NEW PARTICLES AND BREAKING THE COLOUR SYMMETRY*

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In the framework of one-gluon-exchange static forces mediated by a colour octet or nonet of vector gluons, we discuss quark binding in coloured-meson states and its connection with breaking the colour symmetry. A possible identification of ψ (3.1), ψ'(3.7) and the broad bump at 4.1 GeV with some coloured bound states of quarks and antiquarks is pointed out. This identification implies the existence of a second bump in the region of 5 GeV. The general conclusion of the paper is that the colour interpretation of the new particles may be true only if the colour symmetry is badly broken (provided the considered forces are relevant).

1. The recent discoveries of narrow resonances $\psi(3105)$ [1, 2] and $\psi(3695)$ [3] as well as the broad bump in e^-e^+ total cross-section around 4.15 GeV [4] challenged theoretical physicists to search for an adequate explanation of the new phenomena. Among other possible explanations, the colour interpretation of the new particles was already discussed by many people (e.g. in SLAC and CERN informal papers). If these particles are really colour states, then there are a priori two possibilities, namely that they are either elementary (coloured) vector gluons or composite (coloured) vector states of quarks and antiquarks.

2. In a recent paper [5] we discussed the problem of quark binding in coloured states, using as a theoretical laboratory one-gluon-exchange static forces mediated by a colour nonet of vector gluons. If first we restrict ourselves to a colour octet of these gluons, $X_{\mu r}$ ($r = 1, \ldots, 8$), their Han-Nambu coupling to the colour triplet of quarks, q_A ($A = 1, 2, 3$ or R, Y, B), is of the form

$$\sum_{r=1}^{8} g_r \sqrt{\tfrac{3}{2}}\, \bar{q}\gamma_\mu \lambda'_r q X^\mu_r, \tag{1}$$

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where λ'_r denote Gell-Mann λ-matrices acting on SU′(3) indices of q (SU′(3) = colour SU(3)). The charge-conjugation invariance requires that

$$g_1 = g_2, \quad g_4 = g_5, \quad g_6 = g_7. \tag{2}$$

One of the conclusions in Ref. [5] was that, if the one-gluon-exchange static forces following from coupling (1) bind the coloured states

$$M_3 = \frac{1}{\sqrt{2}} \bar{q}\lambda'_3 q = \frac{1}{\sqrt{2}} (\bar{q}_R q_R - \bar{q}_Y q_Y), \quad M_8 = \frac{1}{\sqrt{2}} \bar{q}\lambda'_8 q = \frac{1}{\sqrt{6}} (\bar{q}_R q_R + \bar{q}_Y q_Y - 2\bar{q}_B q_B), \tag{3}$$

the colour symmetry expressed by the relations

$$g_1 = g_2 = \ldots = g_8 \tag{4}$$

must be *badly broken*. On the other hand, the non-coloured state

$$M_0 = \frac{1}{\sqrt{3}} \bar{q}\mathbf{1}' q = \frac{1}{\sqrt{3}} (\bar{q}_R q_R + \bar{q}_Y q_Y + \bar{q}_B q_B) \tag{5}$$

is certainly bound by (1) in the case of colour symmetry.

One-gluon-exchange static potential for a system of n quarks and antiquarks following from (1) is given by formula

$$V(n) = \tfrac{1}{8} \sum_{i \neq j} \sum_{r=1}^{8} v_{rij} \lambda'_{ri} \lambda'_{rj}, \tag{6}$$

where

$$v_{rij} = 6 \frac{g_r^2}{4\pi} \frac{e^{-m_r r_{ij}}}{r_{ij}}, \tag{7}$$

the matrices λ'_{ri} being equal to λ'_r or their charge conjugates λ'^C_r when acting on SU′ (3) indices of i-th quark or antiquark, respectively.

Assuming that the spatial average of v_{rij} (for $n > 2$) does not effectively depend on particle indices i, j,

$$\bar{v}_r \equiv \langle v_{rij} \rangle \geqslant 0, \tag{8}$$

and making use of (2) which gives the equalities

$$\bar{v}_1 = \bar{v}_2, \quad \bar{v}_4 = \bar{v}_5, \quad \bar{v}_6 = \bar{v}_7 \tag{9}$$

we obtain the following formula for $\bar{V}(n) = \langle V(n) \rangle$:

$$\bar{V}(n) = \tfrac{1}{2} \Big[(\bar{v}_2 - \bar{v}_7) \vec{I}'^2 - (\bar{v}_2 - \bar{v}_3) I_3'^2 - (\bar{v}_7 - \bar{v}_8) \tfrac{3}{4} Y'^2 + \bar{v}_7 C$$
$$- \frac{6\bar{v}_2 + 3\bar{v}_3 + 6\bar{v}_7 + \bar{v}_8}{12} n + \frac{2\bar{v}_2 + \bar{v}_3 - 2\bar{v}_5 - \bar{v}_8}{4} n_B + (\bar{v}_5 - \bar{v}_7)(\vec{V}'^2 - V_3'^2 - \tfrac{1}{2} n_R) \Big], \tag{10}$$

where in terms of the SU′ (3) generators F'_r ($r = 1, ..., 8$) we have

$$\vec{I}' \equiv (F'_1, F'_2, F'_3), \quad Y' \equiv \frac{2}{\sqrt{3}} F'_8, \quad \vec{V}' \equiv \left(F'_4, F'_5, \tfrac{1}{2} F'_3 + \frac{\sqrt{3}}{2} F'_8\right),$$

$$C \equiv \sum_{r=1}^{8} F'^2_r, \tag{11}$$

while n_R, n_Y, n_B denote the numbers of quarks and antiquarks of colours R, Y, B, which in general are not diagonal simulteneously with the total number of quarks and antiquarks n, in spite of the relation

$$n = n_R + n_Y + n_B. \tag{12}$$

Average masses and states of hadron SU (3) families with given SU′ (3) characteristics can be approximately calculated as eigenvalues and eigenstates of the spatial-average Hamiltonian

$$\overline{H}(n) = nm_q + n_B \Delta m_q + n_R \delta m_q + \overline{V}(n), \tag{13}$$

where

$$m_q \equiv m_Y, \quad \Delta m_q \equiv m_B - m_Y, \quad \delta m_q \equiv m_R - m_Y. \tag{14}$$

