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LARGE MOMENTUM TRANSFER CORRECTIONS TO THE GLAUBER MODEL IN NONRELATIVISTIC EIKONAL EXPANSION

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Corrections to the Glauber model for hadron-deuteron scattering in the realm of nonrelativistic eikonal expansion are expressed through two-body scattering amplitudes. Numerical calculations including off-shell effects are carried out and the comparison with experimental data is given for elastic π^{-}d scattering at 9 GeV/c. Dependence of the relative values of corrections on parameters of input amplitudes is checked. It turns out that this type of corrections does not exceed 22% at the largest momentum transfer measured, $t \approx -2$ (GeV/c)^2.

1. Introduction

In the last few years a growing interest in calculation of different corrections to the simple Glauber model [1] for hadron-deuteron scattering has developed. The Glauber model is known to be valid for high energies and small scattering angles only. Nevertheless, it has been quite extensively and not without success applied outside the region in which it is supposed to work. The two alternative explanations of this fact are that either the values of different corrections to the Glauber model are still negligible at medium momentum transfers or that there exists some mechanism of cancellation one against the other. On the other hand, cases are known [2] where the simple Glauber model is reported to be incompatible with experimental data. In any case, only a quantitative numerical analysis may give us some insight into the real physical situation. However, although the literature on this problem is quite extensive (see e. g. [3–29]), it consists mostly of theoretical considerations and due to the mathematical complications the direct comparison with experiment is rather rare.

One popular generalisation of the Glauber model is the nonrelativistic eikonal ex-

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pansion (NEE), introduced by Sugar and Blankenbecler [9]. The corrections it involves are of a very fundamental character since they are connected with abandoning the assumptions about small scattering angles. In this paper we shall present the calculation of the hadron-deuteron elastic scattering amplitude in the second approximation in NEE. This amplitude differs from the Glauber amplitude by two terms which we shall call the eikonal and the Saxon-Schiff [9, 4] corrections, respectively. Our formalism will be based in general on that of Sugar and Blankenbecler but with a different choice of eikonal momentum. The reasons and implications of this choice are briefly discussed in Section 2.

Apart from building up a formalism for expressing the correction terms through the two-body amplitudes known from experiment, we also give the method for taking off-shell effects into account. This we are basing on a previous paper [30] by Bartnik together with the present author, where the shapes of the two-body off-shell elastic scattering amplitudes were numerically analysed. In what follows we shall refer to this paper as (I).

For the comparison with experiment we have chosen the case of π^-d elastic scattering at 9 GeV/c [2], where the authors find a disagreement between the simple Glauber model and experimental data of the order of 50% for the largest momentum transfer measured, i.e. for $t \approx -2(\text{GeV}/c)^2$. It turns out that only about half of this discrepancy is due to corrections of the type considered here. The rest must be connected with other corrections (spin, Fermi momentum, recoil etc.). This is discussed in Sections 4 and 5.

This paper is organized as follows. In Section 2 we introduce the NEE for elastic hadron-deuteron scattering and derive the formula for the amplitude which is then written in terms of two-body amplitudes and discussed in Section 3. The numerical results and comparison with experiment is presented in Section 4. In Section 5 we give some conclusions and discuss possible future improvements of this model. Finally, in Appendix A we describe the transformation of the two-body off-shell amplitudes from cms to laboratory frame and the effect this has on their shape.

2. Three-body nonrelativistic eikonal expansion

Throughout this paper we shall use the same notation as in (I) except that in the three-body case the amplitudes and Greens functions will be denoted by capital letters. The sign "′" on any side of t or T-matrices means that the corresponding bra (ket) sandwiching this matrix is off energy shell. The symbol "~" is reserved for amplitudes and Greens functions in the eikonal approximation.

Our considerations will be valid in general in any reference frame, although for simplicity we shall assume that the centre of mass motion has already been factorized out and the numerical calculation will be done in the laboratory frame. The three-body state is fully described by two relative momenta

$$\vec{q}_3 = \frac{m_1\vec{k}_2 - m_2\vec{k}_1}{m_1+m_2}, \tag{2.1}$$

$$p_3 = \frac{(m_1+m_2)\vec{k}_3 - m_3(\vec{k}_1+\vec{k}_2)}{m_1+m_2+m_3}, \tag{2.2}$$

where $\vec{k}_i$ and m_i are the momenta and masses of particles respectively, and the subscript 3 is reserved for the incoming particle[1], whereas 1 and 2 refer to the nucleons of the deuteron. Clearly, $\vec{q}_3$ is the relative momentum of the nucleons and $\vec{p}_3$ is the relative momentum of the incoming particle and the centre of mass of the two nucleons. Introducing the two-body interaction potentials V_{ij} and denoting

$$W = V_{13} + V_{23}, \tag{2.3}$$

we get the full hamiltonian in the form

$$H = \frac{p_3^2}{2\mu_3} + \frac{q_3^2}{2m_{12}} + W + V_{12}, \tag{2.4}$$

where

$$\mu_3 = \frac{(m_1+m_2)m_3}{M}, \qquad m_{12} = \frac{m_1 m_2}{m_1+m_2}, \qquad M = m_1+m_2+m_3. \tag{2.5}$$

