THE GAUGE A-FIELD AND THE STRONG GRAVITY

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The system of the interacting gravitational field and the gauge A-field, introduced by the authors earlier, is considered in the static, spherically symmetric case. In the asymptotical approximation it is shown that the gauge A-field behaves as a short-range one with some effective mass, and that the gravitational potential together with the usual long-range term has also the short-range term, typical for the strong gravity.

The regulating role of gravity in suppressing infinities in the quantum field theory has been discussed by a number of authors [1]. The important regulating effect of the tensor gravity in the quantum electrodynamics has been shown by Salam and his collaborators [2]. For the realization of a similar consideration in the case of the strong interaction these authors have proposed the hypothesis of the existence of the strong gravity with the coupling constant $\kappa_f \sim 1\ \mathrm{GeV}^{-1}$ [1] [3]. In accordance with this hypothesis, together with the usual metric tensor $g_{\mu\nu}$, the metric tensor $f_{\mu\nu}$, connected with the energy-momentum tensor of the strong interacting particles, is also introduced. Since the field $f_{\mu\nu}$ has the rest-mass $m_f \sim \kappa_f^{-1}$ the static gravitational potential also contains, together with the usual long-range term $\left(\sim \frac{1}{r}\right)$, the short-range Yukawa-like term $(\sim e^{-m_f r}/r)$, the presence of which defines the strong gravity of hadrons. It should be noted that from the point of view of the orthodox Einstein theory of gravity the presence of two metric tensors seems to be unnatural, as has been noted in particular in [4]. That is why in [4] the question of the strong gravity has been considered from another point of view on the basis of one of the variants of the scalar-tensor theory.

In the present paper it is shown that the behaviour of the gravitational potential typical for the strong gravity can be obtained in a natural way in a framework of the approach developed by the authors in [5][2]. In [5] we have introduced the gauge field $A^{ik}{}_{\mu}$ (A-field),

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[1] The natural units $\hbar = c = 1$ are used.

[2] In the present paper the notations introduced in [5] are used. The signature is chosen to be equal -2.

by which the local covariance under Lorentz transformations of tetrads was achieved in the tetrad gravity theory [6], in which the local covariance under Lorentz transformations of tetrads was primary absent unlike the orthodox Einstein theory. The A-field has as its source the so-called tetrad spin-angular momentum, which is the Noether invariant corresponding to the group of Lorentz transformations of tetrads. The contribution to the tetrad spin-angular momentum is made by the gravitational field and the A-field itself[3]. The presence of the A-field in the tetrad gravity theory just gives rise to the short-range of the gravitational potential.

In the tetrad theory of gravitation the most general Lagrangian of the gravitational field h_i^{μ} has the following form (without the cosmological term) [6, 7]

$$L_h = \frac{1}{\kappa_1} L_M + \frac{1}{\kappa_2} L_F + \frac{1}{\kappa_3} L_N, \tag{1}$$

where

$$L_M = h(\tfrac{1}{4} g^{\mu\nu} g^{\lambda\sigma} g_{\alpha\beta} \Lambda^{\alpha}_{\mu\lambda} \Lambda^{\beta}_{\nu\sigma} + \tfrac{1}{2} g^{\mu\nu} \Lambda^{\beta}_{\mu\alpha} \Lambda^{\beta}_{\nu\alpha} - g^{\mu\nu} \Lambda^{\alpha}_{\mu\alpha} \Lambda^{\beta}_{\nu\beta}),$$

$$L_F = \varepsilon^{\mu\nu\lambda\sigma} g_{\sigma\beta} \Lambda^{\alpha}_{\mu\alpha} \Lambda^{\beta}_{\nu\lambda},$$

$$L_N = h(\tfrac{1}{2} g^{\mu\nu} g^{\lambda\sigma} g_{\alpha\beta} \Lambda^{\alpha}_{\mu\lambda} \Lambda^{\beta}_{\nu\sigma} - g^{\mu\nu} \Lambda^{\alpha}_{\mu\beta} \Lambda^{\beta}_{\nu\alpha}), \qquad (h = \det h^i{}_{\mu})$$

