SIDEWISE DISPERSION RELATIONS AND STATIC CHARACTERISTICS OF ELECTROMAGNETIC AND WEAK BARYON-BARYON TRANSITIONS

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The method of sidewise dispersion relations is used for the calculation of anomalous magnetic moments (a. m. m.) and constants of "weak magnetism" of electromagnetic and weak $B(J^P = \frac{1}{2}^+) \to B'(J^P = \frac{1}{2}^+)$ transitions. The performed analysis showes that the values of a. m. m. of octet baryons calculated with the method mentioned above, and compared with the experimental data and the ratio of "weak magnetism" constants of different weak baryon-baryon transitions do not contradict the predictions of Cabibbo's model.

1. Introduction

The dispersion relations for the electromagnetic vertex of nucleon as the function of the nucleon mass were tackled for the first time by Bincer [1]. Later on, the approach proposed in [1] was frequently discussed in literature and it proved to be a fruitful method of investigating electromagnetic vertexes of fermions (see, for instance, the bibliography given in [2]).

The method under consideration, as well as the assumption concerning the importance of the low-energy intermediate contributions in the electromagnetic vertex $B(p_i^2 \neq m_B^2) \to B(p_f^2 = m_B^2)+\gamma^*$ (B — baryon with $I^P = \frac{1}{2}^+$, γ^* — virtual γ-quantum) allow one to connect the electromagnetic characteristics of baryons (anomalous magnetic moments (a.m.m.), electromagnetic radii and so on) with the constants of BBM-interaction and the amplitude of process $MB \to B\gamma^*$ in the treshold region of energies (M is a pseudoscalar meson). In [3] to calculate the a.m.m. of baryons the amplitude of process $MB \to B\gamma$ near the threshold was approximated by the contributions of Born's terms with the pseudoscalar — BBM — coupling. It is known that Born's approximation cannot be applied to the process of photoproduction of π^0-mesons [4]. In the latter case the approximation of the model of algebra of currents seems to be more applicable [5]. That is why it seems interesting to calculate the a.m.m. of octet baryons using the amplitudes of processes

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$MB \to B\gamma$ on the threshold found in the model of algebra of currents. Note that the algebra current approximation together with PCAC hypothesis in strange particles processes allow one to describe correctly the experimental data in many cases despite the relatively large K-meson mass [6].

The sidewise dispersion relations may also prove to be fruitful for the investigation of weak vertexes of baryon-baryon transitions. This method was used in [7] to find the axial-vector constant of weak $N \to N$ transition. In the present study we calculate the constants of "weak magnetism" in baryon-baryon transitions with a violation of strangeness by means of the sidewise dispersion relations method. The knowledge of the value of the latter constants may become useful in solving the problem dealing with possible existence of contribution of the second class currents to the processes of β-decav of hyperons [8].

2. *Magnetic moments of baryons.Calculations of absorptive part of amplitude in model of algebra of currents*

Let us consider first the process of transition of virtual baryon $B_1(J^P = \frac{1}{2}^+, (p+l) = W^2)$ into a meson and a baryon on their mass shells ($q^2 = \mu^2$, $k^2 = M_2^2$) with the further radiative transition into a baryon of M_1 mass (Fig. 1).

The general expression for the electromagnetic vertex of virtual baryon is found in paper [1]. The invariant function $K(W^2)$ (further we shall use the notation from paper

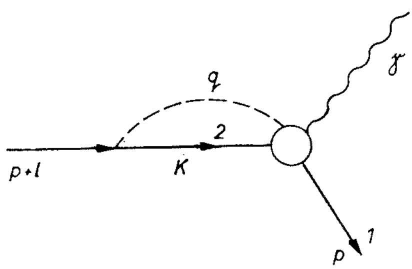

Fig. 1

[3]) of this expression, which coincides with a.m.m. of baryon at $W^2 = M_1^2$, satisfies the unsubtracted dispersion relation [1]. Following [3], suppose that the main contribution to the invariant function $K(W^2)$ for $W^2 = M_1^2$ is made by the pre-threshold region of energies and of all possible two-particles intermediate states (Fig. 1), the once with the minimum summary mass dominate. Then we may confine ourselves to the contribution in the intermediate state of baryon with $J^P = \frac{1}{2}^+$ and of pseudoscalar meson only, and if we approximate the vertex of $\tilde{B} \to BM$ transition ($\tilde{B}$ — is a virtual baryon) with the BBM--coupling constant, then

$$\mu_1 \equiv K(M_1^2) = \frac{1}{\pi} \int_{(M_2+\mu)^2}^{\Lambda(M_2+\mu)^2} dW^2 \frac{\operatorname{Im} K(W^2)}{W^2 - M_1^2}, \tag{1}$$

where Λ is the cutoff.