We shall use two Greens functions: the full Greens function

$$G = (E-H+i\varepsilon)^{-1}, \tag{2.6}$$

where E is the total energy of the system, and the Greens function with only the deuteron potential included

$$G_3 = (E-H+W+i\varepsilon)^{-1}. \tag{2.7}$$

The scattering matrix is given by the Lippmann-Schwinger equation [9]

$$T = W + WG_3T = W + TG_3W. \tag{2.8}$$

This equation is a starting point for the eikonal expansion.

The eikonal approximation will consist of two steps (see (I) for comparison with two-body case):

1. Linearization of the hamiltonian in momentum $\vec{p}$ (from now on we shall drop the subscripts where it does not lead to misunderstanding)

$$\frac{p^2}{2\mu} \to \frac{p_0^2}{2\mu} + \frac{\vec{p}_0(\vec{p}-\vec{p}_0)}{\mu}; \tag{2.9}$$

here $\vec{p}_0$ is the so called eikonal momentum, its choice is not specified so far but obviously the neglected term $(\vec{p}-\vec{p}_0)^2$ must be small in the physical region. From now on we shall always assume that the z axis of our frame is parallel to $\vec{p}_0$.

2. Neglect of all terms describing the bound state

$$V_{12}, \quad \frac{q^2}{2m_{12}}, \quad B \to 0, \tag{2.10}$$

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[1] All considerations in Sections 2 and 3 are valid for any elastic scattering $xd \to xd$ but the practical calculations and comparison with experiment will be given only for $x = \pi^-$.

where B is the binding energy. The choice of the second step is somewhat arbitrary. For instance, one can linearize the hamiltonian in momentum $\vec{q}$ instead of neglecting it. Our choice has the advantage of simplifying the calculations. Note that although the Fermi momentum is neglected here, the deuteron interactions will be taken into account when averaging the scattering matrix with the deuteron wave function.

Now, denoting

$$E_0 = E - B \tag{2.11}$$

we get the eikonal Greens functions in the form

$$\tilde{G} = \left[E_0 - \frac{p_0^2}{2\mu} - \frac{\vec{p}_0(\vec{p}-\vec{p}_0)}{\mu} - W + i\varepsilon \right]^{-1}, \tag{2.12}$$

$$\tilde{G}_3 = \left[E_0 - \frac{p_0^2}{2\mu} - \frac{\vec{p}_0(\vec{p}-\vec{p}_0)}{\mu} + i\varepsilon \right]^{-1}. \tag{2.13}$$

Let us then introduce an operator N defined by the formula [9]

$$N \equiv \tilde{G}^{-1} - G^{-1} = \tilde{G}_3^{-1} - G_3^{-1} = \frac{(\vec{p}_0 - \vec{p})^2}{2\mu} + \frac{\vec{q}^{\,2}}{2m_{12}} + V_{12} - B. \tag{2.14}$$

This operator is in a sense a measure of the validity of the eikonal approximation.[2] In terms of it the Lippmann-Schwinger equation takes the two alternative forms

$$T = \tilde{T} + \tilde{T}\tilde{G}_3 N G_3 T,$$

$$T = \tilde{T} + T G_3 N \tilde{G}_3 \tilde{T}, \tag{2.15}$$

where $\tilde{T}$ is the solution of (2.8) in the eikonal approximation. The iteration of these two equations allows us to expand T in a series of growing powers of the operator N. This is the so called nonrelativistic eikonal expansion. Up to the first order in N the scattering matrix is given by the formula

$$T_1 = \tilde{T} + \tilde{T}\tilde{G}_3 N \tilde{G}_3 \tilde{T}, \tag{2.16}$$

where the neglected part is of the second order in N

$$T - T_1 = \tilde{T}\tilde{G}_3 N (G_3 + G_3 T G_3) N \tilde{G}_3 \tilde{T}. \tag{2.17}$$

These formulae for NEE were first introduced by Sugar and Blankenbecler [9] and were fairly extensively used in nonrelativistic and relativistic approaches, though usually without numerical calculations. The amplitude T_1 in Eq. (2.16) consists of two terms. The first one is the eikonal amplitude which for small scattering angles is equivalent to the Glauber amplitude. However, to get a model valid for higher momentum transfers we must get rid of some additional numerical approximations in the Glauber model (like neglecting the longitudinal momentum transfer) and this, as well as taking into account the off-shell

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[2] For any reasonable choice of $\vec{p}_0$ (linear combination of initial and final momentum) N vanishes in the forward direction and is very small for small scattering angles.

effects, introduces a difference between the Glauber and the eikonal amplitudes which we shall call the eikonal correction. The second term in Eq. (2.16) is called the Saxon--Schiff correction [4, 9].