Using (10) and (13) we can write

$$\overline{H}(n) = \overline{H}_0(n) + \Delta\overline{H}(n) + \delta\overline{H}(n), \tag{15}$$

where

$$\Delta\overline{H}(n) \equiv \varepsilon_1 n_B \text{ or } \varepsilon_2 C \tag{16}$$

and

$$\delta\overline{H}(n) \equiv \frac{\bar{v}_5 - \bar{v}_7}{2} \vec{V}'^2 + \left(\delta m_q - \frac{\bar{v}_5 - \bar{v}_7}{4}\right) n_R. \tag{17}$$

Here

$$\varepsilon_1 \equiv \Delta m_q + \frac{2\bar{v}_2 + \bar{v}_3 - 2\bar{v}_5 - \bar{v}_8}{8}, \quad \varepsilon_2 \equiv \tfrac{1}{2} \bar{v}_7. \tag{18}$$

If we split $\overline{H}(n)$ in this way, we obtain the operator $\overline{H}_0(n)$ which is diagonal simultaneously with the set

$$n, \vec{I}'^2, I'_3, Y', C \quad \text{or} \quad n, \vec{I}'^2, I'_3, Y', n_B, \tag{19}$$

respectively (in general, C and n_B are not diagonal simultaneously).

3. In Ref. [5], we paid special attention to the case, where the colour symmetry SU′ (3) is broken in such a way that the subsymmetries SU′ (2) and U′ (1), generated by the colour-isospin $\vec{I}'$ and colour-hypercharge Y', are preserved. Then

$$\bar{v}_1 = \bar{v}_2 = \bar{v}_3, \quad \bar{v}_4 = \bar{v}_5 = \bar{v}_6 = \bar{v}_7, \quad \delta m_q = 0 \tag{20}$$

and hence

$$\delta\overline{H}(n) = 0. \tag{21}$$

In the case of (20), the main conclusions for the $q\bar{q}$ system drawn in Ref. [5] are the following:

(i) The lowest-lying state $M_{\rm I}$ is always bound and given by

$$M_{\rm I} = M_{\omega'} \cos\theta + M_{\Phi'} \sin\theta, \tag{22}$$

where

$$M_{\omega'} = \frac{\sqrt{2}\,M_0 + M_8}{\sqrt{3}} = \frac{1}{\sqrt{2}}(\bar{q}_{\rm R}q_{\rm R} + \bar{q}_{\rm Y}q_{\rm Y}), \qquad M_{\Phi'} = \frac{M_0 - \sqrt{2}\,M_8}{\sqrt{3}} = \bar{q}_{\rm B}q_{\rm B} \tag{23}$$

and

$$\sin^2\theta = \frac{\varepsilon_2^2}{\varepsilon_2^2 + \frac{1}{2}\left(\varepsilon_1 + \frac{1}{2}\varepsilon_2 + \sqrt{\varepsilon_1^2 + \frac{9}{4}\varepsilon_2^2 + \varepsilon_1\varepsilon_2}\right)^2} \tag{24}$$

and $\sin\theta\cos\theta \geqslant 0$. It is equal to

$$M_{\rm I} \to M_0 \quad \text{if} \quad \varepsilon_1/\varepsilon_2 \to 0, \tag{25}$$

or alternatively to

$$M_{\rm I} \to M_{\omega'} (\text{or } M_{\Phi'}) \quad \text{if} \quad \varepsilon_2/\varepsilon_1 \to 0 \quad \text{and} \quad \varepsilon_1 > 0\,(\text{or } \varepsilon_1 < 0). \tag{26}$$

The mass of $M_{\rm I}$ is

$$m_{\rm I} = m_0 + \varepsilon_1 + \tfrac{3}{2}\varepsilon_2 - \sqrt{\varepsilon_1^2 + \tfrac{9}{4}\varepsilon_2^2 + \varepsilon_1\varepsilon_2}\,, \tag{27}$$

where

$$\begin{aligned} m_0 &= 2m_q - \frac{9\bar{v}_3 + 6\bar{v}_7 + \bar{v}_8}{12} \\ &= 2(m_q + \tfrac{1}{3}\Delta m_q) - \frac{6\bar{v}_3 + 8\bar{v}_7 + 2\bar{v}_8}{12} - \tfrac{2}{3}\varepsilon_1 \\ &\simeq m_{\rm I} - \tfrac{2}{3}\varepsilon_1 \quad (\text{if } \varepsilon_1/\varepsilon_2 \simeq 0) \end{aligned} \tag{28}$$

is the mass of M_0.

