ON THE BRUECKNER–GAMMEL APPROXIMATION FOR THE MAGNETIC SUSCEPTIBILITY OF NEUTRON MATTER

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The magnetic susceptibility of neutron matter is calculated in the lowest order Brueckner theory for the Reid soft core potential. The rearrangement contributions calculated using the approximation of Brueckner and Gammel are important at the highest densities considered. Contrary to the case of liquid ^{3}He, the results obtained using Brueckner and Gammel approximation coincide with those obtained by the other authors within the frame of the Landau theory of Fermi liquids.

It is currently believed that pulsars are rapidly rotating neutron stars possessing very intense magnetic fields [1], and hence the magnetic properties of the neutron star matter are of a considerable interest. In the normal nuclear density region neutron star matter is believed to consist mainly of neutrons, with a small admixture of protons, electrons and muons [2]. Some of the neutron star matter calculations [3] indicate that also at high densities ($k_F > 3\ \mathrm{fm}^{-1}$) the ground state of the cold dense matter could be nearly pure neutron matter. Thus, pure neutron matter appears to be a meaningful approximation of the real neutron star matter near and above normal nuclear density.

Let us consider neutron matter composed of N neutrons. In the presence of an external magnetic field, the two spin populations will no longer be equal in the ground state, and the total energy of the system will be a function of the spin excess parameter

$$\alpha = \frac{N_\uparrow - N_\downarrow}{N}, \tag{1}$$

where $N_\uparrow$ and $N_\downarrow$ are the numbers of neutrons with spin up and spin down with respect to the direction of the applied field. The nuclear energy per particle expanded in powers of α takes the form [4]

$$\frac{E^n(N, \alpha)}{N} = \varepsilon_0 + \tfrac{1}{2}\, \varepsilon_\sigma \alpha^2, \tag{2}$$

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where ε_0 is the energy per neutron in unpolarized neutron matter ($\alpha = 0$), ε_σ is the spin symmetry energy of neutron matter and the spin excess parameter α is assumed to be small, so that the terms proportional to α^n ($n > 2$) can be neglected. The magnetic susceptibility of neutron matter is then given by

$$\chi = \frac{\mu_n^2 \varrho}{\varepsilon_\sigma}, \tag{3}$$

where ϱ is the density of neutron matter and the neutron magnetic moment is denoted by μ_n. For the sake of convenience one introduces usually a ratio of χ and the magnetic susceptibility of the Fermi gas of the free neutrons, χ_F,

$$\frac{\chi_F}{\chi} = \frac{3}{2}\frac{\varepsilon_\sigma}{\varepsilon_F}, \tag{4}$$

where ε_F is the Fermi energy for unpolarized neutron matter with $N_\uparrow = N_\downarrow = \frac{1}{2}N$. The spin symmetry energy ε_σ is given by

$$\varepsilon_\sigma = \frac{1}{N}\left(\frac{\partial^2 E^n}{\partial \alpha^2}\right)_{\varrho,N}\bigg|_{\alpha=0}. \tag{5}$$

The nuclear potential energy of the system, calculated in the lowest order Brueckner theory depends on α in two ways: firstly, through the upper limits of the sums over neutron momenta inside the corresponding Fermi seas, and secondly, through the intrinsic dependence of the effective neutron-neutron interaction in neutron matter (the K matrix) on the two Fermi momenta for neutrons with spin up and spin down,

$$\kappa^3 = k_F^3(1+\alpha), \tag{6a}$$

$$\lambda^3 = k_F^3(1-\alpha). \tag{6b}$$

Hence, when we calculate the second derivative of E^n, Eq. (5), we get two parts of ε_σ,

$$\varepsilon_\sigma = \varepsilon_\sigma^{(0)} + \Delta\varepsilon_\sigma, \tag{7}$$

the first (nonrearrangement) part, $\varepsilon_\sigma^{(0)}$, resulting from the first type of the dependence, and the second (rearrangement) part, $\Delta\varepsilon_\sigma$, resulting from the second type of the dependence of E^n on α. Final expression for ε_σ is given in Ref. [4]. The expression for the rearrangement part of ε_σ contains first and second partial derivatives of the diagonal elements of the K matrix with respect to Fermi momenta, calculated at the point $\kappa = \lambda = k_F$. Although it is possible to calculate the K matrix which depends on two different Fermi momenta, this calculation is very tedious [5]. On the other hand, an approximation introduced by Brueckner and Gammel [6] in their paper on the properties of liquid ^{3}He, generalized subsequently by Brueckner and Dąbrowski [7] to the case of nuclear matter with neutron excess and by Dąbrowski and Haensel to the case of the polarized nuclear matter with neutron excess [8], enables one to calculate $\Delta\varepsilon_\sigma$ using the K matrix for unpolarized neutron matter. The BG approximation for the diagonal elements of the K matrix in the representation of the total spin of the neutron pair has the form

$$K(s = 1, m_s = 1; \kappa\lambda) \approx K(s = 1, m_s = 1; \kappa), \tag{8a}$$

$$\sum_s K(s, m_s = 0; \kappa\lambda) \approx \sum_s K(s, m_s = 0; \omega), \tag{8b}$$

where

$$\omega = 2^{-1/2}(\kappa^2 + \lambda^2)^{1/2},$$

and where the K matrices on the right-hand side of Eqs. (8) are calculated in the case of $\alpha = 0$, with the indicated value of the Fermi momentum.