Our formalism is very similar to that of Sugar and Blankenbecler. However (apart from working in the relative momentum representation which only gives the more symmetric form of the final formulae), there is one important difference, already mentioned in the introduction. Namely, we choose as the eikonal momentum the average of the initial and final momenta

$$\vec{p}_0 = \tfrac{1}{2}(\vec{p}_i+\vec{p}_f) \tag{2.18}$$

and eikonalize twice in the same direction. There are two reasons for this. Firstly, this choice leads to important numerical simplifications[3] and secondly, it has the advantage of diminishing the Saxon-Schiff correction, as will be seen in Section 4, in agreement with theoretical predictions of Kujawski [23].

Our aim is to express the amplitude T_1 through the two-body amplitudes known from experiment, and to calculate it explicitly for the case of π^-d scattering at laboratory momentum 9 GeV/c. This will be done in the next two sections.

3. *Hadron-deuteron scattering amplitude with eikonal and Saxon-Schiff corrections*

Eq. (2.16) is an operator equation, and to calculate the hadron-deuteron scattering amplitude we must carry out all the integrations over the intermediate states and then average both sides with the deuteron wave function. This means that the eikonal three--body amplitudes $\tilde{T}$ will be at least half off-shell. In (I) we have developed the formalism for calculating the half off-shell two-body amplitudes. They were of the following shape [30]

$$'\tilde{f}(\vec{\Delta}) = \frac{i\sigma k}{4\pi}\, e^{-\alpha\Delta^2_\perp - C\Delta^2_z - id\Delta_z}, \tag{3.1}$$

$$\tilde{f}'(\vec{\Delta}) = \frac{i\sigma k}{4\pi}\, e^{-\alpha\Delta^2_\perp - C\Delta^2_z + id\Delta_z}, \tag{3.2}$$

where $\vec{\Delta}_\perp$ and Δ_z are the momentum transfer components perpendicular and parallel to the eikonal direction, respectively. The values of the total cross section σ, half the slope of the differential cross section α and the fitting parameters C and d depend on energy. The amplitudes f differ from the Lippmann-Schwinger amplitudes by a constant factor and are normalized according to the formula

$$|f(\vec{\Delta})|^2 = \frac{d\sigma}{d\Omega}\,. \tag{3.3}$$

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[3] Note, however, that with this choice of eikonal momentum the physical momenta $\vec{p}_i$ and $\vec{p}_f$ are slightly off-shell (see discussion in (I)).

One should bear in mind that although the formulae (3.1) and (3.2) were derived in the eikonal approximation under the assumption of spherical symmetry of potential which allows one to express the potential through the eikonal on-shell amplitude uniquely, for practical calculation one has to substitute the eikonal amplitude by the experimental one. This makes the procedure of the off-shell continuation somewhat ambiguous, especially in large momentum transfer region. However, it should be a good first order approximation and the corrections are of higher order in the two-body N operator.

The formulae (3.1) and (3.2) were derived in the centre of mass frame and to use them in our case we must transform them to the laboratory frame (see Appendix A) which has the effect of changing the values of α, C, and d but does not change their functional shape.

To express the eikonal three-body amplitudes through the two-body ones we use the formula given by Karlsson and Namyslowski [25] which is the generalization of the well known Glauber formula to the full off-shell amplitudes case

$$\begin{aligned}\langle \vec{p}'\vec{q}'|'T'|\vec{p}\vec{q}\rangle &= \langle \vec{p}'\vec{q}'|'\tilde{T}_1'|\vec{p}\vec{q}\rangle + \langle \vec{p}'\vec{q}'|'\tilde{T}_2'|\vec{p}\vec{q}\rangle \\ &- \frac{i\mu}{2\pi p_0}\int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} d\tau \left[\frac{1}{\tau+\frac{1}{2}(p_z-p_{0z})-i\varepsilon} - \frac{1}{\tau-\frac{1}{2}(p_z-p_{0z})+i\varepsilon}\right] \\ &\times \left[\frac{1}{\tau'-\frac{1}{2}(p_z'-p_{0z})+i\varepsilon} - \frac{1}{\tau'+\frac{1}{2}(p_z'-p_{0z})-i\varepsilon}\right] \int d^2\vec{p}_\perp{}'' d^3\vec{q}\,'' \\ &\times \langle \vec{p}_\perp'', \tfrac{1}{2}(p_z'+p_{0z})-\tau', \vec{q}\,''|'\tilde{T}_1'|\vec{p}_\perp, \tfrac{1}{2}(p_z+p_{0z})+\tau, \vec{q}\rangle \\ &\times \langle \vec{p}_\perp', \tfrac{1}{2}(p_z'+p_{0z})+\tau', \vec{q}\,'|'\tilde{T}_2'|\vec{p}_\perp'', \tfrac{1}{2}(p_z+p_{0z})-\tau, \vec{q}\,''\rangle, \end{aligned} \tag{3.4}$$