(ii) The state

$$M_{\rm II} = -M_{\omega'} \sin\theta + M_{\Phi'} \cos\theta \tag{29}$$

is not always bound. It is certainly bound and equal to

$$M_{\rm II} \to M_8 \quad \text{if} \quad \varepsilon_1/\varepsilon_2 \to 0 \quad \text{and} \quad 8\bar{v}_7 < 3\bar{v}_3 + 3\bar{v}_8, \tag{30}$$

or alternatively to

$$M_{\rm II} \to M_{\Phi'} \;(\text{or } M_{\omega'}) \quad \text{if} \quad \varepsilon_2/\varepsilon_1 \to 0 \quad \text{and} \quad \varepsilon_1 > 0 \;(\text{or } \varepsilon_1 < 0). \tag{31}$$

The mass of $M_{\rm II}$ is

$$m_{\rm II} = m_0+\varepsilon_1+\tfrac{3}{2}\,\varepsilon_2+\sqrt{\varepsilon_1^2+\tfrac{9}{4}\,\varepsilon_2^2+\varepsilon_1\varepsilon_2}\,. \tag{32}$$

We shall use also the mass of M_8:

$$\begin{aligned} m_8 &= m_0+3\varepsilon_2 = 2m_q-\frac{9\bar v_3-12\bar v_7+\bar v_8}{12} \\ &= 2(m_q+\tfrac{2}{3}\,\Delta m_q)-\frac{3\bar v_3-8\bar v_7+3\bar v_8}{12}-\tfrac{4}{3}\,\varepsilon_1 \\ &\simeq m_{\rm II}-\tfrac{4}{3}\,\varepsilon_1 \ (\text{if } \varepsilon_1/\varepsilon_2\simeq 0). \end{aligned} \tag{33}$$

(*iii*) The states M_3 and

$$M_{1\pm i2} = \tfrac{1}{2}\,\bar q(\lambda'_1\pm i\lambda'_2)q \tag{34}$$

are bound if (and only if)

$$3\bar v_3 < \bar v_8. \tag{35}$$

Their masses are

$$m_3 = m_{1\pm i2} = m_0+\bar v_3+\varepsilon_2 = 2m_q+\frac{3\bar v_3-\bar v_8}{12}\,. \tag{36}$$

(*iv*) The states

$$M_{4\pm i5} = \tfrac{1}{2}\,\bar q(\lambda'_4\pm i\lambda'_5)q, \qquad M_{6\pm i7} = \tfrac{1}{2}\,\bar q(\lambda'_6\pm i\lambda'_7)q \tag{37}$$

are never bound since their masses are

$$m_{4\pm i5} = m_{6\pm i7} = 2(m_q+\tfrac{1}{2}\,\Delta m_q)+\tfrac{1}{6}\,\bar v_8, \quad \text{where} \quad \tfrac{1}{6}\,\bar v_8 \geqslant 0.$$

(*v*) In the case of exact colour symmetry,

$$\bar v_1 = \bar v_2 = \ldots = \bar v_8, \qquad \Delta m_q = 0, \tag{38}$$

only the state $M_{\rm I} = M_0$ is bound (it is a particular case of $\varepsilon_1 = 0$).

(*vi*) In the case of colour symmetry broken at most by the quark-mass difference Δm_q,

$$\bar v_1 = \bar v_2 = \ldots = \bar v_8, \tag{39}$$

only the state $M_{\rm I}\to M_0$ is bound if

$$\Delta m_q/\tfrac{1}{2}\,\bar v_7 \to 0, \tag{40}$$

or alternatively the states $M_{\rm I}\to M_{\omega'}$ (or $M_{\Phi'}$) and $M_{\rm II}\to M_{\Phi'}$ (or $M_{\omega'}$) if

$$\tfrac{1}{2}\,\bar v_7/\Delta m_q \to 0 \quad \text{and} \quad \Delta m_q>0 \ (\text{or } \Delta m_q<0) \tag{41}$$

(in this case $\varepsilon_1 = \Delta m_q$).

Notice that in the case of $\varepsilon_1 = 0$ we have $\Delta m_q = \frac{1}{8}(-3\bar{v}_3+2\bar{v}_7+\bar{v}_8)$. Hence, if the binding condition (35) is satisfied, we obtain $\Delta m_q > \frac{1}{4}\bar{v}_7 = \frac{1}{2}\varepsilon_2 \geq 0$. On the other hand, the case of $\varepsilon_2 = 0$ means that $\bar{v}_7 = 0$. Then we have $\Delta m_q = \frac{1}{8}(-3\bar{v}_3+\bar{v}_8)+\varepsilon_1$ and, if (35) is true, we get $\Delta m_q > \varepsilon_1$, where $\varepsilon_1 \geq 0$ or $\varepsilon_1 < 0$.

Obviously, all these conclusions concerning quark binding in meson states are valid only if our theoretical laboratory is relevant.

Strong decays of M_3 into ordinary hadrons are (strictly) forbidden by the SU′(2) symmetry, while for M_8 they are allowed because the SU′(3) symmetry is badly broken (if M_3 and M_8 are bound; cf. (35) and (30), respectively).

Radiative decays of M_3 are forbidden (in the first order) if the coloured part of electromagnetic current switches off for the momentum transfer q^2 approaching the real-photon mass shell:

$$\text{total current} \underset{q^2\to 0}{\to} \text{Gell}-\text{Mann current}. \tag{42}$$

Such a property exists e. g. in the case of "vector-gluon dominance" over the coloured part of the photon [6].

Strong and radiative decays of $M_{\Phi'}$ (or $M_{\omega'}$) into a number of $M_{\omega'}$ (or $M_{\Phi'}$) and photons (but *not* into baryon pairs) are damped by "colour Zweig rule" [5] since $M_{\omega'}$ and $M_{\Phi'}$ contain quarks of different colours (cf. (23)).

We can see, therefore, that in the case of (20) and (35) the $q\bar{q}$ vector bound states of the SU′(3)-type M_3 may be good candidates [5] for the narrow resonances ψ(3105) and ψ(3695). As suggested recently [7, 8, 9], these resonances might be ω and Φ states with respect to the ordinary SU(3).

In this interpretation, however, strong and radiative decays of ψ's (represented by M_3 states) into ordinary hadrons and photons would be also strictly forbidden (by the SU′(2) symmetry and the possible vector-gluon-dominance mechanism, respectively). So, the SU′(2) symmetry ought to be also broken, though slightly this time.

