MUONS PRODUCED BY ATMOSPHERIC NEUTRINOS FROM THE VIEW-POINT OF THE PHOTON NEUTRINO WEAK COUPLING THEORY

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The experimental rate of muons produced deep underground by neutrinos and the atmospheric-neutrino spectrum as determined from cosmic ray muon data has been analysed on the basis of the photon-neutrino weak coupling theory. It is shown that theory of weak interactions based on the photon-neutrino weak coupling and dynamical origin of charge, which predicts that the cross section falls off with energy in the high energy region beyond the threshold energy for large mass resonances, is not in contradiction with the observed rate of muons produced by atmospheric neutrinos.

1. Introduction

In recent times several experiments have been performed to determine the rate of muons produced deep underground by atmospheric neutrinos. The experiment performed in the Kolar Gold field by Krishnaswamy et al. [1] at a vertical depth of 7.6×10^5 g cm^{-2} of standard rock gave a neutrino induced muon flux of $(2.6\pm0.7)\times10^{-13}$ cm^{-2} sec^{-1} sr^{-1} for an assumed isotropic angular distribution. Reines et al. [2] have presented the rate of muons observed in the experiment performed in East Rand Proprietory Mine (ERPM) at a depth of 8.74×10^5 g cm^{-2} of standard rock. The observed rate is found to be $(6.5\pm1.1)\times10^{-7}$ sec^{-1}, and the simplifying approximation of an isotropic neutrino distribution leads directly to a flux of $(3.7\pm0.6)\times10^{-13}$ cm^{-2} sec^{-1} sr^{-1}. The rough agreement between the two experiments is an additional evidence for the neutrino origin

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of the signals in each of them, particularly when one considers that the atmospheric muon background at the ERPM sites was about an order of magnitude below that of the Kolar experiment.

Chen et al. [3] have utilized the observed rate of neutrino induced muons to study the neutrino-nucleon cross sections at energies in excess of 10 GeV, and to investigate the production of the hypothetical vector boson ($W\pm$) of weak interactions. Assuming that only atmospheric high energy neutrinos exist, it has been possible to combine the relatively well-known incident neutrino flux with experimental results from inverse β-reactions, available from high energy accelerators, and with the measured rate of neutrino induced muon events deep underground in order to arrive at conclusions regarding 1) the growth of the inverse β-reaction cross section, and 2) the limits on the mass of the intermediate vector boson. If the set of "deep inelastic" (and large mass resonance) inverse β-reaction cross sections $\sigma(\nu p)$, $\sigma(\nu n)$, $\sigma(\bar{\nu} p)$, and $\sigma(\bar{\nu} n)$ are equal, and if one takes 1-standard deviation estimate of errors, it is seen that the saturation of the inverse β-reaction cross section is favored and this saturation would be measurable at a neutrino laboratory energy of about 50 Gev. Furthermore, if the intermediate vector boson exists, the limits on its mass are $45 > M > 2.9$ (GeV/c^2). Again, for vanishing W_3 [1] (the structure-function corresponding to the V and A interference term in the deep inelastic region), and if the variation $\sigma(\nu p)/\sigma(\nu n)$ lies in the range 1/2 to 2, the above statements on saturation of the inverse β-reaction cross section and on mass limits of $W^\pm$ are only modified.

In this paper, we want to analyse the experimental data on the rate of muon produced by atmospheric neutrinos deep underground according to the new theory of weak interactions proposed by Bandyopadhyay based on the photon-neutrino weak coupling [4], and dynamical origin of electric charge [5]. Indeed, in this theory of weak interaction, the most interesting aspects are the facts that in the low energy region and in the energy region near the threshold energy, the cross section is identical with that derived from the current-current coupling theory but in the high energy region, the cross section decreases with energy, the general behaviour being of the form $\sigma \sim \frac{1}{E_\nu}$ where E_ν is the incident laboratory neutrino energy. Evidently, there is no unitarity catastrophe in this theory. Also, as the basic interaction is the photon-neutrino weak coupling, the theory becomes renormalizable.

