2+
11/2−
3/2+
1/2+
1.29
1.12
0.05
0.9
3.59
1.45
1.39
3.09
0.36
0.74
1.14
1.67
10.4
3.02
1.5
(19/2)
(17/2)
(15/2)
15/2
13/2
11/2−
3/2+
1/2+
J^π
Theory
S_l
J^π
Experiment
S_l

Fig. 1

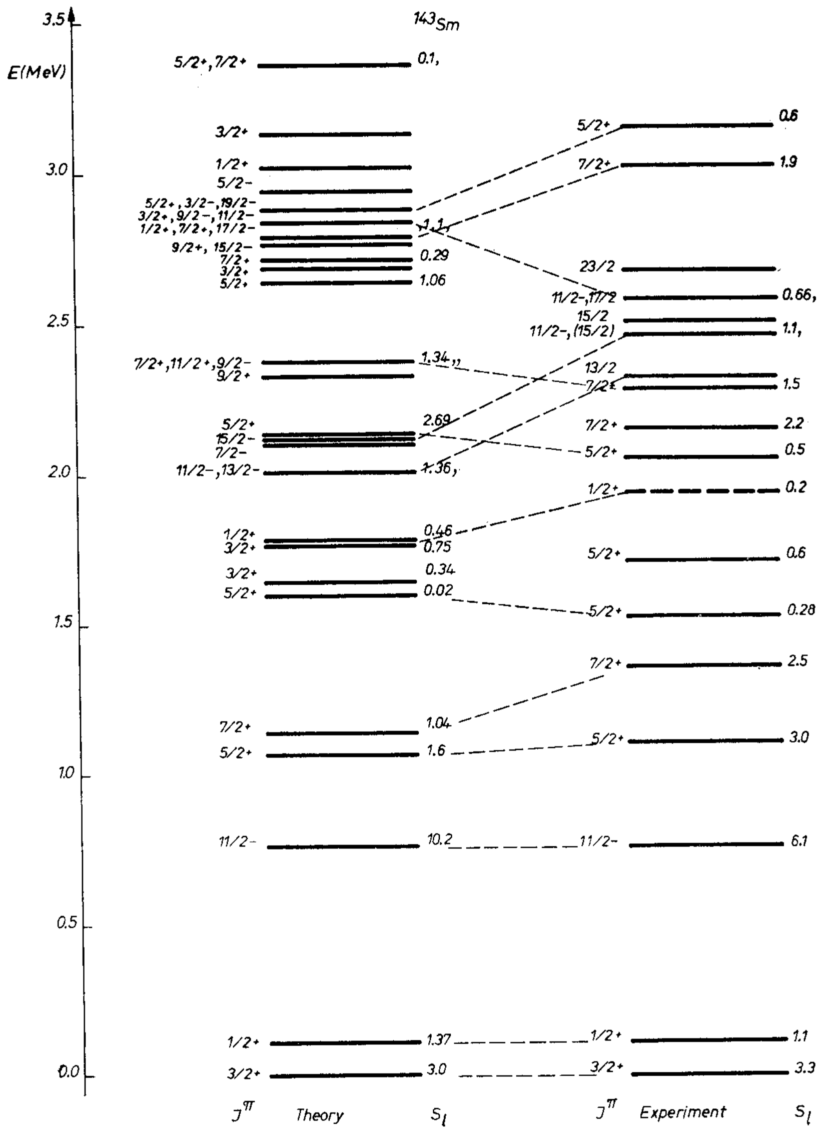

143Sm
E(MeV)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
5/2+, 7/2+
0.1,
3/2+
1/2+
5/2−
5/2+, 3/2−, 19/2−
3/2+, 9/2−, 11/2−
1/2+, 7/2+, 17/2−
9/2+, 15/2−
7/2+
3/2+
5/2+
1.1,
0.29
1.06
7/2+, 11/2+, 9/2−
9/2+
1.34,,
5/2+
15/2−
7/2−
11/2−, 13/2−
2.69
1.36,
1/2+
3/2+
0.46
0.75
3/2+
5/2+
0.34
0.02
7/2+
5/2+
1.04
1.6
11/2−
10.2
1/2+
3/2+
1.37
3.0
5/2+
0.6
7/2+
1.9
23/2
11/2−, 17/2
0.66,
15/2
11/2−, (15/2)
1.1,
13/2
7/2+
1.5
7/2+
2.2
5/2+
0.5
1/2+
0.2
5/2+
0.6
5/2+
0.28
7/2+
2.5
5/2+
3.0
11/2−
6.1
1/2+
1.1
3/2+
3.3
Jπ
Theory
Sl
Jπ
Experiment
Sl

