SEVERAL WAYS OF BREAKING THE COLOUR SYMMETRY

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We discuss some cases of colour-symmetry breaking and its implications for quark binding by one-gluon-exchange forces. We pay special attention to the case, where colour--isospin and colour-hypercharge subsymmetries are preserved. Then, the ω–Φ-like mixing of colour-nonet components 0 and 8 leads to a Zweig-type approximate selection rule for decays of Φ-like meson $= \bar{q}_B q_B$ (q_B is the "blue" quark) into ordinary mesons (and photons).

1. Introduction

If one seriously tries to imagine that the recently discovered narrow resonances ψ (3105) [1, 2] and ψ(3695) [3] are somehow related to quark-antiquark vector bound states transforming under colour SU(3) as components 3 and 8 of an octet [4], one meets immediately the problem of colour symmetry breaking. This breaking should (*i*) allow for the existence of coloured mesons, at least the colour-octet components 3 and 8, (*ii*) split the pair of 3 and 8, and finally (*iii*) not spoil too much the colour selection rules for strong decays of 3 and 8 into non-colour hadrons.

Colour symmetry breaking with preserving colour-isospin subsymmetry was recently discussed by Capps [5] on the ground of baryon spectroscopy. He showed that such a broken colour symmetry can explain the experimentally observed exclusive correspondence of the SU(6) baryon multiplets **56** and **70** to even and odd parities, respectively [6]. He got a picture of the baryon as a bound state of a quark q and a diquark qq (or rather a cluster qq).

In the present paper we discuss several cases of colour-symmetry breaking and its implications for one-gluon-exchange forces and quark binding.

We pay special attention to the case where, as in Capps paper, the colour-isospin and colour-hypercharge subsymmetries are preserved. Under this assumption we discuss the possibility of the ω–Φ-like mixing of the colour-nonet components 0 and 8. Then

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strong (and electromagnetic) decays of Φ-like meson $= \bar{q}_B q_B$ (q_B is the "blue" quark) into ordinary mesons (and photons) are damped by an approximate selection rule of the Zweig type.

2. *One-gluon-exchange potential*

Let us consider the three-triplet quarks q_A ($A = 1, 2, 3$ or R, Y, B is an SU′(3)-triplet index, SU′(3) = colour SU(3)) and assume that they interact strongly with an SU′(3) nonet of vector gluons $X_{\mu r}$ ($r=0, 1, \ldots, 8$), where $X_{\mu 0}$ is an SU(3) singlet and $X_{\mu r}$ ($r = 1,\ldots,8$) form an SU′(3) octet. We will take into account the general case, where the SU′(3) symmetry is arbitrarily broken and q–X coupling has the form

$$\sum_{r=0}^{8} g_r \sqrt{\tfrac{3}{2}}\, \bar{q}\gamma_\mu \lambda'_r q X^\mu_r. \tag{1}$$

Here λ'_r ($r = 0, 1, \ldots, 8$) are Gell-Mann matrices acting on SU′(3) indices of q ($\mathrm{Tr}\,\lambda'_r\lambda'_s = 2\delta_{rs}$, $\lambda'_0 = 1'\sqrt{\tfrac{2}{3}}$). In coupling (1), the charge-conjugation invariance requires that

$$g_1 = g_2, \quad g_4 = g_5, \quad g_6 = g_7. \tag{2}$$

Notice that the gluon $X_{\mu 0}$ is here coupled to the quark number (if $g_0 \neq 0$).

In consequence of (1) the one-gluon-exchange static potential for a system of n quarks and antiquarks is given by the formula

$$V(n) = \tfrac{1}{2} \sum_{i \neq j} \sum_{r=0}^{8} v_{rij} \frac{\lambda'_{ri}}{2} \frac{\lambda'_{rj}}{2}, \tag{3}$$

where

$$\tfrac{1}{6}\, v_{rij} \equiv \frac{g_r^2}{4\pi} \frac{e^{-m_r r_{ij}}}{r_{ij}} \geqslant 0, \tag{4}$$

the matrices λ'_{ri} being equal to λ'_r or λ'^c_r when acting on SU′(3) indices of i-th quark or antiquark, respectively ($\lambda'^c_0 = -\lambda'_0$).

If we assume that the spatial average of v_{rij} does not practically depend on particle indices i, j,

$$\bar{v}_r \equiv \langle v_{rij} \rangle \geqslant 0, \tag{5}$$

then we obtain after simple calculations for $\bar{V}(n) = \langle V(n) \rangle$:

$$\bar{V}(n) = \tfrac{1}{2}\left[\bar{v}_0 \frac{(n_q - n_{\bar{q}})^2}{6} + \sum_{r=1}^{8} \bar{v}_r F'^2_r - \frac{2\bar{v}_0 + 3(\bar{v}_1 + \bar{v}_2 + \bar{v}_3 + \bar{v}_4 + \bar{v}_5) + \bar{v}_8}{12} n_R \right.$$
$$\left. - \frac{2\bar{v}_0 + 3(\bar{v}_1 + \bar{v}_2 + \bar{v}_3 + \bar{v}_6 + \bar{v}_7) + \bar{v}_8}{12} n_Y - \frac{2\bar{v}_0 + 3(\bar{v}_4 + \bar{v}_5 + \bar{v}_6 + \bar{v}_7) + 4\bar{v}_8}{12} n_B \right]. \tag{6}$$

Here F'_r ($r = 1, \ldots, 8$) are the generators of SU'(3) and n_R, n_Y, n_B denote the numbers of coloured quarks and antiquarks which, in general, are not diagonal simultaneously with total numbers of quarks and antiquarks, n_q and $n_{\bar{q}}$, as well as with their sum n, in spite of the relation

$$n = n_q + n_{\bar{q}} = n_R + n_Y + n_B. \tag{7}$$

In the average potential (6), the charge-conjugation invariance requires that

$$\bar{v}_1 = \bar{v}_2, \quad \bar{v}_4 = \bar{v}_5, \quad \bar{v}_6 = \bar{v}_7. \tag{8}$$

Masses of hadrons are approximately given by eigenvalues of the average hamiltonian

$$\overline{H}(n) = m_R n_R + m_Y n_Y + m_B n_B + \overline{V}(n), \tag{9}$$

where m_R, m_Y, m_B are masses of coloured quarks.