4. We shall assume, indeed, that in (10) and (13) the SU′(2) symmetry is broken, but only by the quark-mass difference $dm_q = m_R - m_Y$ which (by assumption) is much smaller than $m_q = m_Y$ and/or $\Delta m_q = m_B - m_Y$. Then

$$\bar{v}_1 = \bar{v}_2 = \bar{v}_3, \quad \bar{v}_4 = \bar{v}_5 = \bar{v}_6 = \bar{v}_7, \quad \delta m_q \neq 0 \tag{43}$$

and

$$\delta\bar{H}(n) = \delta m_q n_R \tag{44}$$

can be considered as a small perturbation of $\bar{H}_0(n)+\Delta\bar{H}(n)$.

In the case of (43), we know from Ref. [5] the exact eigenstates of $\bar{H}_0(n)+\Delta\bar{H}(n)$ for the $q\bar{q}$ system. They are M_I, M_{II}, M_3 and $M_{1\pm i2}$ (the latter not accessible in the e^-e^+ channel except in pairs) described in points (*i*)—(*iii*). (Recall that $M_{4\pm i5}$ and $M_{6\pm i7}$ are never bound). We shall now denote these states with the superscript zero as they will be our zeroth approximation to the eigenstates of the total average hamiltonian $\bar{H}(n)$ for $q\bar{q}$

system. We shall calculate the latter states applying the perturbative method with the operator (44) as perturbation.

In the first order of this perturbative method we obtain the following $q\bar{q}$ states:

$$M_{\mathrm{I}} = M_{\mathrm{I}}^{(0)}+\delta m_q \cos\theta\left(\frac{\sin\theta}{m_{\mathrm{II}}^{(0)}-m_{\mathrm{I}}^{(0)}}M_{\mathrm{II}}^{(0)}-\frac{1}{m_3^{(0)}-m_{\mathrm{I}}^{(0)}}M_3^{(0)}\right),$$

$$M_{\mathrm{II}} = M_{\mathrm{II}}^{(0)}+\delta m_q \sin\theta\left(\frac{\cos\theta}{m_{\mathrm{I}}^{(0)}-m_{\mathrm{II}}^{(0)}}M_{\mathrm{I}}^{(0)}+\frac{1}{m_3^{(0)}-m_{\mathrm{II}}^{(0)}}M_3^{(0)}\right),$$

$$M_3 = M_3^{(0)}+\delta m_q\left(-\frac{\cos\theta}{m_{\mathrm{I}}^{(0)}-m_3^{(0)}}M_{\mathrm{I}}^{(0)}+\frac{\sin\theta}{m_{\mathrm{II}}^{(0)}-m_3^{(0)}}M_3^{(0)}\right),$$

$$M_{1\pm i2} = M_{1\pm i2}^{(0)}. \tag{45}$$

The corresponding masses are

$$m_{\mathrm{I}} = m_{\mathrm{I}}^{(0)}+\delta m_q\cos^2\theta, \quad m_{\mathrm{II}} = m_{\mathrm{II}}^{(0)}+\delta m_q\sin^2\theta,$$

$$m_3 = m_3^{(0)}+\delta m_q, \quad m_{1\pm i2} = m_{1\pm i2}^{(0)}+\delta m_q. \tag{46}$$

Notice that in (46) there is no splitting between M_3 and $M_{1\pm i2}$ because of the fact that in (10) we have $\bar{v}_2 = \bar{v}_3$ and $\bar{v}_5 = \bar{v}_7$ due to (43).

If $\varepsilon_1/\varepsilon_2 \to 0$ then $\sin^2\theta \to 1/3$ and we get from (45)

$$M_{\mathrm{I}} \to M_0^{(0)}+\delta m_q\sqrt{\tfrac{2}{3}}\left(\frac{1/\sqrt{3}}{m_8^{(0)}-m_0^{(0)}}M_8^{(0)}-\frac{1}{m_3^{(0)}-m_0^{(0)}}M_3^{(0)}\right),$$

$$M_{\mathrm{II}} \to M_8^{(0)}+\delta m_q\frac{1}{\sqrt{3}}\left(\frac{\sqrt{2/3}}{m_0^{(0)}-m_8^{(0)}}M_0^{(0)}+\frac{1}{m_3^{(0)}-m_8^{(0)}}M_3^{(0)}\right),$$

$$M_3 \to M_3^{(0)}+\delta m_q\left(-\frac{\sqrt{2/3}}{m_0^{(0)}-m_3^{(0)}}M_0^{(0)}+\frac{1/\sqrt{3}}{m_8^{(0)}-m_3^{(0)}}M_8^{(0)}\right) \tag{47}$$

and

$$m_{\mathrm{I}} \to m_0^{(0)}+\tfrac{2}{3}\,\delta m_q, \quad m_{\mathrm{II}} \to m_8^{(0)}+\tfrac{1}{3}\,\delta m_q, \tag{48}$$

where $m_0^{(0)}$ and $m_8^{(0)}$ are given by (28) and (33), respectively.

If $\varepsilon_2/\varepsilon_1 \to 0$ and $\varepsilon_1 > 0$ (or $\varepsilon_1 < 0$) then $\sin^2\theta \to 0$ (or 1) and we obtain from (45)

$$M_{\mathrm{I}} \to M_{\omega'}^{(0)}-\frac{\delta m_q}{m_3^{(0)}-m_{\omega'}^{(0)}}M_3^{(0)} \text{ (or } M_{\Phi'}^{(0)}),$$

$$M_{\mathrm{II}} \to M_{\Phi'}^{(0)}\left(\text{or } M_{\omega'}^{(0)}-\frac{\delta m_q}{m_3^{(0)}-m_{\omega'}^{(0)}}M_3^{(0)}\right),$$

$$M_3 \to M_3^{(0)}-\frac{\delta m_q}{m_{\omega'}^{(0)}-m_3^{(0)}}M_{\omega'}^{(0)} \tag{49}$$

and

$$m_{\mathrm{I}} \to m_{\omega'}^{(0)} + \delta m_q \text{ (or } m_{\Phi'}^{(0)}), \qquad m_{\mathrm{II}} \to m_{\Phi'}^{(0)} \text{ (or } m_{\omega'}^{(0)} + \delta m_q), \tag{50}$$

where

$$m_{\omega'}^{(0)} = m_0^{(0)}, \qquad m_{\Phi'}^{(0)} = m_0^{(0)} + 2\varepsilon_1 \tag{51}$$

and $m_3^{(0)}$ is given by (36).