and $\Lambda^{\alpha}_{\mu\nu} = h_i^{\alpha}(\partial_\nu h^i{}_\mu - \partial_\mu h^i{}_\nu)$, $\varepsilon^{\mu\nu\lambda\sigma}$ is the completely antisymmetric Levi-Civita symbol, κ_1, κ_2, κ_3 are constants and $\frac{\kappa_1}{2}$ is equal to the Einstein gravitational constant. In the static, spherically symmetric case and in the case of weak gravitational field the solutions of the gravitational equations corresponding to the Lagrangian (1) exactly coincide with the corresponding solutions of the orthodox Einstein theory. Therefore the tetrad gravitational theory, which is based on the Lagrangian (1), is in such agreement with the experimental data, as well as the usual theory.

The introduction of the gauge A-field is achieved by replacing in (1) [5]

$$\partial_\nu h^i{}_\mu \to \partial_\nu h^i{}_\mu - A^{ik}{}_\nu h_{k\mu},$$

that is equally

$$\Lambda^{\alpha}_{\mu\nu} \to \Lambda^{\alpha}_{\mu\nu} - h_i^{\alpha}(A^{ik}{}_\nu h_{k\mu} - A^{ik}{}_\mu h_{k\nu}). \tag{2}$$

The free Lagrangian of the A-field is

$$\mathscr{L}_A = -\frac{h}{4\alpha} F^{ik}{}_{\mu\nu} F_{ik}{}^{\mu\nu}, \tag{3}$$

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[3] Formally the fermion fields can also contribute to the tetrad spin-angular momentum, which depends essentially on the choice of the free Lagrangian of the fermion fields [5].

where

$$F^{ik}{}_{\mu\nu} = \partial_\mu A^{ik}{}_\nu - \partial_\nu A^{ik}{}_\mu + A^{il}{}_\mu A^k{}_{l\nu} - A^{il}{}_\nu A^k{}_{l\mu}$$

and in the used system of units, α is the dimensionless coupling constant of the A-field with sources. The total Lagrangian of the system of the interacting gravity field and A-field is

$$\mathscr{L}_{\text{tot}} = \mathscr{L}_h + \mathscr{L}_A, \tag{4}$$

where $\mathscr{L}_h$ is obtained from L_h by replacing (2). As a consequence, the equations of the gravitational field take the form

$$\frac{1}{\kappa_1}\mathscr{Y}_k{}^\mu + \frac{1}{\kappa_2}\mathscr{F}_k{}^\mu + \frac{1}{\kappa_3}\mathscr{N}_k{}^\mu = -\mathscr{T}_{(A)k}{}^\mu, \tag{5}$$

where

$$\mathscr{Y}_k{}^\mu = \frac{1}{h}\frac{\delta\mathscr{L}_M}{\delta h^k{}_\mu}, \qquad \mathscr{F}_k{}^\mu = \frac{1}{h}\frac{\delta\mathscr{L}_F}{\delta h^k{}_\mu}, \qquad \mathscr{N}_k{}^\mu = \frac{1}{h}\frac{\delta\mathscr{L}_N}{\delta h^k{}_\mu}$$

and $\mathscr{T}_{(A)k}{}^\mu = \dfrac{1}{h}\dfrac{\delta\mathscr{L}_A}{\delta h^k{}_\mu}$ is the energy-momentum tensor of the A-field. In accordance with (4) equations of the A-field are

$$F_{ik}{}^{\mu\nu}{}_{;\nu} = \frac{\alpha}{h}\left(J^{(h)\mu}_{ik} + J^{(A)\mu}_{ik}\right), \tag{6}$$

where the tetrad spin-angular momenta of the gravitational field and the A-field are equal

$$J^{(h)\mu}_{ik} = -\frac{\partial\mathscr{L}_h}{\partial(\partial_\mu h_l{}^\nu)}(\eta_{li}h_k{}^\nu - \eta_{lk}h_i{}^\nu), \qquad J^{(A)\mu}_{ik} = -\frac{\partial\mathscr{L}_A}{\partial(\partial_\mu A^{ik}{}_\nu)}(\delta^l_i A_k{}^m{}_\nu + \delta^m_k A_i{}^l{}_\nu),$$

respectively.

