VOLUME CONSERVING PAIRING AND ROTATIONAL BANDS OF ^{156}Dy, ^{156}Er, AND ^{162}Er

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Volume-conserving pairing was used to determine the Coriolis-antipairing effect on the rotational bands of ^{156}Dy, ^{156}Er, and ^{162}Er. The results follow the experimental moments of inertia up to the region of the critical spin values quite well. In particular they do not show the too rapid rise of the moment of inertia at low spin values which is difficult to avoid in the "standard pairing" calculation.

The anomalous behaviour of the moment of inertia J (usually referred to as "back-bending") observed in several rare-earth nuclei [1]–[6] may be the result of many different processes occurring in a rotating system at high angular velocity ω. Among them, the most studied are the Coriolis-antipairing (CAP) [7] and the alignment (AL) [8] effects. However, it is very difficult to reproduce quantitatively the experimental situation by either of them. Realistic calculations based on CAP or AL [9]–[13] tend to give too fast an increase in J with increasing spin I and too low a critical spin value I_{crit}.

One of the reasons for this difficulty may be the use of the constant matrix element approximation for the pairing force. The pairing strength G should, in fact, be considered as the average value of the matrix elements $G_{\nu\nu'}$, taken over the pairs of the single-particle states ν, ν', with the weights $U_\nu V_\nu$ and $U_{\nu'} V_{\nu'}$, where U and V are the BCS pairing amplitudes. The strong dependence of the weight functions UV on the energy gap Δ should lead to the Δ-dependence of the average value G. One can argue (see Ref. [14]) that for decreasing Δ, G should also decrease. This would make the pairing contribution $G(\Sigma\, UV)^2$ to the pairing potential decrease faster for decreasing Δ, leading to a much steeper pairing

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potential and a slower decrease of Δ with I. It is exactly what is needed to slow the change of the moment of inertia with I.

With the aim of checking this conclusion, the volume-conserving pairing (VCP) of Refs [14] and [15] was applied in calculating the moment of inertia for the rotational states of ^{156}Dy, ^{156}Er, and ^{162}Er. In the framework of the VCP method the effective strength of the pairing force depends on Δ and is obtained from the condition of the constant volume by means of a constrained variational procedure. Thus, the only input parameters of the method are those of the s. p. potential. This is another advantage of VCP over the standard pairing for which G_p and G_n appear as two additional parameters.

The calculation was performed with the Nilsson s. p. potential. Single-particle energies for given deformations ε_2 and ε_4 were used to construct the s. p. part of the total energy. The contribution of the pairing field, Coulomb energy, and rotational energy were added. Rotational energy was taken in the form $\frac{\hbar^2}{2J}[I(I+1)-\langle \hat{I}^2\rangle]$ with J calculated according to the cranking model formula. The explicit form of the expression for the total energy and the parameters of the Nilsson s. p. potential may be found in Ref. [14]. The total

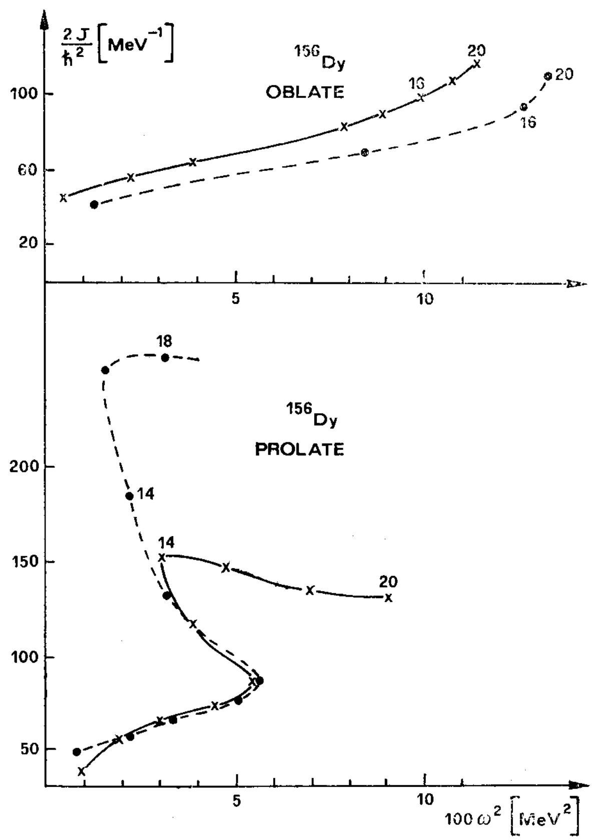

Fig. 1. The calculated moments of inertia (broken lines) for oblate and prolate states of ^{156}Dy are compared with the experimental values (full lines) for the g. s. and "β-vibrational" rotational bands