3. *Case of* $\bar{v}_1 = \bar{v}_2 = \ldots = \bar{v}_8$

In the special case of exact SU'(3) symmetry, i. e. when

$$\bar{v}_1 = \bar{v}_2 = \ldots = \bar{v}_8, \tag{10}$$

the average potential (6) takes the form (cf. [7, 8])

$$\overline{V}(n) = \tfrac{1}{2}\left[\bar{v}_0 \frac{(n_q - n_{\bar{q}})^2}{6} + \bar{v}_8 C - \frac{v_0 + 8\bar{v}_8}{6} n\right], \tag{11}$$

where

$$C = \sum_{r=1}^{8} F_r'^2 \tag{12}$$

is the quadratic Casimir operator of SU'(3).

In this case, we get from (9) and (11) for non-coloured mesons and baryons (i. e. for SU'(3)-singlet states of $q\bar{q}$ and qqq) the following masses

$$m_{M_0} = 2m_q - \frac{\bar{v}_0 + 8\bar{v}_8}{6}, \quad m_{B_0} = 3m_q - \frac{-\bar{v}_0 + 4\bar{v}_8}{2}, \tag{13}$$

where we assumed

$$m_R = m_Y = m_B \equiv m_q. \tag{14}$$

From (13) we obtain

$$2m_{B_0} - 3m_{M_0} = \tfrac{3}{2}\,\bar{v}_0. \tag{15}$$

Since experimentally $2m_{B_0} - 3\, m_{M_0} \simeq 0$, we have

$$\bar{v}_0 \simeq 0. \tag{16}$$

TABLE I

Average masses of simplest quark systems in the case of exact SU′(3) symmetry (and $\bar{v}_0 = 0$)

System	SU′(3) repr.	C	$\bar{H}(n)$	Interpretation
q	3	$\frac{4}{3}$	m_q	quark
$q\bar{q}$	1	0	$2m_q - \frac{4}{3}\bar{v}_8 = m_{M_0}$	non-coloured mesons
	8	3	$2m_q + \frac{1}{6}\bar{v}_8$	unbound (no coloured mesons)
qq	$\bar{3}$	$\frac{4}{3}$	$2m_q - \frac{2}{3}\bar{v}_8 = m_q + \frac{1}{2}m_{M_0}$	diquark with mass > m_q
	6	$\frac{10}{3}$	$2m_q + \frac{1}{3}\bar{v}_8$	unbound
qqq	1	0	$3m_q - 2\bar{v}_8 = m_{B_0} = \frac{3}{2}m_{M_0}$	non-coloured baryons
	8	3	$3m_q - \frac{1}{2}\bar{v}_8 = \frac{9}{4}m_q + \frac{1}{4}m_{B_0}$	coloured baryons with mass > $2m_q$
	10	6	$3m_q + \bar{v}_8$	unbound
$qq\bar{q}$	3	$\frac{4}{3}$	$3m_q - \frac{4}{3}\bar{v}_8 = m_q + m_{M_0}$	quark + meson
$q\bar{q}q\bar{q}$	1	0	$4m_q - \frac{8}{3}\bar{v}_8 = 2m_{M_0}$	two mesons (no exotic mesons)
$qqqq$	3	$\frac{4}{3}$	$4m_q - 2\bar{v}_8 = m_q + m_{B_0}$	quark + baryon
$qqq\bar{q}$	$\bar{3}$	$\frac{4}{3}$	$4m_q - 2\bar{v}_8 = m_q + m_{B_0}$	antiquark + baryon
$qqqq\bar{q}$	1	0	$5m_q - \frac{10}{3}\bar{v}_8 = m_{B_0} + m_{M_0}$	baryon + meson (no exotic baryons)

Masses of simplest quark systems, as they follow from (9) and (11), are given in Table I, where we assumed (14) (and $\bar{v}_0 = 0$). The table shows that in the case of exact SU′(3) symmetry (and $\bar{v}_0 = 0$) there are no coloured mesons (i. e. no SU′(3)-octet states of $q\bar{q}$). Also the phenomenon of saturation of interquark forces in SU′(3)-singlet states of $q\bar{q}$ and qqq is visible from the Table.

4. *Case of* $\bar{v}_1 = \bar{v}_2 = \bar{v}_4 = \bar{v}_5 = \bar{v}_6 = \bar{v}_7 = 0$, $\bar{v}_3 = \bar{v}_8$

It is worthwhile to notice that this phenomenon of saturation of interquark forces persists in another special case, where the SU′(3) symmetry is terribly broken in such a way that only couplings to gluons $X_{\mu 3}$ and $X_{\mu 8}$ are relevant and have equal strength. Then

$$\bar{v}_1 = \bar{v}_2 = \bar{v}_4 = \bar{v}_5 = \bar{v}_6 = \bar{v}_7 = 0,\ \bar{v}_3 = \bar{v}_8. \tag{17}$$

In this case the average potential (6) has the form (cf. [9, 10])

$$\bar{V}(n) = \frac{1}{2}\left[\bar{v}_0 \frac{(n_q - n_{\bar{q}})^2}{6} + v_8 \frac{1}{3} |Z|^2 - \frac{v_0 + 2\bar{v}_8}{6} n\right], \tag{18}$$

where the "complex charge"

$$Z = \sqrt{3}(-F_8' + iF_3') \tag{19}$$

TABLE II

Average masses of simplest quark systems in the case of (17) (and $\bar{v}_0 = 0$)