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[1] In the electromagnetic case, the structure functions W_1 and W_2 are defined by the relation:

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left[W_2 \cos^2 \frac{\theta}{2} + \left(2W_1 + \frac{E_\mu + E_\nu}{m_p} W_3 \right) \sin^2 \frac{\theta}{2} \right].$$

In the current-current coupling theory, the differential inelastic β-reaction cross section is as follows:

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{g^2}{2\pi} \frac{E_\mu}{m_p E_\nu} \left[W_2 \cos^2 \frac{\theta}{2} + \left(2W_1 + \frac{E_\mu E_\nu}{m_p} W_3 \right) \sin^2 \frac{\theta}{2} \right],$$

where θ is the angle between incident and outgoing lepton in the laboratory frame $\nu = E_\nu - E_\mu$.

In Sec. 2, we shall outline the methods of rate and flux calculations. In Sec. 3, we shall discuss the neutrino-nucleon cross section according to the present theory, and in Sec. 4, we shall present a comparison of the theoretical and experimental results for the observed muon rate.

2. *Methods of rate and flux calculation*

In this section, we outline the calculations of the rate of neutrino induced muons and underground horizontal neutrino induced muon flux. The rate R_i and the flux F_i are given by the following integrals over neutrino or antineutrino energy E_ν:

$$R_i = \iint I(E_\nu)J(E_\nu, \theta) \frac{dA}{d\theta} N(E_\nu)\sigma_i(E_\nu)d\theta dE_\nu = \int I(E_\nu)\varepsilon(E_\nu)N(E_\nu)\sigma_i(E_\nu)dE_\nu, \quad (1)$$

and

$$F_i = \int I(E_\nu)N(E_\nu)\sigma_i(E_\nu)dE_\nu. \quad (2)$$

Excepting $\sigma_i(E_\nu)$ which will be derived in the next section according to the present theory, all other parameters are taken from the sources mentioned in Chen et al. [3]. However, for the convenience, we here recapitulate these following Chen et al. [3].

a) $I(E_\nu)$ $J(E_\nu, \theta)$ is the incident neutrino or antineutrino flux with $I(E_\nu)$ being the horizontal flux. For energies between 0.1 and 1 GeV, the total neutrino intensity calculated by Wolfendale et al. [6] was used. For energies between 1 and 100 GeV, we used the neutrino intensities calculated by Osborne et al. [7] and Cowsik et al.[8] which are based on a model of the production and propagation of pions and kaons in the atmosphere. Cowsik et al. [8] present separately the neutrino and antineutrino intensities which when summed give a result similar to the spectrum of Osborne et al. [7]. The errors quoted by Osborne et al. [7], are based on a K/π ratio of 0.2 ± 0.2. The main contributing factor in our theory will be near the threshold energy for various resonances (including the large mass resonances).

b) $\varepsilon(E_\nu)$ is detector efficiency for observing muons or electrons produced in the neutrino or antineutrino induced interaction. It is given by the integral over angle of the product of the detector differential aperture $dA/d\theta$ with the resultant muon or electron angular distribution. In view of the large neutrino momenta involved, this angular distribution was taken to be that of the incident neutrino $J(E_\nu, \theta)$.

c) $N(E_\nu)$ is the number of target nucleons per unit volume multiplied by the range of the muon or electron in rock. The ranges for muons <100 Gev were obtained from LRL High Energy Particle Data Series [9]. It should be noted that since the calculated rate and flux are both proportional to the range of the resultant particle, the fraction of the incident neutrino energy taken by this particle becomes a sensitive parameter.

d) $\sigma_i(E_\nu)$ is the cross section per nucleon per incident neutrino or antineutrino for the process under consideration. We shall consider this cross section in details in the next section, It is pointed out that electrons contribute only about 4% to the total calculated rate, primarily because the range of the electron is so much less than the muon range in the energy interval of interest.