Making use of the perturbative formulae (45) we derive the following expression for the total width of M_3:

$$\Gamma(M_3 \to \text{all}) = \Gamma(M_3 \to \gamma \to \text{all}) + \Gamma(M_3 \to M_3^{\text{gr}} + \text{all})$$

$$+(\delta m_q)^2 \left[\frac{\cos^2\theta}{(m_{\mathrm{I}} - m_3)^2} \Gamma(M_{\mathrm{I}} \to \text{all}) + \frac{\sin^2\theta}{(m_{\mathrm{II}} - m_3)^2} \Gamma(M_{\mathrm{II}} \to \text{all}) \right]. \tag{52}$$

Here, the sum of partial widths of M_3 for the processes $M_3 \to \gamma \to$ all and $M_3 \to M_3^{\text{gr}} +$ all (where M_3^{gr} denotes the ground state in the SU′(3)-family M_3) approximates the total width of $M_3^{(0)}$ because of the absence of strong and (first-order) radiative decays of $M_3^{(0)}$ into (ordinary) hadrons and photons.

In formula (52) M_{I} can be identified with an ordinary meson of the same Lorentz- and SU(3)-characteristics as the coloured meson M_3, while M_{II} is a third meson of the same characteristics as M_3, whose existence we here predict. It should decay strongly into hadrons because the SU′ (2) symmetry does not work for its stability. Its decay rate, however, might be slightly damped by the broken SU′(3) or by the colour Zweig rule.

5. In an obvious notation indicating the SU(3)⊗SU′(3) characteristics of meson states, we impose tentatively the following identification of the new particles appearing in the e^-e^+ channel (for comparison see [7–9]):

$$\omega = M_{\omega,\mathrm{I}}, \qquad \psi(3.1) = M_{\omega,3}, \qquad \chi(4.1) = M_{\omega,\mathrm{II}},$$

$$\Phi = M_{\Phi,\mathrm{I}}, \qquad \psi(3.7) = M_{\Phi,3}, \qquad ''?'' = M_{\Phi,\mathrm{II}}, \tag{54}$$

where by $\chi(4.1)$ we denote the bump at 4.15 GeV. In particular, we have here the indices

$$\mathrm{I} = 0 \quad \text{and} \quad \mathrm{II} = 8 \quad \text{if} \quad \varepsilon_1 = 0 \tag{55}$$

and

$$\mathrm{I} = \omega' (\text{or } \Phi') \quad \text{and} \quad \mathrm{II} = \Phi' (\text{or } \omega') \quad \text{if} \quad \varepsilon_2 = 0$$

$$\text{and } \varepsilon_1 > 0 \text{ (or } \varepsilon_1 < 0). \tag{56}$$

Notice that the state $M_{\Phi,\mathrm{II}}$ should give a new bump in the total e^-e^+ cross section at the centre-of-mass energy

$$\sim m_{\chi(4.1)} + (m_{\psi(3.7)} - m_{\psi(3.1)}) \sim 5 \text{ GeV}.$$

In the case of (54) we obtain from (52)

$$\Gamma(\psi(3.1) \to \text{all}) = \Gamma(\psi(3.1) \to \gamma \to \text{all})$$

$$+(\delta m_q)^2 \left[\frac{\cos^2\theta}{(m_{\psi(3.1)} - m_\omega)^2}\,\Gamma(\omega \to \text{all}) + \frac{\sin^2\theta}{(m_{\chi(4.1)} - m_{\psi(3.1)})^2}\,\Gamma(\chi(4.1) \to \text{all})\right] \quad (57)$$

and the analogical formula for ψ (3.7) with ω and $\chi(4.1)$ replaced by Φ and "?" $= M_{\Phi,\mathrm{II}}$, respectively, and with the additional cascade-decay term $\Gamma(\psi(3.7) \to \psi(3.1)+\text{all})$. Putting in (57)

$$\Gamma(\omega \to \text{all}) = 10\ \text{MeV}, \quad \Gamma(\chi(4.1) \to \text{all}) \simeq 300\ \text{MeV},$$

$$\Gamma(\psi(3.1) \to \gamma \to \text{all}) \simeq 20\ \text{keV} \quad (58)$$

we get the estimate

$$\Gamma(\psi(3.1) \to \text{all}) \simeq \left[20 + \left(\frac{\delta m_q}{\text{MeV}}\right)^2 (2\cos^2\theta + 300\sin^2\theta) \times 10^{-3}\right] \text{keV}, \quad (59)$$

where

$$\Gamma(\psi(3.1) \to \text{all}) \simeq 100\ \text{keV}. \quad (60)$$

From (59) and (60) we obtain the following magnitude of the quark-mass difference $\delta m_q = m_{\mathrm{R}} - m_{\mathrm{Y}}$:

$$|\delta m_q| \simeq \begin{cases} 30\ \text{MeV} & \text{if} \quad \varepsilon_1 = 0, \\ 200\ \text{MeV} & \text{if} \quad \varepsilon_2 = 0 \quad \text{and} \quad \varepsilon_1 > 0, \\ 20\ \text{MeV} & \text{if} \quad \varepsilon_2 = 0 \quad \text{and} \quad \varepsilon_1 < 0. \end{cases} \quad (61)$$

In consistence with our perturbative assumption, δm_q of this magnitude can be really considered as a small correction to the quark "true" mass $m_q = m_{\mathrm{Y}}$ and also to the quark--mass difference $\Delta m_q = m_{\mathrm{B}} - m_{\mathrm{Y}}$ which, due to the strong breaking of SU′(3) into SU′(2)⊗⊗U′(1) (cf. (35)), should be of a similar order of magnitude as m_q.