System	$\lvert Z\rvert^2$	$H(n)$	Interpretation
q_A	1	m_q	quark
$q_A\bar{q}_B$ $(A = B)$	0	$2m_q - \frac{1}{3}\bar{v}_8 = m_{M_0}$	"Z-neutral" mesons
$q_A\bar{q}_B$ $(A \neq B)$	3	$2m_q + \frac{1}{6}\bar{v}_8$	unbound (no "Z-charged" mesons)
q_Aq_B $(A \neq B)$	1	$2m_q - \frac{1}{6}\bar{v}_8 = m_q + \frac{1}{2}m_{M_0}$	diquark with mass $> m_q$
q_Aq_B $(A = B)$	4	$2m_q + \frac{1}{3}\bar{v}_8$	unbound
$q_Aq_Bq_C$ (A, B, C — different)	0	$3m_q - \frac{1}{2}\bar{v}_8 = m_{B_0} = \frac{3}{2}m_{M_0}$	"Z-neutral" baryons
$q_Aq_Bq_C$ (A, B, C — two equal)	3	$3m_q$	unbound (no "Z-charged" baryons)
$q_Aq_Bq_C$ (A, B, C — equal)	9	$3m_q + \bar{v}_8$	unbound (no "Z-charged" baryons)
$q_Aq_B\bar{q}_C$ $(A \neq B = C)$	1	$3m_q - \frac{1}{3}\bar{v}_8 = m_q + m_{M_0}$	quark + meson
$q_A\bar{q}_Bq_C\bar{q}_D$ $(A = B,\ C = D)$	0	$4m_q - \frac{2}{3}\bar{v}_8 = 2m_{M_0}$	two mesons (no exotic mesons)
$q_Aq_Bq_Cq_D$ ($A, B, C\ D$ — three different)	1	$4m_q - \frac{1}{2}\bar{v}_8 = m_q + m_{B_0}$	quark + baryon
$q_Aq_Bq_C\bar{q}_D$ $(A \neq B,\ C = D)$	1	$4m_q - \frac{1}{2}\bar{v}_8 = m_q + m_{B_0}$	antiquark + baryon
$q_Aq_Bq_Cq_D\bar{q}_F$ (A, B, C — different, $D = F$)	0	$5m_q - \frac{5}{6}\bar{v}_8 = m_{B_0} + m_{M_0}$	baryon + meson (no exotic baryons)

describes the diagonal set of SU′(3) generators and, therefore, may be called the "colour". Its eigenvalues in one-quark states are cubic roots of unity,

$$Z_R = \frac{-1+i\sqrt{3}}{2}, \quad Z_Y = \frac{-1-i\sqrt{3}}{2}, \quad Z_B = 1 \tag{20}$$

and satisfy the "neutralization conditions" in $q_R\bar{q}_R$, $q_Y\bar{q}_Y$, $q_B\bar{q}_B$ and $q_Rq_Yq_B$ states. It implies the phenomenon of saturation of interquark forces in "Z-neutral" states [9, 10].

Masses of simplest quark systems following from (9) and (18) are listed in Table II, where we assumed (14) (and $\bar{v}_0 = 0$). Now, the "Z-neutral" coloured mesons $= \frac{1}{\sqrt{2}}\bar{q}\lambda'_3 q$ and $= \frac{1}{\sqrt{2}}\bar{q}\lambda'_8 q$ exist and are degenerate with the non-coloured mesons $= \frac{1}{\sqrt{2}}\bar{q}\lambda'_0 q = \frac{1}{\sqrt{3}}\bar{q}q$. However, the "Z-charged" coloured mesons $= \frac{1}{\sqrt{2}}\bar{q}\lambda'_r q$ $(r = 1, 2, 4, 5, 6, 7)$ do not appear.

5. *Case of* $\bar{v}_1 = \bar{v}_2 = \bar{v}_3, \bar{v}_4 = \bar{v}_5 = \bar{v}_6 = \bar{v}_7$

Now, we turn to the discussion of the third special case, where SU′(3) symmetry is broken in such a way that the SU′(2)×U′(1) subsymmetry generated by $\vec{I}' = (F_1', F_2', F_3')$ and $Y' = \frac{2}{\sqrt{3}} F_8'$ persists as an exact symmetry. Then

$$\bar{v}_1 = \bar{v}_2 = \bar{v}_3, \bar{v}_4 = \bar{v}_5 = \bar{v}_6 = \bar{v}_7. \tag{21}$$

In this case the average potential (6) can be written as follows:

$$\bar{V}(n) = \tfrac{1}{2}\Bigg[\bar{v}_0 \frac{(n_q - n_{\bar{q}})^2}{6} + (\bar{v}_3 - \bar{v}_7)\vec{I}'^2 - (\bar{v}_7 - \bar{v}_8)\tfrac{3}{4} Y'^2 + \bar{v}_7 C - \frac{2\bar{v}_0 + 9\bar{v}_3 + 6\bar{v}_7 + \bar{v}_8}{12} n + \frac{3\bar{v}_3 - 2\bar{v}_7 - \bar{v}_8}{4} n_B\Bigg]. \tag{22}$$

The average hamiltonian (9) has the form

$$\bar{H}(n) = m_q n + \Delta m_q n_B + \bar{V}(n), \tag{23}$$

where we assumed

$$m_R = m_Y \equiv m_q, \qquad m_B \equiv m_q + \Delta m_q. \tag{24}$$

In (22) and (23) the operators n, $n_q - n_{\bar{q}}$, $\vec{I}'^2$, I_3', Y' and *either* C *or* n_B always can be taken as simultaneously diagonal. Thus the hamiltonian (23) can be treated in two different perturbation schemes, where *either*

$$\bar{H}_1(n) \equiv \left(\Delta m_q + \frac{3\bar{v}_3 - \bar{v}_7 - \bar{v}_8}{8}\right) n_B \equiv \varepsilon_1 n_B \tag{25}$$

is considered as a perturbation and

$$\bar{H}_2(n) \equiv \tfrac{1}{2}\bar{v}_7 C \equiv \varepsilon_2 C \tag{26}$$

included into the unperturbed hamiltonian *or* vice versa. The former or latter of these schemes is applicable in the case, where *either* ε_1 *or* ε_2 can be considered as a small quantity.