3. *High energy inverse β-reaction cross-section*

It has been suggested by several authors that a certain lepton-hadron relation is solicited to understand their systematics in terms of only a limited number of fundamental particles. In previous papers [5] it has been shown that if μ^-, ν_μ (and their antiparticles) are taken to be associated with baryonic matter to form all types of hadrons, it is possible to explain certain characteristic features of weak interactions. For the proton and neutron, the following configurations were taken

$$p = (\nu_\mu B^+), \qquad n = (\mu^- B^+),$$

where B^+ represents the collection of all other fundamental constituents. Again, it has been suggested that in view of the photon-neutrino weak coupling [4], we can consider the dynamical origin of charge for the electron and muon when nonlocal field theory is taken into account [10]. According to this view, we can represent the electron and muon as $(\nu_e S)$ and $(\nu_\mu S)$, respectively, where S represents the system of photons interacting weakly with ν_e and ν_μ [5]. The interactions which give charge as well as mass also add two more components, transforming a two-component spinor to a four-component one [11]. It is noted that photons are here taken as fundamental field quanta. Also, we have shown that this view of charge helps us to explain the $V-A$ form of the current-current coupling for leptonic decay processes. However, for weak scattering processes, unlike the current-current coupling theory, this model gives cross sections consistent with unitarity, and no unitarity catastrophe occurs at high energy [5].

Having considered these points, we shall now study the neutrino absorption cross section for the process

$$\nu_\mu + N \to N^* + \mu^-, \tag{3}$$

where N^* represents either a nucleon or a nucleon resonance. In view of the above configurations for the neutron and proton, and the dynamical origin of charge, the process can be described as follows (Figs 1a and 1b).

$$\nu_\mu + N = \nu_\mu + (\mu^- B^+) \to \nu_\mu + (\nu_\mu(S)B^+)$$

$$(\nu_\mu S) + (\nu_\mu B^+) = \mu^- + N' \tag{4a}$$

or

$$(\nu_\mu S) + (\nu_\mu B^+) = \mu^- + N^*, \tag{4b}$$

where B^{*+} is an excited level of B^+.

To calculate the cross section according to these diagrams, we note that we can introduce here, considering certain approximations as to the exchange of the system of photons, a photon propagator propagating between the current $\bar{\psi}_{N'}\gamma_\mu(1+\gamma_5)\psi_N$ and

$\bar{\psi}_\mu \gamma_\mu (1+\gamma_5)\psi_{\nu_\mu}$ [5]. The four-momentum K of the propagator is given by $K = p - p' = q' - q$ where p, p' are the four-momenta of the initial and final nucleon and q, q' are the four momenta of neutrino and muon, respectively. Therefore the matrix element for the process will be modified by the factor $1/(p'-p)^2$ which is to be introduced in the expression obtained in the conventional current-current coupling theory. Evidently this will give cross section $\sigma \propto \frac{1}{E_\nu}$, where E_ν is the laboratory neutrino energy [5].

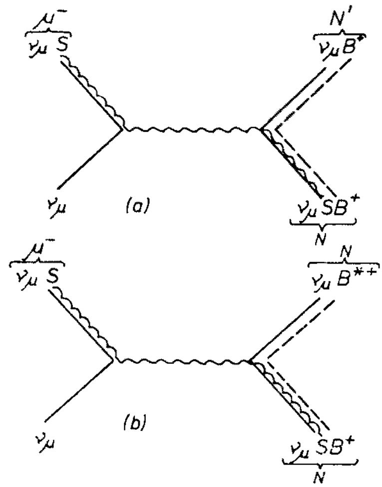

Fig. 1. Diagrams for the muon-neutrino nucleon scattering process. a) represents the process $\nu_\mu + N \to N' + \mu^-$ (N' being a different charge state of N), b) represents the process $\nu_\mu + N \to N^* + \mu^-$.

However, it should be observed that in low energy scattering, and in the energy region corresponding to the threshold energy for nucleon resonances, the cross section will be given by the conventional current-current coupling theory and the concept of nonlocality as introduced here will not be effective. In fact, in that case, the kinetic energy of the outgoing charged lepton can be taken to be negligible and the factor $K^2 = (q'-q)^2$ approaches zero. But in this limit, the system of photons responsible for the charge of the system and propagating between the hadron and the lepton concerned behave as real photons, and this just amounts to the unphysical charge non-conserving process $n \to p + \gamma$. So, the concept of nonlocality breaks down in this case, and the process can take place only in the local limit. This is true in the low energy inverse β-decay experiment $\nu_e + p \to n + e^+$ performed by Reines and Cowan, and the result is found to be in good agreement with the predictions of the current-current coupling theory.