Fig. 2

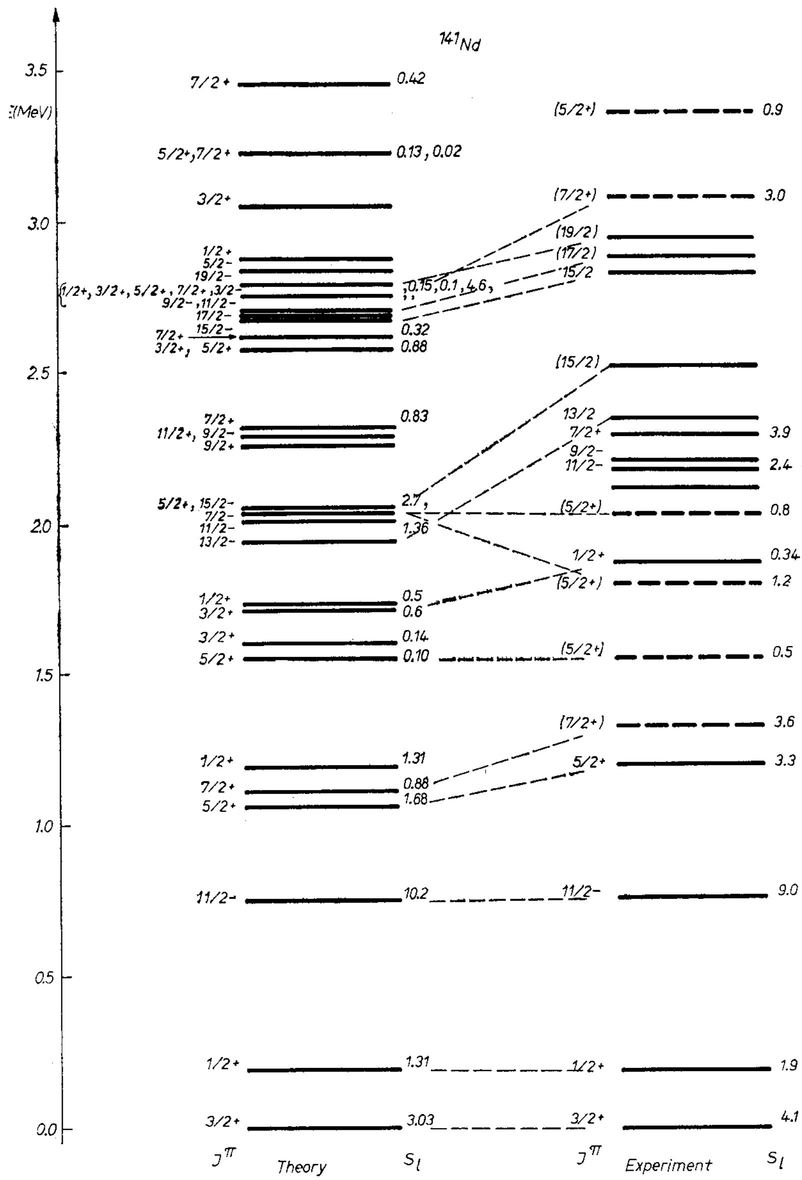


Fig. 3