5.1. Case of diagonal C and small ε_1

In the first perturbation scheme, the unperturbed $q\bar{q}$ states are (here C is diagonal!):

$$M_0^{(0)} = \frac{1}{\sqrt{2}} \bar{q}\lambda_0' q = \frac{1}{\sqrt{3}} (\bar{q}_R q_R + \bar{q}_Y q_Y + \bar{q}_B q_B),$$

$$M_3^{(0)} = \frac{1}{\sqrt{2}} \bar{q}\lambda_3' q = \frac{1}{\sqrt{2}} (\bar{q}_R q_R - \bar{q}_Y q_Y),$$

$$M_8^{(0)} = \frac{1}{\sqrt{2}} \bar{q}\lambda_8' q = \frac{1}{\sqrt{6}} (\bar{q}_R q_R + \bar{q}_Y q_Y - 2\bar{q}_B q_B) \tag{27}$$

and correspond to the unperturbed masses:

$$m_{M_0}^{(0)} = 2m_q - \frac{2\bar{v}_0 + 9\bar{v}_3 + 6\bar{v}_7 + \bar{v}_8}{12},$$

$$m_{M_3}^{(0)} = 2m_q - \frac{2\bar{v}_0 - 3\bar{v}_3 + \bar{v}_8}{12},$$

$$m_{M_8}^{(0)} = 2m_q - \frac{2\bar{v}_0 + 9\bar{v}_3 - 12\bar{v}_7 + \bar{v}_8}{12}. \tag{28}$$

The binding conditions for $M_3^{(0)}$ and $M_8^{(0)}$ are

$$2\bar{v}_0 - 3\bar{v}_3 + \bar{v}_8 > 0, \; 2\bar{v}_0 + 9\bar{v}_3 - 12\bar{v}_7 + \bar{v}_8 > 0 \tag{29}$$

($M_0^{(0)}$ is bound automatically).

Since due to (27) we have

$$\langle n_B \rangle_{00} = \tfrac{2}{3}, \quad \langle n_B \rangle_{33} = 0, \quad \langle n_B \rangle_{88} = \tfrac{4}{3}, \quad \left(\langle n_B \rangle_{08} = -\frac{\sqrt{8}}{3} \right), \tag{30}$$

we obtain in the first-order perturbation calculation (with respect to the term (25)):

$$m_{M_0} = m_{M_0}^{(0)} + \tfrac{2}{3}\varepsilon_1 = 2(m_q + \tfrac{1}{3}\Delta m_q) - \frac{2\bar{v}_0 + 6\bar{v}_3 + 8\bar{v}_7 + 2\bar{v}_8}{12},$$

$$m_{M_3} = m_{M_3}^{(0)} = 2m_q - \frac{2\bar{v}_0 - 3\bar{v}_3 + \bar{v}_8}{12} = m_{M_0}^{(0)} + \bar{v}_3 + \varepsilon_2,$$

$$m_{M_8} = m_{M_8}^{(0)} + \tfrac{4}{3}\varepsilon_1 = 2(m_q + \tfrac{2}{3}\Delta m_q) - \frac{2\bar{v}_0 + 3\bar{v}_3 - 8\bar{v}_7 + 3\bar{v}_8}{12} = m_{M_0}^{(0)} + \tfrac{4}{3}\varepsilon_1 + 3\varepsilon_2. \tag{31}$$

The binding conditions for M_3 and M_8 are

$$2\bar{v}_0 - 3\bar{v}_3 + \bar{v}_8 > 0, \quad 2\bar{v}_0 + 3\bar{v}_3 - 8\bar{v}_7 + 3\bar{v}_8 > 0 \tag{32}$$

(M_0 is bound automatically).

Notice that if in particular $\bar{v}_3 = \bar{v}_7$ then

$$m_{M_3} = m_{M_0} + \tfrac{3}{2}\bar{v}_3 - \tfrac{2}{3}\left(\Delta m_q + \frac{\bar{v}_3 - \bar{v}_8}{8}\right),$$

$$m_{M_8} = m_{M_0} + \tfrac{3}{2}\bar{v}_3 + \tfrac{2}{3}\left(\Delta m_q + \frac{\bar{v}_3 - \bar{v}_8}{8}\right) \tag{33}$$

and the binding conditions (32) reduce to

$$2\bar{v}_0 - 3\bar{v}_3 + \bar{v}_8 > 0, \quad 2\bar{v}_0 - 5\bar{v}_3 + 3\bar{v}_8 > 0, \tag{34}$$

where the first condition implies already the second if $\bar{v}_3 \leqslant \bar{v}_8$ or vice versa if $\bar{v}_3 \geqslant \bar{v}_8$. Clearly, this is the case of SU′(3) symmetry broken only by $\bar{v}_8$ and Δm_q.

Notice further that if we have $\bar{v}_3 = \bar{v}_7 = \bar{v}_8$ then

$$m_{M_3} = m_{M_0}+\tfrac{3}{2}\,\bar{v}_8-\tfrac{2}{3}\,\Delta m_q,$$
$$m_{M_8} = m_{M_0}+\tfrac{3}{2}\,\bar{v}_8+\tfrac{2}{3}\,\Delta m_q \tag{35}$$

and the binding conditions for M_3 and M_8 reduce to

$$\bar{v}_0 > \bar{v}_8. \tag{36}$$

Obviously, it is the case of exact SU′(3) symmetry broken only by the quark mass difference Δm_q.