where

$$\langle \vec{p}'\vec{q}'|'\tilde{T}_1'|\vec{p}\vec{q}\rangle = \langle \vec{p}'|'\tilde{t}_1'|\vec{p}\rangle \delta^3[\tfrac{1}{2}(\vec{p}'-\vec{p})+(\vec{q}'-\vec{q})], \tag{3.5}$$

$$\langle \vec{p}'\vec{q}'|'\tilde{T}_2'|\vec{p}\vec{q}\rangle = \langle \vec{p}'|'\tilde{t}_2'|\vec{p}\rangle \delta^3[\tfrac{1}{2}(\vec{p}'-\vec{p})-(\vec{q}'-\vec{q})]. \tag{3.6}$$

Here p_{0z} denotes the on-shell value of the z-component of momentum $\vec{p}$[4] and the amplitudes $'\tilde{t}_1'$ and $'\tilde{t}_2'$ are the off-shell scattering amplitudes for the reactions $xp \to xp$ and $xn \to xn$. Using the δ functions one may carry out six of the seven integrations in Eq. (3.4) and with the additional assumption that the full off-shell amplitudes depend like the half off-shell

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[4] Note that p_{0z} is not the z component of vector $\vec{p_0}$ (which is equal to $|\vec{p_0}|$ since $\vec{p_0}$ is parallel to z axis). It has an extra contribution coming from the fact that in general $E_0 \neq \dfrac{p_0^2}{2\mu}$:

$$p_{0z} = p_0 + \eta,$$

where

$$\eta = \frac{\mu E_0}{2p_0} - \frac{p_0}{2}$$

(see (I), Eq. (2.6)).

ones only on the difference of their arguments (which implies here that they are independent of the integration variable τ), applying the Cauchy theorem, we finally get

$$\langle\vec{p}'\vec{q}'|'\tilde{T}'|\vec{p}\vec{q}\rangle = '\tilde{t}_1'(\vec{\Delta})\delta^3(\tfrac{1}{2}\vec{\Delta}+\vec{\delta})+'\tilde{t}_2'(\vec{\Delta})\delta^3(\tfrac{1}{2}\vec{\Delta}-\vec{\delta})$$

$$+\frac{\mu}{p_0}\left[\frac{1}{\delta_z-\frac{1}{2}(p_z'+p_z)+p_{0z}+i\varepsilon}-\frac{1}{\delta_z+\frac{1}{2}(p_z'+p_z)-p_{0z}-i\varepsilon}\right]'\tilde{t}_1'(\tfrac{1}{2}\vec{\Delta}-\vec{\delta})'\tilde{t}_2'(\tfrac{1}{2}\vec{\Delta}+\vec{\delta}), \tag{3.7}$$

where

$$\vec{\Delta} = \vec{p}'-\vec{p}, \quad \vec{\delta} = \vec{q}'-\vec{q} \tag{3.8}$$

also for unphysical values of the momenta.

The formula (3.7) which may be symbolically written as

$$\tilde{T} = \tilde{T}_1+\tilde{T}_2+\tilde{T}_3 \tag{3.9}$$

must be now substituted in Eq. (2.16) which after using the locality of Greens functions and N operator in momentum space takes the form

$$T_1(\vec{\Delta}) = \int d^3\vec{q}_f d^3\vec{q}_i \psi^*(\vec{q}_f)\Bigg\{\langle\vec{p}_f\vec{q}_f|\tilde{T}|\vec{p}_i\vec{q}_i\rangle + \frac{\mu^2}{p_0^2}\int d^3\vec{p}d^3\vec{q}\langle\vec{p}_f\vec{q}_f|\tilde{T}|\vec{p}\vec{q}\rangle\frac{1}{(p_0+\eta-p_z+i\varepsilon)^2}$$
$$\times\left[\frac{(\vec{p}-\vec{p}_0)^2}{2\mu}+\frac{q^2}{2m_{12}}+V_{12}-B\right]\langle\vec{p}\vec{q}|\tilde{T}|\vec{p}_i\vec{q}_i\rangle\Bigg\}\psi(\vec{q}_i), \tag{3.10}$$

where ψ is the deuteron wave function and from the Schrödinger equation

$$(V_{12}-B)\psi(\vec{q}_i) = -\frac{q_i^2}{2m_{12}}\psi(\vec{q}_i).$$

In practical calculations we have induced D-wave in sigle scattering terms but for the sake of simplicity ψ was parametrized as simple gaussian in double scattering and correction terms. Although the input amplitudes (3.1) and (3.2) have a very simple form, some of the integrations must be done numerically. In view of the very complicated shape of the final formulae (computer input) we shall not present them here, classifying instead schematically the different terms of corrections and giving their general description.