A question arises whether δm_q of the magnitude (61) can be produced by the electromagnetic breaking of SU′(3). This would be an attractive conjecture indeed.

6. If we apply the mass formulae (27), (32) and (36) to masses $m_{\mathrm{I}} = m_\omega$, $m_{\mathrm{II}} = m_{\chi(4.1)}$ and $m_3 = m_{\psi(3.1)}$ and neglect the small correction δm_q, we obtain the following relations:

$$\varepsilon_1^2 + \tfrac{9}{4}\varepsilon_2^2 + \varepsilon_1\varepsilon_2 = \left(\frac{m_{\chi(4.1)} - m_\omega}{2}\right)^2,$$

$$m_0 + \varepsilon_1 + \tfrac{3}{2}\varepsilon_2 = \frac{m_{\chi(4.1)} + m_\omega}{2},$$

$$m_0 + \bar{v}_3 + \varepsilon_2 = m_{\psi(3.1)}. \quad (62)$$

From this system of equations we get ε_1, ε_2 and m_0 as functions of $\bar{v}_3$:

$$\varepsilon_1 = -\frac{2m_{\psi(3.1)}-m_\omega-m_{\chi(4.1)}}{2}+\bar{v}_3-\tfrac{1}{2}\,\varepsilon_2(\bar{v}_3),$$

$$\varepsilon_2 = \varepsilon_2(\bar{v}_3) \equiv \frac{1}{\sqrt{2}}\sqrt{(m_{\psi(3.1)}-m_\omega-\bar{v}_3)\,(m_{\chi(4.1)}-m_{\psi(3.1)}+\bar{v}_3)}\,,$$

$$m_0 = m_{\psi(3.1)}-\bar{v}_3-\varepsilon_2(\bar{v}_3), \tag{63}$$

where

$$0 \leqslant \bar{v}_3 \leqslant m_{\psi(3.1)}-m_\omega. \tag{64}$$

Hence

$$0 \leqslant \varepsilon_2 \leqslant \frac{m_{\chi(4.1)}-m_\omega}{2\sqrt{2}} = 1.19\ \text{GeV}, \tag{65}$$

where the lower and upper bound for ε_2 corresponds to $\bar{v}_3 = m_{\psi(3.1)}-m_\omega$ and $\bar{v}_3 = \tfrac{1}{2}(2m_{\psi(3.1)}-m_\omega-m_{\chi(4.1)})$, respectively.

The value of $\bar{v}_3$ might be tentatively determined by the additional conjecture that SU′(3) symmetry is broken (by strong interactions) at most in such a way that not only the component M_3 but also the components M_0 and M_8 of the meson colour nonet survive as eigenstates if $\overline{H}(n)$ if we neglect the small correction $\delta\overline{H}(n)$. This conjecture implies that

$$\Delta m_q+\frac{3\bar{v}_3-2\bar{v}_7-\bar{v}_8}{8} \equiv \varepsilon_1 = 0, \tag{66}$$

and

$$\tfrac{1}{2}\,\bar{v}_7 \equiv \varepsilon_2 > 0, \tag{67}$$

where, of course, the binding conditions (30) and (35) for M_8 and M_3 must be satisfied. In this case, making use of (62) (or (63)) we determine all quantities ε_1, ε_2, m_0 and $\bar{v}_3$:

$$\varepsilon_2 = \frac{m_{\chi(4.1)}-m_\omega}{3} = 1.12\ \text{GeV}, \qquad m_0 = m_\omega = 0.78\ \text{GeV},$$

$$\bar{v}_3 = \frac{3m_{\psi(3.1)}-2m_\omega-m_{\chi(4.1)}}{3} = 1.20\ \text{GeV}. \tag{68}$$

Hence, using (66) and (67) we obtain

$$\bar{v}_7 = \frac{2(m_{\chi(4.1)}-m_\omega)}{3} = 2.24\ \text{GeV},$$

$$\bar{v}_8 = \frac{24\Delta m_q+9m_{\psi(3.1)}-2m_\omega-7m_{\chi(4.1)}}{3} = 8\Delta m_q-0.88\ \text{GeV}. \tag{69}$$

Then, from the binding condition (30)

$$\bar{v}_8 > \tfrac{8}{3}\,\bar{v}_7 - \bar{v}_3 = 4.78\ \text{GeV}, \qquad \Delta m_q = \tfrac{1}{8}\,\bar{v}_8 + 0.11\ \text{GeV} > 0.71\ \text{GeV}, \tag{70}$$

while from the binding condition (35)

$$\bar{v}_8 > 3\bar{v}_3 = 3.60\ \text{GeV}, \qquad \Delta m_q = \tfrac{1}{8}\,\bar{v}_8 + 0.11\ \text{GeV} > 0.56\ \text{GeV}. \tag{71}$$

Thus, the condition (30) is here more restrictive than (35). Notice that using (61) we confirm that

$$\Delta m_q > 0.71\ \text{GeV} \gg |\delta m_q| \simeq 0.03\ \text{GeV}, \tag{72}$$

in consistency with our perturbative assumption. We can see from (68), (69) and (70) that $\bar{v}_3 < \bar{v}_7 < \bar{v}_8$.