The last case, however, is excluded by the binding conditions (32) which if $\bar{v}_0 \simeq 0$ are consistent only with a badly broken SU′(3) symmetry. In fact, making use of (22) and (23) we get for noncoloured baryons (i.e. for SU′(3)-singlet states of qqq) the mass

$$m_{B_0} = 3(m_q+\tfrac{1}{3}\,\Delta m_q)-\frac{-2\bar{v}_0+3\bar{v}_3+4\bar{v}_7+\bar{v}_8}{4}. \tag{37}$$

This result is exact, i.e. independent of the perturbation scheme. From (31) and (37) we obtain the relation

$$2m_{B_0}-3m_{M_0} = \tfrac{3}{2}\,\bar{v}_0. \tag{38}$$

Assuming as an experimental result that $2m_{B_0}-3m_{M_0} \simeq 0$ we obtain from (38) and (32):

$$\bar{v}_0 \simeq 0, \quad 3\bar{v}_3 < \bar{v}_8. \tag{39}$$

Our perturbation assumption

$$\Delta m_q+\frac{3\bar{v}_3-2\bar{v}_7-\bar{v}_8}{8} \equiv \varepsilon_1 \simeq 0 \tag{40}$$

is satisfied if

$$\frac{-3\bar{v}_3+2\bar{v}_7+\bar{v}_8}{8} = \Delta m_q-\varepsilon_1 \simeq \Delta m_q. \tag{41}$$

Hence, by (39) we get

$$\tfrac{1}{2}\,\varepsilon_2 \equiv \tfrac{1}{4}\,\bar{v}_7 < \Delta m_q, \quad \Delta m_q > 0. \tag{42}$$

We conclude that in the case characterized by (21) and (40) the SU′(3) symmetry must be in fact badly broken (as seen from (39)) in order to provide the binding of $q\bar{q}$ in the states M_3 and M_8.

5.2. Case of diagonal n_B and small ε_2

In the second perturbation scheme, the unperturbed $q\bar{q}$ states are (here n_B is diagonal!):

$$M_\omega^{(0)} = \frac{1}{\sqrt{2}}\,(\bar{q}_R q_R+\bar{q}_Y q_Y),$$

$$M_3^{(0)} = \frac{1}{\sqrt{2}}(\bar{q}_R q_R - \bar{q}_Y q_Y),$$

$$M_\Phi^{(0)} = \bar{q}_B q_B \tag{43}$$

and correspond to the unperturbed masses:

$$m_{M_\omega}^{(0)} = 2m_q - \frac{2\bar{v}_0 + 9\bar{v}_3 + 6\bar{v}_7 + \bar{v}_8}{12} = m_{M_0}^{(0)},$$

$$m_{M_3}^{(0)\prime} = 2m_q - \frac{2\bar{v}_0 - 3\bar{v}_3 + 18\bar{v}_7 + \bar{v}_8}{12} = m_{M_3}^{(0)} - 3\varepsilon_2,$$

$$m_{M_\Phi}^{(0)} = 2(m_q + \Delta m_q) - \frac{2\bar{v}_0 + 12\bar{v}_7 + 4\bar{v}_8}{12} = m_{M_8}^{(0)} + 2\varepsilon_1 - 3\varepsilon_2. \tag{44}$$

The binding condition for $M_3^{(0)}$ is

$$2\bar{v}_0 - 3\bar{v}_3 + 18\bar{v}_7 + \bar{v}_8 > 0 \tag{45}$$

($M_\omega^{(0)}$ and $M_\Phi^{(0)}$ are bound automatically).

Since due to (27) and (43) we have

$$M_\omega^{(0)} = \frac{\sqrt{2}\, M_0^{(0)} + M_8^{(0)}}{\sqrt{3}}, \qquad M_\Phi^{(0)} = \frac{M_0^{(0)} - \sqrt{2}\, M_8^{(0)}}{\sqrt{3}} \tag{46}$$

and consequently

$$\langle C \rangle_{\omega\omega} = 1, \quad \langle C \rangle_{33} = 3, \quad \langle C \rangle_{\Phi\Phi} = 2, \quad (\langle C \rangle_{\omega\Phi} = -\sqrt{2}), \tag{47}$$

we obtain in the first order perturbation calculation (with respect to the term (26)):

$$m_{M_\omega} = m_{M_\omega}^{(0)} + \varepsilon_2 = 2m_q - \frac{2\bar{v}_0 + 9\bar{v}_3 + \bar{v}_8}{12},$$

$$m_{M_3} = m_{M_3}^{(0)\prime} + 3\varepsilon_2 = 2m_q - \frac{2\bar{v}_0 - 3\bar{v}_3 + \bar{v}_8}{12} = m_{M_\omega}^{(0)} + \bar{v}_3 + \varepsilon_2,$$

$$m_{M_\Phi} = m_{M_\Phi}^{(0)} + 2\varepsilon_2 = 2(m_q + \Delta m_q) - \frac{2\bar{v}_0 + 4\bar{v}_8}{12} = m_{M_\omega}^{(0)} + 2\varepsilon_1 + 2\varepsilon_2. \tag{48}$$

The binding condition for M_3 is

$$2\bar{v}_0 - 3\bar{v}_3 + \bar{v}_8 > 0 \tag{49}$$

(M_ω and M_Φ are bound automatically).

Notice that masses (48) happen to be independent of $\bar{v}_7$ (as far as $\varepsilon_2 = \frac{1}{2}\bar{v}_7$ is small). It means that, for the term in (22) proportional to $\bar{v}_7$, the diagonal matrix elements between states (43) are zero.