Now, to account for the accelerator neutrino experiments by Budagov et al. [12], it is noted that there may be large mass nucleon resonances such as N^* with mass 3.939 GeV which has the production threshold near 10 GeV. neutrino laboratory energy and so, in this energy region, the cross section will rise linearly with laboratory neutrino energy consistent with the prediction of the current-current coupling theory as well as the present theory as discussed above. However, if we take that there is no large mass resonance which has reaction threshold greater than 10 GeV laboratory energy, then for energy region > 10 GeV, the present theory requires that the cross section should fall off with energy. Thus for $E_\nu \leqslant 10$ GeV, we take that the cross section is given by

$$\sigma = (0.8 \pm 0.2) \times 10^{-38} E_\nu \text{ cm}^2 \text{ GeV}^{-1} \text{ nucleon}^{-1}, \tag{5}$$

and for $E_\nu > 10$ GeV, we should have

$$\sigma = (0.8 \pm 0.2) \times 10^{-38}/E_\nu \text{ cm}^2 \text{ GeV}^{-1} \text{ nucleon}^{-1}. \tag{6}$$

The energy transfer ratio, i.e. the ratio of laboratory energies of the produced muon (average energy) to that of the incident neutrino is taken to be a constant [13]

$$K = 0.62 \pm 0.12.$$

The uncertainties given for K lies in the region $0.30 \leqslant K \leqslant 0.74$. Results from CERN indicate that $K = 0.56$.

4. *Calculations of muon rate*

Using the results of the accelerator neutrino experiments up to 10 GeV and taking that the total neutrino cross section decreases with energy $\sigma \propto \frac{1}{E_\nu}$ in the energy region $E_\nu > 10$ GeV as predicted according to the present theory of weak interaction discussed above, we here derive the muon rate and horizontal muon flux according to equations (1) and (2) and compare these results with the experimental values. Of course, in evaluating this, we utilize here certain tacit assumptions which are as follows [3].

1) The neutrino cross section per nucleon deduced from a propane bubble chamber in the CERN experiment can be applied directly to the "standard rock" (i.e. Z/A of propane (Standard rock) is 0.59 (0.50)), and this difference in composition is taken to be unimportant.

2) Antineutrino cross sections per nucleon are equal to neutrino cross sections per nucleon.

Indeed, if one assumes that the hadronic weak strangeness conserving currents are charge symmetric, i.e. they obey $|\Delta I| = 1$ rule, then we should have

$$\sigma(\nu p) = \sigma(\bar{\nu} n), \quad \sigma(\nu n) = \sigma(\bar{\nu} p), \quad E \to \infty. \tag{8}$$

However, in the deep inelastic region, for electron-proton scattering, the present experiments exclude the diffraction model as it is found that $\frac{\sigma(en)}{\sigma(ep)} < 1$. Indeed the present experiments suggest that $\frac{\sigma(en)}{\sigma(ep)} \simeq 2/3$ [14]. It is expected that according to the present model of weak interactions, which is supposed to be mediated by photons, a similar relation would hold good, i.e. $\frac{\sigma(\nu n)}{\sigma(\nu p)} \simeq 2/3$, in the processes $\nu + N \to \nu + \text{anything}$.

However, it is not known whether the ratio $\frac{\sigma(\nu n)}{\sigma(\nu p)}$ satisfies this value in the processes $\nu+N \rightarrow \mu^- +$anything. Present experimental results indicate $\sigma(\nu n) > \sigma(\nu p)$ [15]. Thus, it seems likely that our assumption (2) will not be much deviated from the experimental value yet to be determined. Again, if the ratio $\frac{\sigma(\nu n)}{\sigma(\nu p)}$ is taken to be 1.5 instead of 1, then also our conclusions will not be significantly different.