Having determined the values of $\bar{v}_3$, $\bar{v}_7$ and m_0 as well as the lower bound of $\bar{v}_8$ we can also evaluate from (28) the lower bound of the quark mass m_q:

$$m_q = \tfrac{1}{2}\,m_0 + \frac{9\bar{v}_3 + 6\bar{v}_7 + \bar{v}_8}{24} > 1.60\ \text{GeV}. \tag{73}$$

Hence, for the average quark mass in the non-coloured baryon state

$$B_0 = \varepsilon_{ABC} q_A q_B q_C \tag{74}$$

we have

$$m_q + \tfrac{1}{3}\,\Delta m_q > 1.84\ \text{GeV}. \tag{75}$$

The mass of B_0, which is an exact eigenstate of $\overline{H}(n)$ for qqq system, is given in the case of (20) by

$$m_{B_0} = \tfrac{3}{2}\,m_0 + \varepsilon_1. \tag{76}$$

Hence, if we use (68) we get

$$m_{B_0} = 1.17\ \text{GeV} \tag{77}$$

which is a reasonable value for the average mass of ordinary baryons.

In conclusion, we can say that the possible identification (54) of the new particles is quite consistent with our theoretical laboratory provided by one-gluon-exchange static forces mediated between quarks and antiquarks by a colour octet of vector gluons. The colour symmetry, however, is then forced to be badly broken.

7. Now, we should like to stress that the above discussion of quark binding and colour symmetry breaking changes much, if the forces between quarks and antiquarks are mediated by a colour nonet of vector gluons [5]. Then, coupling (1) contains the additional term

$$g_0 \sqrt{\tfrac{3}{2}}\,\bar{q}\gamma_\mu \lambda'_0 q X_0^\mu, \tag{78}$$

where $\lambda_0' = \sqrt{\frac{2}{3}}\,\mathbf{1}'$. In consequence, the term

$$\tfrac{1}{2}\left[\bar{v}_0\,\frac{(n_q-n_{\bar{q}})^2}{6} - \frac{2\bar{v}_0}{12}\,n\right] \tag{79}$$

has to be added to the average potential $\bar{V}(n)$ given by (10), where n_q and $n_{\bar{q}}$ are numbers of quarks and antiquarks, respectively, and we have

$$n = n_q+n_{\bar{q}} = n_{\rm R}+n_{\rm Y}+n_{\rm B}. \tag{80}$$

If we consider the case of (20), formulae (27) and (32) for $m_{\rm I}$ and $m_{\rm II}$ do not change, while formulae (28), (33), (36) and (76) should be replaced by

$$m_0 = 2m_q - \frac{2\bar{v}_0+9\bar{v}_3+6\bar{v}_7+\bar{v}_8}{12}$$

$$= 2(m_q+\tfrac{1}{3}\,\Delta m_q) - \frac{2\bar{v}_0+6\bar{v}_3+8\bar{v}_7+2\bar{v}_8}{12} - \tfrac{2}{3}\,\varepsilon_1$$

$$\simeq m_{\rm I}-\tfrac{2}{3}\,\varepsilon_1 \text{ (if } \varepsilon_1/\varepsilon_2 \simeq 0), \tag{28'}$$

$$m_8 = m_0+3\varepsilon_2 = 2m_q - \frac{2\bar{v}_0+9\bar{v}_3-12\bar{v}_7+\bar{v}_8}{12}$$

$$= 2(m_q+\tfrac{2}{3}\,\Delta m_q) - \frac{2\bar{v}_0+3\bar{v}_3-8\bar{v}_7+3\bar{v}_8}{12} - \tfrac{4}{3}\,\varepsilon_1$$

$$\simeq m_{\rm II}-\tfrac{4}{3}\,\varepsilon_1 \text{ (if } \varepsilon_1/\varepsilon_2 \simeq 0), \tag{33'}$$

$$m_3 = m_{1\pm i2} = m_0+\bar{v}_3+\varepsilon_2 = 2m_q - \frac{2\bar{v}_0-3\bar{v}_3+\bar{v}_8}{12} \tag{36'}$$

and

$$m_{B_0} = \tfrac{3}{2}\,m_0+\tfrac{3}{4}\,\bar{v}_0+\varepsilon_1, \tag{76'}$$

where $\varepsilon_1 \equiv \Delta m_q+\frac{1}{8}(3\bar{v}_3-\bar{v}_7-\bar{v}_8)$ and $\varepsilon_2 \equiv \frac{1}{2}\,\bar{v}_7$ are the same as before. Thus, the binding conditions (30) and (35) for M_8 and M_3 take now the form

$$8\bar{v}_7 < 2\bar{v}_0+3\bar{v}_3+3\bar{v}_8 \quad (\text{if } \varepsilon_1 = 0) \tag{30'}$$

and

$$3\bar{v}_3 < 2\bar{v}_0+\bar{v}_8. \tag{35'}$$

Now the states $M_{4\pm i5}$ and $M_{6\pm i7}$ also can be bound, since their masses become

$$m_{4\pm i5} = m_{6\pm i7} = 2(m_q+\tfrac{1}{2}\,\Delta m_q) - \frac{\bar{v}_0-\bar{v}_8}{6} \tag{81}$$

thus they are bound if (and only if)

$$\bar{v}_0 > \bar{v}_8. \tag{82}$$

We can see that, in the new situation, the case of exact colour symmetry (38) allows for the existence not only of the colour singlet $M_{\rm I} = M_0$ but also, if the binding condition (82) is satisfied, of the colour octet $M_{\rm II} = M_8,\ M_3,\ M_{1\pm i2},\ M_{4\pm i5}$ and $M_{6\pm i7}$.