From (37) and (48) we obtain the following relation between masses of non-coloured baryons B_0 and the non-coloured combination

$$M_0 \simeq \frac{\sqrt{2}\,M_\omega + M_\Phi}{\sqrt{3}} \simeq \frac{1}{\sqrt{3}}(\bar{q}_R q_R + \bar{q}_Y q_Y + \bar{q}_B q_B) \tag{50}$$

of mesons M_ω and M_Φ:

$$2m_{B_0} - 3m_{M_0} \simeq \tfrac{3}{2}\,\bar{v}_0 - 4\varepsilon_2 \simeq \tfrac{3}{2}\,\bar{v}_0, \tag{51}$$

where

$$m_{M_0} \simeq \frac{2m_{M_\omega} + m_{M_\Phi}}{3} = m_{M_0} + \tfrac{4}{3}\,\varepsilon_2. \tag{52}$$

Assuming as an experimental result that $2m_{B_0} - 3m_{M_0} \simeq 0$, we have from (51) and (49)

$$\bar{v}_0 \simeq 0, \quad 3\bar{v}_3 < \bar{v}_8. \tag{53}$$

Besides, our perturbation assumption

$$\tfrac{1}{2}\,\bar{v}_7 \equiv \varepsilon_2 \simeq 0 \tag{54}$$

must be satisfied.

We conclude that in the case characterized by the relations (21) and (54) the SU′(3) symmetry must be badly broken (as seen from (53) and (54)) in order to provide the binding of $q\bar{q}$ in the states M_3 and

$$M_8 \simeq \frac{M_\omega - \sqrt{2}\,M_\Phi}{\sqrt{3}} \simeq \frac{1}{\sqrt{6}}(\bar{q}_R q_R + \bar{q}_Y q_Y - 2\bar{q}_B q_B). \tag{55}$$

The latter mix in fact with M_0 to form the more stable states M_ω and M_Φ (if $\varepsilon_2 = \frac{1}{2}\bar{v}_7$ is small).

We would like to point out that "blue" quarks q_B and antiquarks $\bar{q}_B$ do not appear in the mesons

$$M_\omega \simeq \frac{\sqrt{2}\,M_0 + M_8}{\sqrt{3}} \simeq \frac{1}{\sqrt{2}}(\bar{q}_R q_R + \bar{q}_Y q_Y), \quad M_3 = \frac{1}{\sqrt{2}}(\bar{q}_R q_R - \bar{q}_Y q_Y), \tag{56}$$

whereas "red" and "yellow" ones are absent in the meson

$$M_\Phi \simeq \frac{M_0 - \sqrt{2}\,M_8}{\sqrt{3}} \simeq \bar{q}_B q_B. \tag{57}$$

Thus, an approximate selection rule of the Zweig type [11] should work toward damping strong (and electromagnetic) decays of M_Φ into an arbitrary number of M_ω and M_3 (and photons). There is no obvious reason, however, why such a selection rule should work in decays of M_Φ into channels with at least one baryon-antibaryon pair (containing q_B and $\bar{q}_B$). (See Appendix.)

On the other hand, strong decays of M_3 into an arbitrary number of M_ω and M_Φ and non-coloured baryons B_0 are strictly forbidden by SU′(2) symmetry generated by $\vec{I}'$.

5.3. General case

The average hamiltonian $\overline{H}(n)$ given by (22) and (23) can be also exactly diagonalized by means of standard methods. For a $q\bar{q}$ system, its exact eigenvalues are

$$m_1 = m_{M_0}^{(0)}+\varepsilon_1+\tfrac{3}{2}\,\varepsilon_2-\sqrt{\varepsilon_1^2+\tfrac{9}{4}\,\varepsilon_2^2+\varepsilon_1\varepsilon_2}\,,$$
$$m_2 = m_{M_0}^{(0)}+\varepsilon_1+\tfrac{3}{2}\,\varepsilon_2+\sqrt{\varepsilon_1^2+\tfrac{9}{4}\,\varepsilon_2^2+\varepsilon_1\varepsilon_2}\,,$$
$$m_3 = m_{M_3}^{(0)}, \tag{58}$$

where ε_1 is given by (25), ε_2 by (26), and $m_{M_0}^{(0)}$ and $m_{M_3}^{(0)}$ by (28). (See Appendix.)

The state $|3\rangle$ is obviously the meson M_3 (cf. (31)), whereas the states $|1\rangle$ and $|2\rangle$ are the mesons M_0 and M_8 if $0 \leqslant \varepsilon_1/\varepsilon_2 \ll 1$ or the mesons M_ω and M_Φ if $0 \leqslant \varepsilon_2/\varepsilon_1 \ll 1$. Indeed, in the first case (cf. (31))

$$m_1 \simeq m_{M_0}^{(0)}+\tfrac{2}{3}\,\varepsilon_1 = m_{M_0},$$
$$m_2 \simeq m_{M_0}^{(0)}+\tfrac{4}{3}\,\varepsilon_1+3\varepsilon_2 = m_{M_8} \tag{59}$$

or in the second case (cf. (48))

$$m_1 \simeq m_{M_0}^{(0)}+\varepsilon_2 = m_{M_\omega},$$
$$m_2 \simeq m_{M_0}^{(0)}+2\varepsilon_1+2\varepsilon_2 = m_{M_\Phi}. \tag{60}$$

In the case of $0 \leqslant \varepsilon_1/\varepsilon_2 \ll 1$ we can write

$$|1\rangle = M_0 \cos\theta_1+M_8 \sin\theta_1,$$
$$|2\rangle = -M_0 \sin\theta_1+M_8 \cos\theta_1. \tag{61}$$