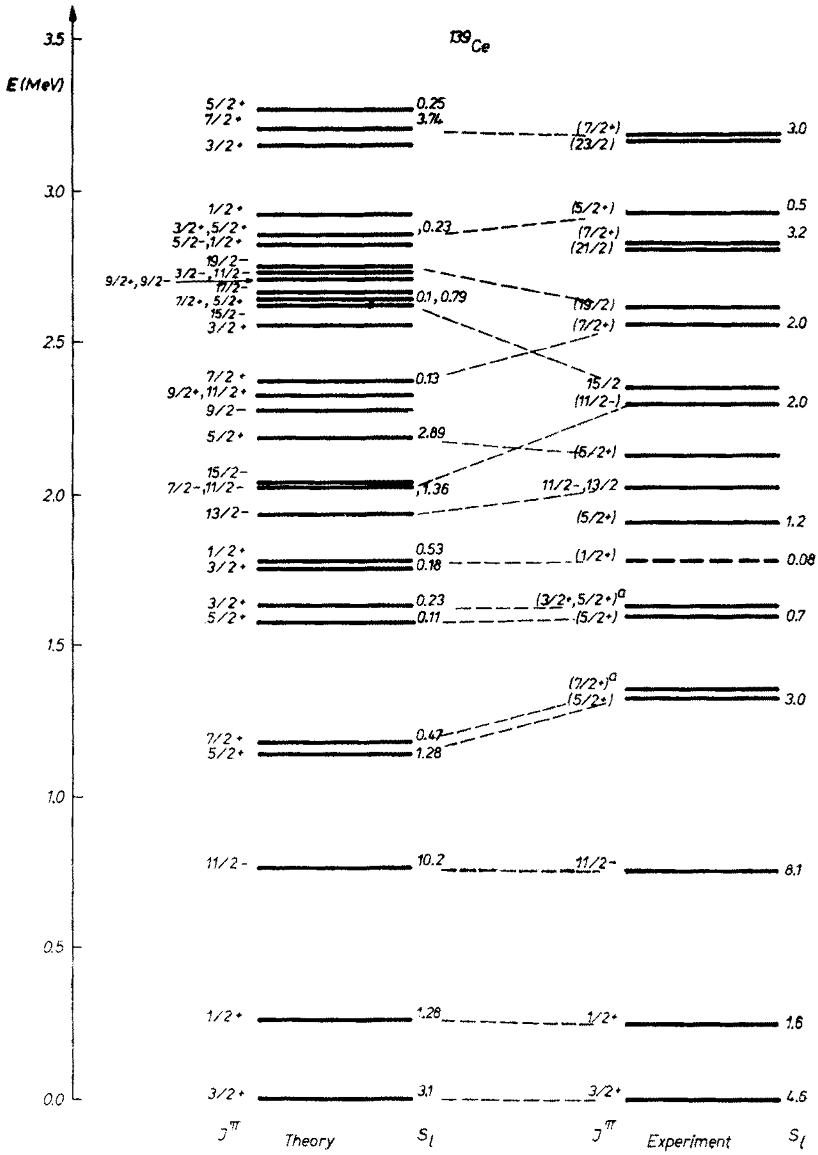
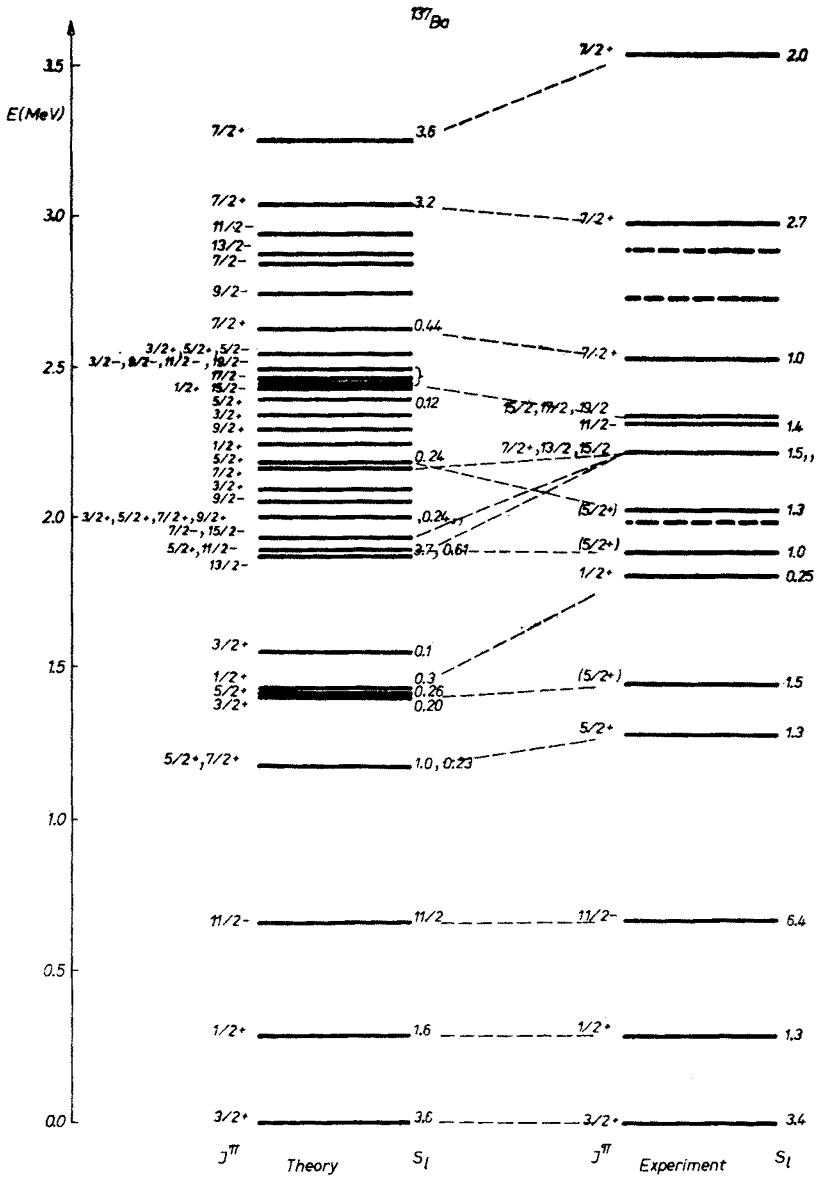