This condition remains true if we break the colour symmetry at most by the quark-mass difference Δm_q and so have (39) (in this case $\varepsilon_1 = \Delta m_q$). The only difference from the previous case is that now the states M_0 and M_8 mix (if $\Delta m_q \neq 0$) to form more stable states $M_{\rm I}$ and $M_{\rm II}$ given by (22) and (29), respectively.

Notice, however, that the binding condition (82) may be easily in contradiction with formula (76′) for m_{B_0} since for ordinary baryons and mesons the relation $m_{B_0} \simeq \frac{3}{2} m_{\rm I}$ is in reasonable agreement with experiment (here $m_{\rm I} \simeq m_0 + \frac{2}{3}\varepsilon_1$ if $\varepsilon_1/\varepsilon_2 \simeq 0$). It is so if (35′) implies a value of $\bar{v}_0$ which is too large for (76′).

In the new situation, the identification (54) of the recently discovered particles leads to the same discussion as described before by formulae (62)–(65).

If we now make the same conjecture (66) as before, then also formulae (67)–(69) remain true though the binding conditions (30) and (35) for M_8 and M_3 have to be replaced by (30′) and (35′). So, formulae (70), (71), (73), (75) and (77) take now the form:

$$\bar{v}_8 > \tfrac{8}{3}\bar{v}_7 - \bar{v}_3 - \tfrac{2}{3}\bar{v}_0 = 4.78\ \text{GeV} - \tfrac{2}{3}\bar{v}_0, \quad \Delta m_q > 0.71\ \text{GeV} - \tfrac{1}{12}\bar{v}_0, \tag{70′}$$

$$\bar{v}_8 > 3\bar{v}_3 - 2\bar{v}_0 = 3.60\ \text{GeV} - 2\bar{v}_0, \quad \Delta m_q > 0.56\ \text{GeV} - \tfrac{1}{4}\bar{v}_0, \tag{71′}$$

$$m_q = \tfrac{1}{2} m_0 + \frac{2\bar{v}_0 + 9\bar{v}_3 + 6\bar{v}_7 + \bar{v}_8}{24} > 1.60\ \text{GeV} + \tfrac{1}{12}\bar{v}_0, \tag{73′}$$

$$m_q + \tfrac{1}{3}\Delta m_q > 1.84\ \text{GeV} + \tfrac{1}{18}\bar{v}_0 \tag{75′}$$

and

$$m_{B_0} = 1.17\ \text{GeV} + \tfrac{3}{4}\bar{v}_0. \tag{77′}$$

Here $\bar{v}_0 > 0$ has an unknown value. We can see from (68) and (69) that $\bar{v}_3 < \bar{v}_7$.

Another conjecture than (66) seems to be appealing in the present situation, namely that the colour symmetry is broken at most by the quark-mass difference Δm_q and so (39) is satisfied. This conjecture implies that

$$\varepsilon_1 = \Delta m_q \tag{83}$$

and

$$\varepsilon_1 \equiv \tfrac{1}{2}\bar{v}_7 = \tfrac{1}{2}\bar{v}_3 = \tfrac{1}{2}\bar{v}_8, \tag{84}$$

where, of course, the binding condition (82) for M_8 and M_3 must be satisfied (it is also the binding condition for $M_{1\pm i2}$, $M_{4\pm i5}$ and $M_{6\pm i7}$). In this case we obtain from (63)

$$\varepsilon_1 = \Delta m_q = -\frac{2m_{\psi(3.1)} - m_\omega - m_{\chi(4.1)}}{2} + \tfrac{3}{4}\bar{v}_8 = 0.69\ \text{GeV},$$

$$\varepsilon_2 = \tfrac{1}{2}\bar{v}_8 = \tfrac{1}{6}\big[2m_{\psi(3.1)} - m_\omega - m_{\chi(4.1)}$$
$$+ \sqrt{(2m_{\psi(3.1)} - m_\omega - m_{\chi(4.1)})^2 + 6(m_{\psi(3.1)} - m_\omega)(m_{\chi(4.1)} - m_{\psi(3.1)})}\big] = 0.89\ \text{GeV},$$
$$m_0 = m_{\psi(3.1)} - \tfrac{3}{2}\bar{v}_8 = 0.45\ \text{GeV}, \quad \bar{v}_3 = \bar{v}_7 = \bar{v}_8 = 1.78\ \text{GeV}. \tag{85}$$

From (24) and (85) we calculate

$$\sin^2 \theta = 0.16 \tag{86}$$

and then from (59) and (60)

$$|\delta m_q| \simeq 40 \text{ MeV}. \tag{87}$$

Thus

$$\Delta m_q = 0.69 \text{ GeV} \gg |\delta m_q| \simeq 0.04 \text{ GeV}, \tag{88}$$

in consistency with our perturbative assumption.

From (28′), (82) and (85) we obtain the lower bound for the quark mass m_q

$$m_q = \tfrac{1}{2} m_0 + \frac{\bar{v}_0 + 8\bar{v}_8}{12} = 1.41 \text{ GeV} + \tfrac{1}{12} \bar{v}_0 > 1.56 \text{ GeV}. \tag{89}$$

Finally, from (76′), (82) and (85) we get the lower bound for the mass m_{B_0} of non--coloured baryon B_0

$$m_{B_0} = 1.37 \text{ GeV} + \tfrac{3}{4} \bar{v}_0 > 2.70 \text{ GeV} \tag{90}$$

which is in contradiction with experiment. It shows that the conjecture of the colour symmetry (39) is wrong if we take the identification (54) for granted, or vice versa.

Concluding we can say that the possible identification (54) of the recently discovered particles is consistent with our theoretical laboratory based on one-gluon-exchange static forces mediated between quarks and antiquarks by a colour nonet of vector gluons. However, the colour symmetry must then be broken and the coupling of the ninth gluon restricted to a moderate strength.

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