Hence

$$m_1 = m_{M_0} \cos^2\theta_1+m_{M_8} \sin^2\theta_1,$$
$$m_2 = m_{M_0} \sin^2\theta_1+m_{M_8} \cos^2\theta_1, \tag{62}$$

where

$$\sin^2\theta_1 = \frac{m_{M_0}m_2-m_{M_8}m_1}{m_{M_0}^2-m_{M_8}^2} \tag{63}$$

and (by 58))

$$m_{M_0}+m_{M_8} = m_1+m_2 = m_{M_0}^{(0)}+2\varepsilon_1+3\varepsilon_2. \tag{64}$$

Since the first-order perturbation values (31) of m_{M_0} and m_{M_8} satisfy already (64) we can put in general

$$m_{M_0} = m_{M_0}^{(0)}+\tfrac{2}{3}\,\varepsilon_1-\Delta_1,$$
$$m_{M_8} = m_{M_0}^{(0)}+\tfrac{4}{3}\,\varepsilon_1+3\varepsilon_2+\Delta_1, \tag{65}$$

where Δ_1 depends on ε_1 in such a way that $\Delta_1/\varepsilon_1 \to 0$ when $\varepsilon_1 \to 0$. The higher-order correction Δ_1 can be calculated from $\overline{H}(n)$ by the perturbation method. Using (63) and (65) we obtain

$$\sin^2\theta_1 = \tfrac{1}{2}-\tfrac{3}{2}\,\frac{\sqrt{\varepsilon_1^2+\frac{9}{4}\,\varepsilon_2^2+\varepsilon_1\varepsilon_2}}{\varepsilon_1+\frac{9}{2}\,\varepsilon_2+3\Delta_1}\,. \tag{66}$$

Obviously, for $\varepsilon_1 = 0$ we have $\sin\theta_1 = 0$ and hence $|1\rangle = M_0^{(0)}$ and $|2\rangle = M_8^{(0)}$.

Similarly, in the case of $0 \leqslant \varepsilon_2/\varepsilon_1 \ll 1$ we can put

$$|1\rangle = M_\omega\cos\theta_2+M_\Phi\sin\theta_2,$$
$$|2\rangle = -M_\omega\sin\theta_2+M_\Phi\cos\theta_2 \tag{67}$$

and

$$m_{M_\omega} = m_{M_0}^{(0)}+\varepsilon_2-\Delta_2,$$
$$m_{M_\Phi} = m_{M_0}^{(0)}+2\varepsilon_1+2\varepsilon_2+\Delta_2, \tag{68}$$

where Δ_2 depends on ε_2 in such a way that $\Delta_2/\varepsilon_2 \to 0$ when $\varepsilon_2 \to 0$. We obtain here

$$\sin^2\theta_2 = \tfrac{1}{2}-\tfrac{1}{2}\,\frac{\sqrt{\varepsilon_1^2+\frac{9}{4}\,\varepsilon_2^2+\varepsilon_1\varepsilon_2}}{\varepsilon_1+\frac{1}{2}\,\varepsilon_2+\Delta_2}\,. \tag{69}$$

Clearly, for $\varepsilon_2 = 0$ we have $\sin\theta_2 = 0$ and hence $|1\rangle = M_\omega^{(0)}$ and $|2\rangle = M_\Phi^{(0)}$.

6. *Case of* $\bar{v}_1 = \bar{v}_2,\ \bar{v}_4 = \bar{v}_5,\ \bar{v}_6 = \bar{v}_7$

In the general case of the relation (8) which is induced only by the charge-conjugation invariance, the SU′(3) symmetry is so badly broken that no SU′(2) subsymmetry survives. If this is the case, the most extended set of operators appearing in the average hamiltonian which always can be taken as simultaneously diagonal consists of n, $n_q-n_{\bar{q}}$, I_3', Y' and n_R, n_Y, n_B. If in addition the remainder of $\overline{H}(n)$ can be considered as a small perturbation, this set approximately diagonalizes $\overline{H}(n)$.

In this perturbation scheme, the unperturbed $q\bar{q}$ states are (here n_R, n_Y, n_B are diagonal!):

$$M_R^{(0)} = \bar{q}_R q_R,$$
$$M_Y^{(0)} = \bar{q}_Y q_Y,$$
$$M_B^{(0)} = \bar{q}_B q_B. \tag{70}$$

They are "red", "yellow" and "blue" analogues of Φ meson. So, three aproximate selection rules of the Zweig type should work here toward slowing down strong (and electromagnetic) decays of any of these mesons into other mesons (and photons).

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APPENDIX

The exact formulae (58) for masses of a $q\bar{q}$ system in the case of (21) can be obtained by writing

$$|1\rangle = M_0^{(0)}\cos\vartheta_1 + M_8^{(0)}\sin\vartheta_1,$$
$$|2\rangle = -M_0^{(0)}\sin\vartheta_1 + M_8^{(0)}\cos\vartheta_1 \quad \text{(A1)}$$

or

$$|1\rangle = M_\omega^{(0)}\cos\vartheta_2 + M_\Phi^{(0)}\sin\vartheta_2,$$
$$|2\rangle = -M_\omega^{(0)}\sin\vartheta_2 + M_\Phi^{(0)}\cos\vartheta_2 \quad \text{(A2)}$$

and solving the eigenvalue equation

$$\overline{H}(q,\bar{q})\,|s\rangle = m_s \quad (s = 1, 2) \quad \text{(A3)}$$

in the basis $\{M_0^{(0)}, M_8^{(0)}\}$ or $\{M_\omega^{(0)}, M_\Phi^{(0)}\}$.