Consider the system of equations (5) and (6) in the static, spherically symmetric case. The gravitational field and the A-field are chosen in the form[4]

$$h^k{}_\mu = \left\| \begin{matrix} e^{\frac{\nu}{2}} & 0 & 0 & 0 \\ 0 & e^{\frac{\lambda}{2}}\sin\vartheta\cos\varphi & r\cos\vartheta\cos\varphi & -r\sin\vartheta\sin\varphi \\ 0 & e^{\frac{\lambda}{2}}\sin\vartheta\sin\varphi & r\cos\vartheta\sin\varphi & r\sin\vartheta\cos\varphi \\ 0 & e^{\frac{\lambda}{2}}\cos\vartheta & -r\sin\vartheta & 0 \end{matrix} \right\|, \tag{7}$$

[4] The tetrad indices are noted with a "roof" above in the individual components of the field.

$$A^{\hat{0}\hat{1}}{}_0 = ae^{\frac{v}{2}} \sin \vartheta \cos \varphi, \qquad A^{\hat{3}\hat{2}}{}_2 = cr \sin \varphi,$$

$$A^{\hat{0}\hat{2}}{}_0 = ae^{\frac{v}{2}} \sin \vartheta \sin \varphi, \qquad A^{\hat{1}\hat{3}}{}_2 = -cr \cos \varphi,$$

$$A^{\hat{0}\hat{3}}{}_0 = ae^{\frac{v}{2}} \cos \vartheta, \qquad A^{\hat{0}\hat{1}}{}_3 = dr \sin \vartheta \cos \vartheta \cos \varphi,$$

$$A^{\hat{3}\hat{2}}{}_0 = be^{\frac{v}{2}} \sin \vartheta \cos \varphi, \qquad A^{\hat{0}\hat{2}}{}_3 = dr \sin \vartheta \cos \vartheta \sin \varphi,$$

$$A^{\hat{1}\hat{3}}{}_0 = be^{\frac{v}{2}} \sin \vartheta \sin \varphi, \qquad A^{\hat{0}\hat{3}}{}_3 = -dr \sin^2 \vartheta,$$

$$A^{\hat{1}\hat{2}}{}_0 = -be^{\frac{v}{2}} \cos \vartheta, \qquad A^{\hat{3}\hat{2}}{}_3 = cr \sin \vartheta \cos \vartheta \cos \varphi,$$

$$A^{\hat{0}\hat{1}}{}_2 = dr \sin \varphi, \qquad A^{\hat{1}\hat{3}}{}_3 = cr \sin \vartheta \cos \vartheta \sin \varphi,$$

$$A^{\hat{0}\hat{2}}{}_2 = -dr \cos \varphi, \qquad A^{\hat{1}\hat{2}}{}_3 = cr \sin^2 \vartheta,$$

$$A^{\hat{0}\hat{1}}{}_1 = A^{\hat{0}\hat{2}}{}_1 = A^{\hat{0}\hat{3}}{}_1 = A^{\hat{3}\hat{2}}{}_1 = A^{\hat{1}\hat{3}}{}_1 = A^{\hat{1}\hat{2}}{}_1 = A^{\hat{0}\hat{3}}{}_2 = A^{\hat{1}\hat{2}}{}_2 = 0, \tag{8}$$

where the functions v, λ, a, b, c, d depend on r only. Then the metric tensor $g_{\alpha\beta}$ is expressed in the following way

$$g_{\alpha\beta} = \mathrm{diag}\,(e^{v}, -e^{\lambda}, -r^2, -r^2 \sin^2 \vartheta).$$