From Eqs. (3.9) and (3.10) it is clear that the Saxon-Schiff correction consists of nine terms. The full amplitude (with normalisation (3.3)) may be schematically written as the sum

$$F(\Delta) = \tilde{F}(\vec{\Delta})+\sum_{l=1}^{3}\sum_{n=1}^{3}F_{ln}(\vec{\Delta}). \tag{3.11}$$

The first term here, $\vec{F}(\vec{\Delta})$, is the eikonal amplitude. One can see from Eq. (3.7) that it consists of two single scattering terms and a double scattering term which in the forward direction reduces to the Glauber [1] formula but in general differs by the eikonal correction. For the sake of discussion the amplitudes may be grouped as follows (we omit single scattering amplitudes since they are exactly the same in our and Glauber models)

1. F_1 — double scattering amplitude in the Glauber model. It has slope $\frac{\alpha}{2}$ in the variable $\Delta_\perp^2$ [5].
2. F_2 — double scattering amplitude with eikonal correction — same slope.
3. $F_3 = F_{11}+F_{22}$ — double scattering on the same particle. According to expectations they are very small in the forward direction and proportional to the deuteron form factor, therefore their contribution is negligible everywhere.
4. $F_4 = F_{12}+F_{21}$ — double scattering in Saxon-Schiff correction. It has slope $\alpha/2$ in $\Delta_\perp^2$ but for $t = 0$ is much smaller than F_2.
5. $F_5 = F_{13}+F_{23}+F_{31}+F_{32}$ — triple scattering and $F_6 = F_{33}$ — quadruple scattering have slopes less than $\alpha/2$ and, therefore, one could expect them to dominate for large t. However, only numerical analysis may tell us whether F_4-F_6 dominate in some t region since if such a region exists, Δ_z and η [5] effects are there comparable with $\Delta_\perp$ (see next section).

4. *Numerical results and comparison with experiment*

As mentioned in the introduction, for comparison with experiment we have chosen the case of elastic π^-d scattering at laboratory momentum 9 GeV/c [2]. The differential cross section for this reaction was measured up to $t = -2.28$ (GeV/c)^2. From the considerations of Bradamante et al. [2] it follows that the results of calculations of Alberi and Bertocchi [31] in the Glauber model based on the Barger-Phillips [32] parametrization of π^-p and π^-n amplitudes disagree with experimental data by about 50% at the highest momentum transfer measured. It should be stressed here that our aim was not to get a better fit to the data[6] but to check to what extent the eikonal and Saxon-Schiff corrections may be responsible for this discrepancy. Therefore, being not interested so much in the absolute value of the full amplitude as in the relative values of corrections, we have carried out the calculations for the three sets of parameters α, σ and ϱ (slope, total cross section and Re $f(0)$/Im $f(0)$; see (I)) describing π^-p and π^-n on-shell amplitudes. All values of parameters are given in Table I. The set A is the one quoted in Ref. [2], the set B is based on our own fit to π^-p scattering data up to $t = -0.7$ (GeV/c)^2 and the set C was taken to reproduce the Phillips amplitudes [2] but only in the double scattering region.

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[5] The variable $\Delta_\perp^2$ gives the main dependence on t (especially in the laboratory system). However, one must bear in mind that for large momentum transfers, the factors including Δ_z and η^4 may also give a substantial contribution.

[6] It is possible that with some reasonable change of two-body amplitudes one can get much better agreement between the Glauber model predictions and experimental data than the one reported in Ref. [2] (see [29]).

TABLE I

Three sets of parameters describing π^-p and π^-n elastic scattering at 9 GeV/c taken for numerical calculations of π^-d amplitude (see Sec. 4). The letters from the first column are used to differentiate between the sets of curves in Fig. 1 and 2

	α_{π^-p} (GeV/c)$^{-2}$	α_{π^-n} (GeV/c)$^{-2}$	σ_{π^-p} (mb)	σ_{π^-n} (mb)	ϱ_{π^-p}	ϱ_{π^-n}
A	4.25	4.25	26.9	25.3	−0.126	−0.230
B	3.92	3.92	26.9	25.3	−0.126	−0.230
C	3.35	3.35	24.4	24.4	0	0

Our results are presented in Figs 1–3. In Fig. 1a and b there is the π^-d differential cross section for different sets of parameters. In all three sets curve number 1 corresponds to the Glauber model, curve number 2 is Glauber with eikonal correction and curve number 3 is the full cross section with the Saxon-Schiff and eikonal corrections. In Fig. 1b the single scattering contribution is subtracted and the comparison with experiment is done only for large t, since the set C of parameters does not reproduce single scattering