energy was varied with respect to Δ^n, Δ^p and ε_2. The variations with respect to Δ^n and Δ^p were constrained by the volume conservation condition in the form of the mean square radius conservation. The mean square radius was kept constant by compensating the effect of the Δ^p and Δ^n variations with the appropriate changes of the s. p. potential oscillator strength (see Ref. [14] for details). In this way the equilibrium values ε_2^{eq}, Δ^p_{eq}, Δ^n_{eq} and the corresponding values of the moment of inertia were obtained for each value of I.

The results for ^{156}Dy are of particular interest. In this nucleus, two sets of rotational $K = 0$ states are known up to the $I = 20$. They may be interpreted as two rotational bands, one built on the ground state, the other on the 655 keV 0^+ excited state, usually interpreted as the β-vibrational state. The two bands cross at $I \approx 16$. The separation of the two $I = 16$ states is only 23 keV. This shows that the coupling of the two bands is very weak. Thus, the results of the Coriolis-antipairing calculation may be directly compared with the data for each band.

The calculated energy-versus-deformation curve for ^{156}Dy has two well-pronounced minima. The oblate one, at $\varepsilon_2 = -0.300$, is lower in energy and should correspond to the ground state, while the prolate, at $\varepsilon_2 \approx 0.173$, should give rise to an excited 0^+ state with

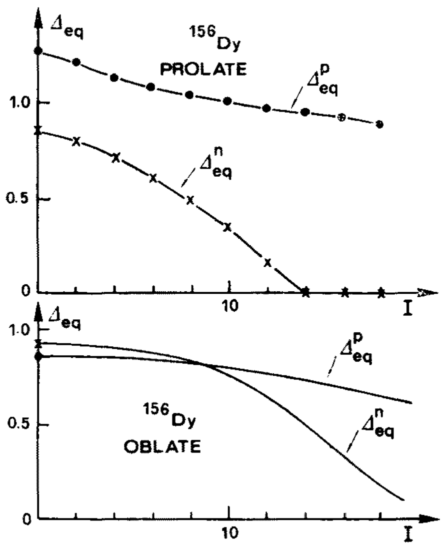

Fig. 2. Equilibrium values of Δ^n and Δ^p for the oblate and prolate states of ^{156}Dy

excitation energy of about 700 keV. The calculated moments of inertia for rotational bands built on these two states are shown in Fig. 1 (broken lines) together with the experimental values (full lines). The similarity of the shapes of the calculated and experimental curves strongly supports the interpretation of the data assuming an oblate ground state and a prolate 0^+ excited state. This would also explain the weakness of the coupling between the two rotational bands.

Fig. 2 shows the I dependence of the energy gaps. For the highest values of I the neutron gap falls to zero and with a flat pairing potential the BCS approximation breaks down. This explains the differences between the calculated and experimental J-values for the prolate $I > 14$ states.

The case of ^{156}Er is very similar to that of ^{156}Dy. An oblate ground state is obtained at $\varepsilon_2 = -0.300$, while the prolate minimum at $\varepsilon_2 \approx 0.125$ corresponds to an excited 0^+ state. The experimental data on the yrast levels (full line in Fig. 3) should thus be com-

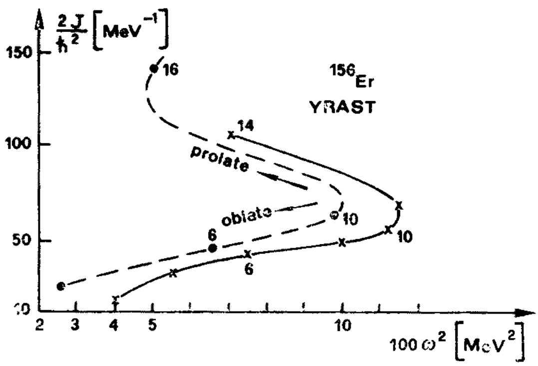

Fig. 3. Calculated (broken line) and experimental (full line) moments of inertia for the yrast levels of ^{156}Er

pared with the result for the oblate-prolate bands crossing which occurs at $I \approx 10$. The moment of inertia calculated in this way is shown in Fig. 3 by the broken line.