Utilizing the above assumptions (1) and (2), and the cross sections (5) and (6), we calculate the muon rate and horizontal muon flux from equations (1) and (2). Our calculated values are

$$\text{muon rate } R = 6.04 \times 10^{-7}\ \text{sec}^{-1}, \tag{9}$$

and

$$\text{horizontal muon flux} = 3.9 \times 10^{-13}\ \text{cm}^{-2}\ \text{sec}^{-1}\ \text{sr}^{-1}. \tag{10}$$

These results are to be compared with the experimental value of the muon rate [2].

$$R = (6.5 \pm 1.1) \times 10^{-7}\ \text{sec}^{-1}. \tag{11}$$

It may be pointed out here that with k-value slightly increased, the result becomes better.

In view of the existence of large errors, the predicted result can be considered to be consistent with experiments. Also we observe that the resulting horizontal muon flux as deduced here is slightly larger than that deduced from assuming an isotropic muon flux determined in ERPM and Kolar Gold field experiments. These results were $(3.7 \pm 0.6) \times 10^{-13}$ $\text{cm}^{-2}\ \text{sec}^{-1}\ \text{sr}^{-1}$ (ERPM) and $(2.6 \pm 0.7) \times 10^{-13}\ \text{cm}^{-2}\ \text{sec}^{-1}\ \text{sr}^{-1}$ (Kolar).

It is noted that if we take the ratio $\frac{\sigma(\nu n)}{\sigma(\nu p)} = 1.5$ instead of 1, then we get with assumption (1),

$$\text{muon rate } R \simeq 7.5 \times 10^{-7}\ \text{sec}^{-1}, \tag{12}$$

and

$$\text{horizontal muon flux} = 4.77 \times 10^{-13}\ \text{cm}^{-2}\ \text{sec}^{-1}\ \text{sr}^{-1}. \tag{13}$$

This result for muon rate is also found to be within the limit of the experimental value $(6.5 \pm 1.1) \times 10^{-6}\ \text{sec}^{-1}$ observed in ERPM experiment [2].

It may be pointed out here that in these calculations, we have derived the cross sections assuming that there is no large mass resonance which has reaction threshold larger than 10 GeV neutrino laboratory energy, and so, above 10 GeV the cross section begins to decrease. The sensitivity of the assumption can be tested by quoting results calculated on the basis of the larger reaction threshold. Indeed, if we take that the reaction threshold of a "large mass resonance" is nearly 15 GeV neutrino laboratory energy, and only above this energy the cross section begins to decrease, then with the same energy transfer ratio k(i. e. $k = 0.62$), we get

$$\text{muon rate } R \simeq 6.2 \times 10^{-7}\ \text{sec}^{-1} \tag{14}$$

and

$$\text{horizontal muon flux} \simeq 4.02 \times 10^{-13}\ \text{cm}^{-2}\ \text{sec}^{-1}\ \text{sr}^{-1}. \tag{15}$$

Thus, the results are found to be only slightly modified for a reasonable value of the threshold energy for large mass resonances.

5. *Discussion*

From the observed muon rate, Chen et al. [3] have argued that the naive form of the current-current coupling theory which predicts that the cross section should increase with energy (thus violating unitarity) is not at all favoured and the cross section must saturate at a certain energy. This saturation should be measurable at a neutrino laboratory energy of about 50 GeV. However, to make this saturation effective, the existence of intermediate vector bosons is generally solicited. Again, it has been pointed by these authors that if the W-boson of weak interaction exists, it can affect the muon flux underground in two ways: 1) The W-boson participates in all weak interactions and 2) the W-besons can be directly produced. Taking into account these aspects, Chen et al. [3] have observed that the calculated muon rate, for all boson masses, is larger than the observed rate, a result which would imply that the W-boson does not exist.

From our discussions in the above sections, it is evident that our theory of weak interactions is consistent with the observed muon rate. Also, we must observe here that the standard theory of weak interactions (current-current coupling) is also not in contradiction with the experimental results. In fact, we must keep in mind that both the cosmic ray intensities and the coefficient of E in the total inelastic cross section formula from CERN are too large to allow a firm conclusion in support or against a theory.

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