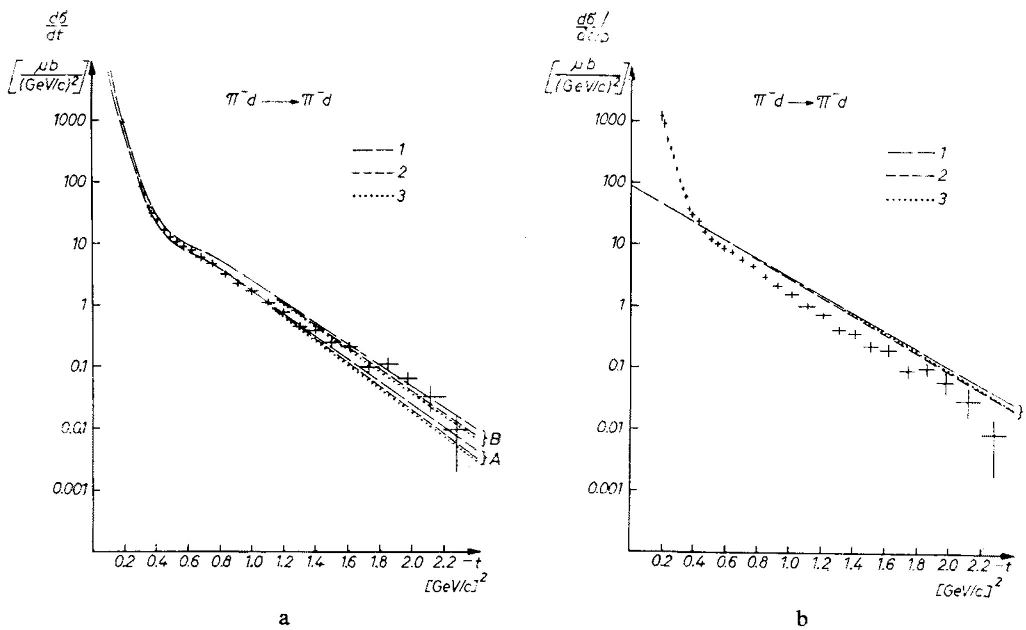

Fig. 1. The differential cross section for the elastic π^-d scattering at laboratory momentum 9 GeV/c. The effect of eikonal and Saxon-Schiff correction is shown explicitly. Letters A, B, C correspond to different sets of input parameters presented in Table I. Numbers 1, 2, 3 in each set of curves correspond to Glauber amplitude, Glauber + eikonal correction and Glauber + eikonal + Saxon–Schiff correction, respectively. In the set C in Fig. 1b only double and higher order scatterings are taken into account. The experimental data are from Ref. [2]

in a reasonable way. We used this set only to reproduce Bradamante's curves in the most interesting region[7] and to test the dependence of relative values of corrections on the parameters used.

Fig. 2 shows a comparison of the contributions from subsequent terms of the amplitude to the differential cross section. The numbers correspond to $F_1 - F_6$ from Sec. 3 and the interference is completely ignored $\left(\frac{d\sigma}{dt}\Big|_i \sim |F_i|^2\right)$. Fig. 3a and b illustrates the t

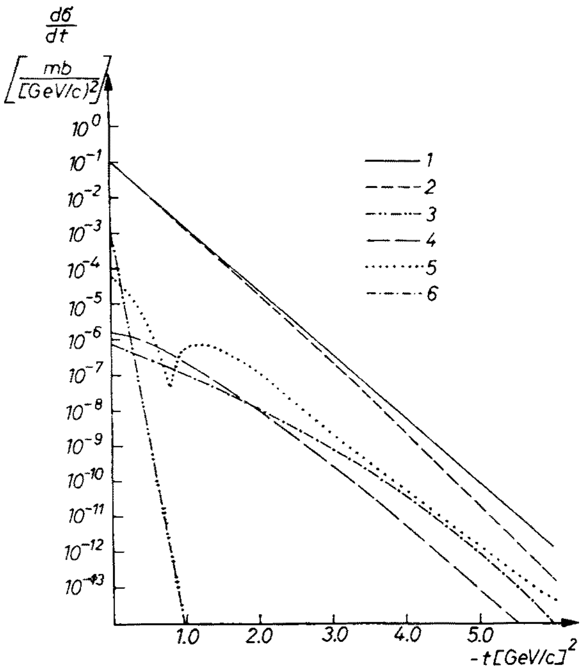

Fig. 2. The contributions from different terms F_i ($i = 1, ..., 6$) to the differential cross section as discussed in Sec. 3 $\left(\text{without interference: } \frac{d\sigma}{dt}\Big|_i \sim |F_i|^2\right)$

dependence of real and imaginary parts of F_i ($i = 1, ..., 6$). From this figure a mechanism of cancellations between different terms may be deduced.