Substituting (7) and (8) in the equations (5) and (6) it is easy to obtain the system of the nonlinear differential equations for the functions v, λ, a, b, c, d. We shall not write this system because of its bulky form. Setting

$$e^{-\lambda} = 1+U(r), \qquad e^{v} = 1+V(r)$$

we assume that for the large r the functions U and V decrease as $\frac{1}{r}$, and the functions a, b, c, d decrease faster than $\frac{1}{r}$. Then the system of equations for the functions a, b, c, v, λ in the asymptotic approximation takes the form:

$$\begin{aligned} \Delta a &= 2\mu_0^2(3nb+c), \\ \Delta b &= 2\mu_0^2[(6m-1)b-na+2nc], \\ \Delta c &= \mu_0^2(a-6nb-c), \end{aligned} \tag{9}$$

$$\lambda' + \frac{e^{\lambda}-1}{r} - 2r(3nb'+c') - 4(3nb+c) = 0,$$

$$v' - \frac{e^{\lambda}-1}{r} - 2(a-6nb-c) = 0,$$

$$v'' - \frac{\lambda' v'}{2} + \frac{v'^2}{2} + \frac{v'-\lambda'}{r} - \frac{2}{r}(a-6nb-c) - 2(a'-6nb'-c') = 0, \tag{10}$$

where $\mu_0^2 = \frac{\alpha}{\kappa_1}$, $n = \frac{\kappa_1}{\kappa_2}$, $m = \frac{\kappa_1}{\kappa_3}$ and prime denotes derivation about r. (In the above approximation $d = b$).

The solution of the system (9) can be written as follows:

$$a = A\frac{e^{-\mu r}}{r}, \quad b = B\frac{e^{-\mu r}}{r}, \quad c = C\frac{e^{-\mu r}}{r}, \tag{11}$$

where A, B and C are some dimensionless constants, two of which are expressed by the third one; $\mu^2 = \kappa\mu_0^2$ and the constant κ is the root of some algebraic cubic equation and has the following values: $\kappa_1 = 1$, $\kappa_{2,3} = 6m-2\pm 6\sqrt{m^2-n^2}$. It should be noted that when the condition $m^2 > n^2$ is not fulfilled and when the roots κ_2 and κ_3 are negative, the physical meaning has only the root $\kappa_1 = 1$. Then the gauge A-field is the short-range field with the effective mass $\mu_0 = \sqrt{\frac{\alpha}{\kappa_1}}$. Appearance of mass in the A-field is connected with the presence in the $J_{ik}^{(h)\mu}$ of the terms which contain the A-field and directly define the interaction between the gravitational field and the A-field. It should be noted that according to the procedure of introduction of the A-field the consistent consideration of this field is possible in the presence of the gravitational interactions only [5].

Substituting (11) into (10) we get

$$e^{\nu} = 1+\frac{M}{r}+\frac{L}{\mu}\frac{e^{-\mu r}}{r},$$

$$e^{-\lambda} = 1+\frac{M}{r}+Ke^{-\mu r}, \tag{12}$$

where M, L and K are constants and L and K are expressed by A, B, C. From the relation (12) it is seen that the gravitational potential together with usual long-range term contains the short-range term, connected with the short-range behaviour of the A-field. If we take into account Salam's evaluation [3] of the strong gravity, and if we assume $\mu_0 \sim 1$ GeV, we shall get for the coupling constant of the A-field with sources the following estimation: $\alpha \sim 10^{-36}$. It is interesting to note that as a result of the relation $\mu_0 = \sqrt{\frac{\alpha}{\kappa_1}}$, Planck's mass m_g is connected with the proton mass m_p in the following way $\sqrt{\alpha}\, m_g \sim m_p$.

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