^{162}Er has the prolate equilibrium deformation which changes very slowly with increasing I. Experimental data show a strong back-bending effect at $I_{\text{crit}} \approx 16$. The calculated energy versus Δ^{n} curves have minima at $\Delta^{\text{n}} = 0$ and $\Delta_1^{\text{n}} \neq 0$ with a barrier between them. For $I < 14$ the Δ_1^{n} minimum is the lower one. The change of Δ_1^{n} with I is shown in Fig. 4.

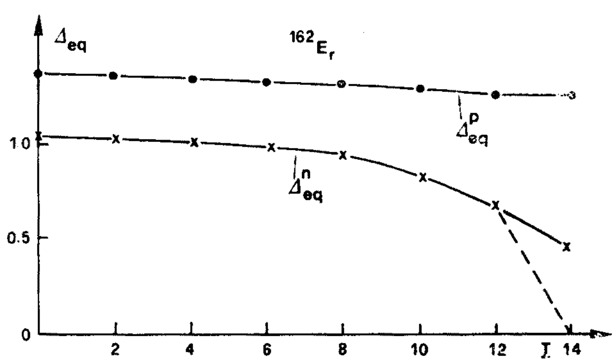

Fig 4. Equilibrium values of Δ^{n} and Δ^{p} for ^{162}Er

It is very slow because the existence of the barrier makes the slope of the pairing potential large. At $I = 14$ the two minima have about the same energy, the $\Delta^{\text{n}} = 0$ being slightly lower. The corresponding jump in the yrast $\Delta_{\text{eq}}^{\text{n}}$ value is shown in Fig. 4 by the broken line. The resulting yrast J values are shown in Fig. 5. Since the cranking formula gives the g. s. moment of inertia for ^{162}Er smaller by about 20% than its experimental value, a constant was added to fit the experimental position of the 2^+ state.

The change of J is slow and there is plenty of room for the effect of the coupling between the $\Delta^n = 0$ and Δ_1^n states. With the barrier between the two minima the coupling can be considered in terms of the mixing of the two BCS solutions. The coupling was not calculated here but it has already been shown by one of the authors that with a pheno-

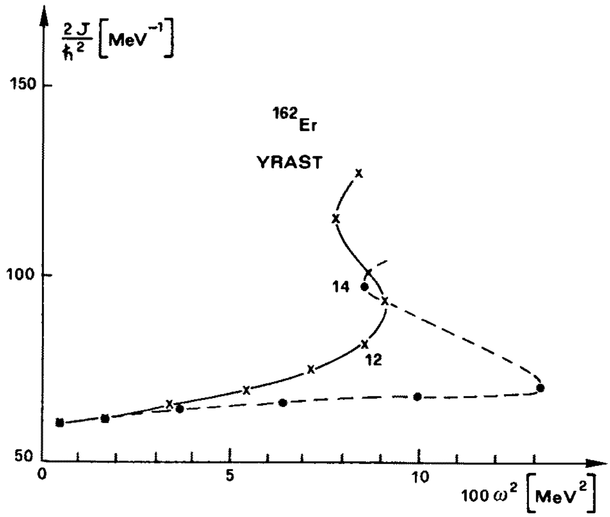

Fig. 5. Calculated (broken line) and experimental (full line) moments of inertia for the yrast levels of ^{162}Er

menological one-parameter coupling the experimental values of J can be fitted extremely well [16].

The following conclusions can be drawn from the results presented above:

a) The VCP reproduces quite well the experimental moments of inertia in the nuclei considered here and may be used as a good starting point for a realistic description of the intrinsic structure of rotating nuclei.

b) Away from the stability line, the crossing of the rotational bands corresponding to prolate and oblate deformation should be considered as one of the possible mechanisms of the back-bending effect in the yrast band.

c) The shapes of the $J(\omega^2)$ curves for ^{156}Dy strongly suggest that the oblate state is the best candidate for the ground state of this nucleus.

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