Then, we get the eigenvalues $m_s\,(s = 1, 2)$ given by (58) (of course, independently of the basis) and the formulae for the expansion coefficients:

$$\sin^2\vartheta_1 = \frac{\varepsilon_1^2}{\varepsilon_1^2 + \frac{9}{8}\left(\frac{1}{3}\varepsilon_1 + \frac{3}{2}\varepsilon_2 + \sqrt{\varepsilon_1^2 + \frac{9}{4}\varepsilon_2^2 + \varepsilon_1\varepsilon_2}\right)^2} \quad \text{(A4)}$$

or

$$\sin^2\vartheta_2 = \frac{\varepsilon_2^2}{\varepsilon_2^2 + \frac{1}{2}\left(\varepsilon_1 + \frac{1}{2}\varepsilon_2 + \sqrt{\varepsilon_1^2 + \frac{9}{4}\varepsilon_2^2 + \varepsilon_1\varepsilon_2}\right)^2}, \quad \text{(A5)}$$

where $\operatorname{sgn}(\sin\vartheta_1\cos\vartheta_1) = +$ and $\operatorname{sgn}(\sin\vartheta_2\cos\vartheta_2) = +$. If $0 \leqslant \varepsilon_1/\varepsilon_2 \ll 1$ or $0 \leqslant \varepsilon_2/\varepsilon_1 \ll 1$ we have from (A4) or (A5), respectively:

$$\sin^2\vartheta_1 \simeq \frac{8}{81}\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2, \quad \sin^2\vartheta_2 \simeq \frac{1}{2}\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2. \quad \text{(A6)}$$

Thus, for $\varepsilon_1 = 0$ or $\varepsilon_2 = 0$ we get from (A1) or (A2), respectively:

$$|1\rangle = M_0^{(0)}, \quad |1\rangle = M_\omega^{(0)} = \frac{\sqrt{2}\,M_0^{(0)} + M_8^{(0)}}{\sqrt{3}},$$
$$|2\rangle = M_8^{(0)}, \quad |2\rangle = M_\Phi^{(0)} = \frac{M_0^{(0)} - \sqrt{2}\,M_8^{(0)}}{\sqrt{3}}. \quad \text{(A7)}$$

Notice the following relations between the exact masses (m_1, m_2) and the first-order perturbation masses $(m_{M_0}, m_{M_8}$ or $m_{M_\omega}, m_{M_\Phi})$:

$$m_1 = m_{M_0}\cos^2\vartheta_1 + m_{M_8}\sin^2\vartheta_1 - \frac{2\sqrt{8}}{3}\,\varepsilon_1\cos\vartheta_1\sin\vartheta_1$$
$$= m_{M_\omega}\cos^2\vartheta_2 + m_{M_\Phi}\sin^2\vartheta_2 - \sqrt{8}\,\varepsilon_2\cos\vartheta_2\sin\vartheta_2,$$

$$m_2 = m_{M_0} \sin^2 \vartheta_1 + m_{M_8} \cos^2 \vartheta_1 + \frac{2\sqrt{8}}{3} \varepsilon_1 \cos \vartheta_1 \sin \vartheta_1$$
$$= m_{M_\omega} \sin^2 \vartheta_2 + m_{M_\Phi} \cos^2 \vartheta_2 + \sqrt{8}\, \varepsilon_2 \cos \vartheta_2 \sin \vartheta_2. \tag{A8}$$

These relations differ, of course, from (62) and similar relations for m_{M_ω} and m_{M_Φ}, where masses (65) and (68) appear including higher-order corrections.

An attractive conjecture may be made that meson families M_ω, M_3 and M_Φ really exist as physical particles, where M_ω are identical with ordinary mesons, whereas M_3 and M_Φ represent new mesons. The recently discovered $\psi(3105)$ and ψ (3695) might be related to vector mesons of the types M_3 and M_Φ. Their decay would be then damped by strict SU′(2) invariance in the first case and the Zweig "blue" selection rule in the second. According to our analysis, such a possibility could be realized in the case, where

$$\bar{v}_7 \ll 2\Delta m_q + \frac{3\bar{v}_3 - 2\bar{v}_7 - \bar{v}_8}{4}, \quad 3\bar{v}_3 < 2\bar{v}_0 + \bar{v}_8 \tag{A9}$$

(and $\bar{v}_1 = \bar{v}_2 = \bar{v}_3, \bar{v}_4 = \bar{v}_5 = \bar{v}_6 = \bar{v}_7$). The conditions (A9) mean that the SU′(3) symmetry is badly broken, especially when $\bar{v}_0$ is small in comparison with $\bar{v}_3$ and/or $\bar{v}_8$. The last is certainly true if m_q (i.e. the "true" quark mass) is considerably bigger than m_{M_0} and m_{B_0}.

Other members of the meson SU′(3)-nonet (not accessible in the e^-e^+ channel, except in pairs), $M_{1\pm i2}$, $M_{4\pm i5}$ and $M_{6\pm i7}$, are like M_3 exact eigenstates of $\overline{H}(q, \bar{q})$. They correspond to masses

$$m_{1\pm i2} = m_3, \quad m_{4\pm i5} = m_{6\pm i7} = 2(m_q + \tfrac{1}{2}\Delta m_q) - \frac{\bar{v}_0 - \bar{v}_8}{6}. \tag{A10}$$

So, $M_{4\pm i5}$ and $M_{6\pm i7}$ are not bound (if $\bar{v}_0$ is small).

If the colour-symmetry breaking is caused only by the quark-mass difference Δm_q (i.e. if $\bar{v}_1 = \bar{v}_2 = \ldots = \bar{v}_8, \Delta m_q \neq 0$), then the first condition (A9) still can be satisfied though the second cannot (if $\bar{v}_0$ is small). In this case, only the states M_ω and M_Φ of the meson SU′(3)-nonet can be bound. Then, M_Φ might represent new mesons.

Obviously, all results concerning quark binding presented in this paper are valid only if the theoretical laboratory used here is relevant.

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