From our numerical results the following observations can be made:

1. The effect of eikonal and Saxon-Schiff corrections is rather small in the momentum transfer region up to $-2(\text{GeV}/c)^2$ and does not exceed 22%, therefore (if the Bradamante-Phillips amplitudes are the only proper ones[6]) these corrections can not be the only ones responsible for discrepancies between the Glauber model and experiment.[8]

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[7] With our simple parametrization of amplitudes we can not reproduce the very complicated amplitudes of Barger and Phillips in all momentum transfer regions.

[8] It is obvious that the other effects neglected here like spin of nucleons, Fermi motion or deuteron recoil should also give some contribution.

2. The relative values of the corrections depend rather weakly on the shape parameters of the two-body amplitudes.

3. The corrections diminish the value of the differential cross section. If one believes Phillips' parametrization to be the best one, then the corrections go in the right direction (see Fig. 1b).

4. The eikonal correction is much bigger than the Saxon-Schiff one. This effect is probably connected with our choice of eikonal momentum [23] since then some part of the usual Saxon-Schiff correction is already contained in the eikonal approximation.

5. Even the numerical analysis does not uniquely answer the question, where the different terms start to dominate (see Figs 2, 3). The slopes of their contributions to the

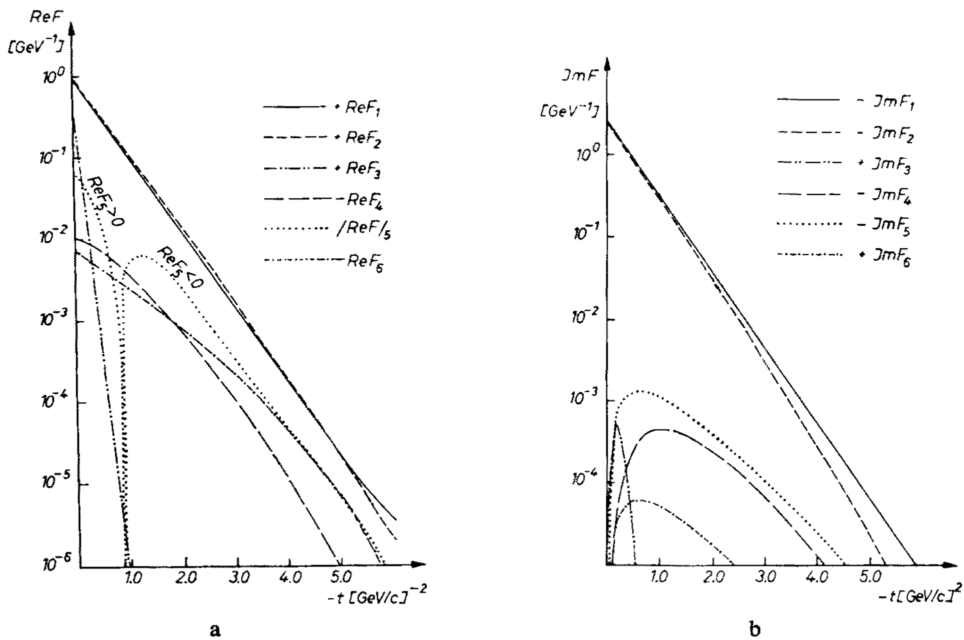

Fig. 3. The shape of a) real and b) imaginary parts of subsequent terms in the amplitude. The numbers correspond to the numbers from Sec. 3. From this figure the mechanism of cancellations may be deduced

differential cross section are changing with t, therefore qualitative analysis like the one sketched in Sec. 3 may give the wrong predictions. On the other hand our model is still too simple to be valid quantitatively in a much larger momentum transfer domain than the Glauber model (see next section).

5. Conclusions and remarks

To end our discussion the following remarks should be made:

1. It is not clear which of the two-body amplitude parametrizations is the best one. Though Phillips' parametrization works exceptionally well for π^{-}p up to the highest

energies measured [33], for π^-d one can get better results with simple gaussian parametrization (see Fig. 1a, b) or some others proposed recently (see [29] and references therein). Since there is so much freedom in the Glauber model itself, the actual value of corrections calculated in some formalism and the problem, whether they explain the existing disagreement between the Glauber model and experiment or not, can not be an argument for or against the formalism used. We can only decide whether the kind of correction considered is still negligible or should be taken into account.

2. The small value of the Saxon-Schiff correction around $t = -2(\text{GeV}/c)^2$ is an argument for using the eikonal approximation (with eikonal correction) in this region. However, the choice of eikonal momentum $\frac{1}{2}(\vec{p}_i+\vec{p}_f)$ should be recommended if the approximation is to work well in the medium momentum transfer region.

3. Parametrization of two-body amplitudes in a simple gaussian form in momentum transfer is a somewhat rough approximation and does not allow us to push the validity of our formalism into the region of much larger momentum transfers than for the Glauber model. Unfortunately, for non-gaussian functions our formalism does not work so well because of serious mathematical complications.