139Ce
E (MeV)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
5/2+ 0.25
7/2+ 3.74
3/2+
1/2+
3/2+,5/2+ ,0.23
5/2−,1/2+
19/2−
9/2+,9/2−
3/2−,11/2−
11/2−
7/2+, 5/2+ 0.1, 0.79
15/2−
3/2+
7/2+ 0.13
9/2+,11/2+
9/2−
5/2+ 2.89
15/2−
7/2−,11/2− ,1.36
13/2−
1/2+ 0.53
3/2+ 0.18
3/2+ 0.23
5/2+ 0.11
7/2+ 0.47
5/2+ 1.28
11/2− 10.2
1/2+ 1.28
3/2+ 3.1
(7/2+) 3.0
(23/2)
(5/2+) 0.5
(7/2+) 3.2
(21/2)
(19/2)
(7/2+) 2.0
15/2
(11/2−) 2.0
(5/2+)
11/2−,13/2
(5/2+) 1.2
(1/2+) 0.08
(3/2+,5/2+)a
(5/2+) 0.7
(7/2+)a
(5/2+) 3.0
11/2− 8.1
1/2+ 1.6
3/2+ 4.6
Jπ
Theory
Sl
Jπ
Experiment
Sl

Fig. 4



Figs 1 — 5. Theoretical spectra of the excited levels are compared with the experimental data which are taken from the paper [5], a) — [6], [7]. Calculated spectroscopic factors smaller than 0.1 are not given

In conclusion one can say that for the considered $N = 81$ nuclei the intermediate coupling model with anharmonic correction gives quite good agreement with the experiment for the states with the low spin and even parity as well as for the odd parity states up to spin 19/2.

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