The magnetic susceptibility of neutron matter can be also calculated within the frame of the Landau theory of the normal Fermi liquids [9]. In this approach the quasi-particle interaction is calculated as the second functional derivative at the Fermi surface of the potential energy of the system with respect to the distribution of particles. The magnetic susceptibility can be expressed with the help of the zero-order Landau parameter of the spin dependent part of the quasi-particle interaction, g_0.

These two approaches have been previously applied to the calculation of the magnetic susceptibility of liquid ^{3}He [10, 11]. The results of these calculations are in complete disagreement with the experimental value of the magnetic susceptibility of liquid ^{3}He. Both calculations have been reviewed critically by Bäckman [12], who concludes that the divergences between the Landau theory values of Bertsch [10] and the values obtained with BG approximation by Østgaard [11] result from the incorrectness of BG approximation. Let us notice that contrary to the opinion expressed in Ref. [10] the use of the densities of the spin up and spin down particles instead of the two different Fermi momenta does not change the final results: these two sets of variables are mathematically equivalent. In our opinion, these divergences result mainly from the fact that both authors use the lowest order Brueckner theory expression for the potential energy of the system. The lowest order (standard) Brueckner theory with zero potential energy in the intermediate states gives only the two-body contribution to the potential energy per particle. The higher order contributions from Goldstone diagrams with three and more hole lines should be calculated explicitly and added to the two-body contribution. The importance of these higher order terms is determined by the defect parameter κ [13]. However, the value of κ is of the order 0.4 for liquid ^{3}He [11] and hence the lowest order Brueckner theory expression for the potential energy of the system is inadequate [14].

In contrast to the case of liquid ^{3}He the values of κ for neutron matter are quite small even for the densities much higher than the normal nuclear density [2]. E.g., the value of κ for the Reid soft core (RSC) potential [15] is less than its nuclear matter value ($\kappa = 0.13$ at the normal nuclear density $\varrho_0 = 0.17\ \text{fm}^{-3}$) even at the density of neutron matter $\varrho = 3.2\ \varrho_0$ [16]. However, up to now, no attempt of a complete calculation of χ_F/χ in Brueckner theory with RSC n–n interaction was made (in Clark's calculation [17] the rearrangement contribution to χ_F/χ was neglected). In the present paper the magnetic susceptibility of neutron matter is calculated in the lowest order Brueckner theory with the RSC potential, the rearrangement contribution being calculated in the BG approximation. The calculational procedure has been essentially the same as that presented in Ref. [18]. The differentiation of the K matrix was carried out numerically, by making finite shifts in the Fermi momentum and repeating the whole selfconsistent calculation. Then the derivatives were determined from the resulting shifts in the K matrix. The contribution to χ_F/χ from $J > 2$ partial waves was estimated in the first Born approximation for OPEP.

The quotient χ_F/χ can be splitted into two parts,

$$\chi_F/\chi = (\chi_F/\chi)_0 + (\chi_F/\chi)_R,$$

TABLE I

Results for magnetic susceptibility of neutron matter with the RSC potential

k_F (fm^{-1})	$\left(\frac{\chi_F}{\chi}\right)_0$	$\left(\frac{\chi_F}{\chi}\right)_R$	$\frac{\chi_F}{\chi}$
1.4	1.89	0.06	1.95
1.7	1.99	0.21	2.20
2.0	2.10	0.34	2.44
2.2	2.18	0.41	2.59
2.5	2.29	0.45	2.74

where $(\chi_F/\chi)_0$ and $(\chi_F/\chi)_R$ are, respectively, the nonrearrangement and rearrangement parts of χ_F/χ. The results for several values of k_F are given in Table I. The rearrangement contribution is negligible at low density, but becomes quite important at highest densities considered. In Fig. 1 the values of $(\chi_F/\chi)_0$ and χ_F/χ are plotted versus k_F. Our values of

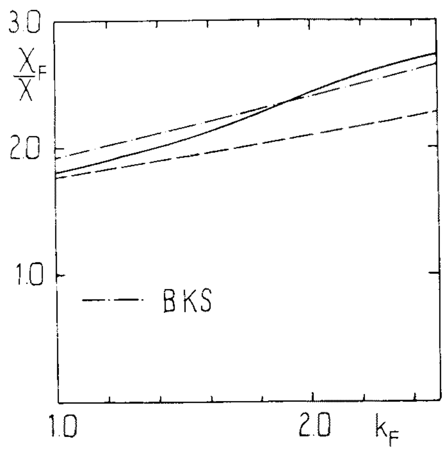

Fig. 1. Results for the magnetic susceptibility of neutron matter with the RSC potential (solid line). Dashed line corresponds to our $(\chi_F/\chi)_0$ and dash-dotted line represents the results of Ref. [19]

$(\chi_F/\chi)_0$ coincide with those obtained by Clark [17], who performed his calculation in the density region corresponding to $0.78\ \text{fm}^{-1} < k_F < 2.02\ \text{fm}^{-1}$. The RSC results of Bäckman, Källman and Sjöberg [19], obtained within the frame of the Landau theory of the normal Fermi liquids, are also shown for comparison. In contrast to the situation in the case of liquid ^{3}He, the results obtained using BG approximation for $(\chi_F/\chi)_R$ agree very well with those obtained in the Landau theory. Thus, we conclude that BG approximation gives quite good estimate of the rearrangement contribution to the magnetic susceptibility of neutron matter. On the other hand, the formulae of Ref. [12] lead to the χ_F/χ values which are substantially higher than those obtained in Ref. [19].

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