4. Among the different correction terms there are many cancellations as may be deduced from Fig. 3a, b. They may be the main source of the unexpectedly big success of the simple Glauber model[9].

5. The future improvement of this model should go in three directions:
   a) inclusion of the phase of the two-body amplitudes (e.g. via the Regge model);
   b) parametrization of the shape of the full off-shell amplitudes and calculation of further terms in NEE;
   c) relativistic generalization of all this formalism[10].

All these changes will be rather difficult to introduce, especially if one wants to take into account also the spin and Fermi motion effects. However, we hope that the relatively simple formalism presented in this paper may serve as a basis for such future generalizations.

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[9] Other examples are also known with cancellations among different corrective terms in generalizations of the Glauber model (see e.g. [10]). Although their mechanism is much different from the one in our case, the final effect is the same.

[10] At first sight it may seem senseless that we use a nonrelativistic formalism for the description of ultrarelativistic energies. However, it is the characteristic feature of all eikonal theories that the non-relativistic and relativistic approaches lead to the same type of final formula (e.g. Glauber model has been derived in both approaches). There are hints [18, 34] that also in the case of eikonal expression the two formalisms may be easily translated into each other with the help of some simple prescription. And all kinematics in our calculations are, of course, fully relativistic. Therefore, our formulae should find application also in relativistic formalism, though the numerical values of amplitudes may be somewhat changed by relativistic corrections.

APPENDIX A

Transformation of the off-shell amplitudes to the laboratory frame

We start from the cms amplitudes (3.1) and (3.2). The relation between laboratory and cms amplitudes is [35] (subscripts c and l refer to cms and laboratory frame respectively)

$$f_l(\Delta_\perp, \Delta_{zl}) = \frac{[\sin^2\theta_c + (\gamma_c \cos\theta_c + \lambda\gamma_{c1})^2]^{3/4}}{[\gamma_c + \lambda\gamma_{c1}\cos\theta_c]^{1/2}} f_c(\Delta_\perp, \Delta_{zc}(\Delta_{zl})), \tag{A.1}$$

where

$$\lambda = \frac{m_1}{m_2}, \tag{A.2}$$

and $\left(\text{with } \gamma_0 = \dfrac{E_{11}}{m_1}\right)$

$$\gamma_{c1} = \frac{\lambda + \gamma_0}{\sqrt{1+2\lambda\gamma_0+\lambda^2}}, \tag{A.3}$$

$$\gamma_c = \frac{1+\lambda\gamma_0}{\sqrt{1+2\lambda\gamma_0+\lambda^2}}. \tag{A.4}$$

γ_0, γ_{c1} and γ_c are the Lorentz coefficients for the incoming particle in the laboratory frame, for the incoming particle in the cms frame and for the target particle in the cms frame respectively. From the transformation of the momentum components we get for the left off-shell amplitude

$$'\tilde{f}_l(\vec{\Delta}) = \frac{ik_l\sigma}{4\pi}\beta(t)e^{-\alpha\Delta^2_\perp - C'\Delta^2_{zl} - id_1\Delta_{zl}}, \tag{A.5}$$

where

$$C' = \frac{C}{\gamma_c^2}, \quad d_1 = \frac{d}{\gamma_c}, \tag{A.6}$$

$$\beta(t) = \frac{\left\{-\frac{t}{k_c^2}\left(1+\frac{t}{4k_c^2}\right) + \left[\gamma_c\left(1+\frac{t}{2k_c^2}\right)+\lambda\gamma_{c1}\right]^2\right\}^{3/4}}{\left[\gamma_c+\lambda\gamma_{c1}\left(1+\frac{t}{2k_c^2}\right)\right]^{1/2}[\gamma_c+\lambda\gamma_{c1}]}. \tag{A.7}$$

Of course, the form (A.5) would be extremely inconvenient in further calculations. Therefore, we tried a two parameter fit according to the formula

$$\beta(t)e^{-\alpha\Delta^2_\perp - C'\Delta^2_{zl}} = e^{-\alpha_1\Delta^2_\perp - C_1\Delta^2_l}, \tag{A.8}$$

where α_1 and C_1 are the new effective values of slopes which already involve the effect of the β function.

The fit turned out to be extremely good, with both sides of Eq. (A.8) equal up to the four first digits from $t = 0$ to $t = -2(\text{GeV}/c)^2$. Moreover, it does not change the values of α very prominently (e.g. starting from $\alpha = 4.25\,(\text{GeV}/c)^{-2}$ we got $\alpha_1 = 4.31$ for π^-p at 9 GeV/c). This procedure gives us the final shape of the amplitude in exactly the same form as the initial one, namely in the form (3.1), very convenient for further calculations. Note that this procedure has nothing to do with the Glauber approximate similarity of shapes [36] of laboratory and cms amplitudes since in our case the values of parameters are changed and no assumption of small scattering angles is necessary.

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