Taking into account the said assumptions, we find the imaginary part [3]:

$$\mathrm{Im}\, K(W^2) = \frac{M_1 M_2}{4\pi} \frac{q^*}{W} \int_{-1}^{1} dx\, \mathrm{Sp}\, \{\bar{u}(p) J_\mu u(k) \bar{u}(k) i g \gamma_5 v_\mu^{(2)}\}, \tag{2}$$

where M_1, M_2 are the masses of final and intermediate baryons, q^* is an absolute value of 3-impulse of meson in the c.m.s., $v_\mu^{(2)}$ is a projection operator determined in [1], g is the constant of strong interactions in the BBM-vertex, x is the cosine of scattering angle in the c.m.s.

As it has been mentioned in the introduction the algebra current approximation may prove to be a satisfactory approximation of $MB \to B\gamma$ amplitude at threshold energy region. In the chosen kinematical situation (threshold region) the resonance contributions

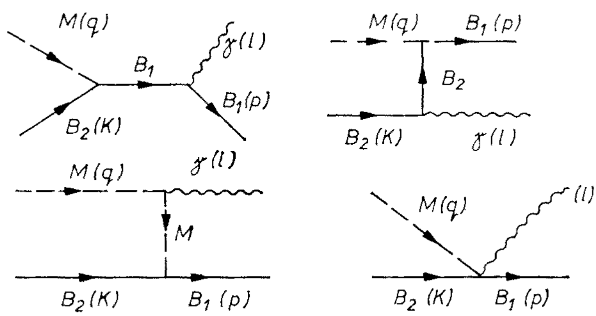

Fig. 2

are suppressed and the amplitude of the precess under consideration is determined by the contribution of pole diagrams with pseudovector BBM-interaction and the contact diagram (Fig. 2). After the simple though bulky calculation we find

$$\mathrm{Im}\, K_{12}(W^2) = \frac{g^2}{8\pi} \frac{q^*}{W} F_{12}(W^2),$$

$$F_{12}(W^2) = e_1 E_1(W^2) + e_2 E_2(W^2) + \mu_1 K_1(W^2) + \mu_2 K_2(W^2). \tag{3}$$

Analytic expressions for $E_{1,2}(W^2)$ and $K_{1,2}(W^2)$ are given in the Appendix I.

Due to the fact that we considered real masses of baryons, we can hardly expect coincidence of the value of the a.m.m. of baryons determined by the method described above with the predictions of SU(3)-symmetry.

First, let us find out what new information we can get from the approximation mentioned above as compared with the similar calculations in Born's model [3]. Let us consider a case when the masses of baryons are equal $M_1 = M_2 = M$. On the threshold $(W = M+\mu)$ the intermediate state baryon-pseudoscalar meson possesses the negative P-parity and, therefore, on emitting an electric dipole γ-quantum, it can turn into a baryon with a positive parity. The radiation of E1 γ-quantum in the process $B(J^P = \frac{1}{2}^+) \to B'(J^P = \frac{1}{2}^+) + \gamma$ is possible only in the case when one of the baryons is off the mass shell, and the amplitude of such a transition is proportional to a "power of virtuality" of the baryon, i.e. in lowest order to the value $W = M \sim \mu$ [9]. According to what was previously mentioned it is possible to represent the amplitude of E1 transition $\tilde{B} \to B\gamma$ as follows

$$M(\tilde{B} \to B\gamma) \sim \frac{\mu}{M} M(MB \to B\gamma). \tag{4}$$

The main contribution in (4) when $\mu \to 0$ gives the singular terms in amplitude $M(MB \to B\gamma)$. It is known that the predictions of both pseudoscalar and pseudovector Born approximation for $MB \to B\gamma$ amplitude differ only in terms of a.m.m. baryons [10]. The latter have a small coefficient μ/M in $MB \to B\gamma$ amplitude at threshold. For the singular terms at $W \to M$ in $M(MB \to B\gamma)$ the predictions of both approximations are identical. Therefore, both approximations under consideration must differ only in terms of the order μ/M, and the difference becomes apparent only in the multipliers in (3) which are proportional to a.m.m. of baryons.

Making an expansion of value $E_{1,2}(W^2)$, $K_{1,2}(W^2)$ determined in the model of algebra of currents and in Born's approximation[1] [3] in the small parameter $\alpha = \mu/M$ we find

a. Born's approximation	b. Model of algebra of currents
$E_1^B = 1-\frac{3}{2}\alpha+O(\alpha)$	$E_1 = 1-\frac{3}{2}\alpha+O(\alpha)$
$E_2^B = -1+\frac{1}{2}\alpha+O(\alpha)$	$E_2 = -1+\frac{1}{2}\alpha+O(\alpha)$
$K_1^B = K_2^B = -\frac{\alpha}{2}+O(\alpha)$	$K_1 = -\frac{1}{4}\alpha+O(\alpha)$
	$K_2 \sim O(\alpha)$,

where $0(\alpha)$ are the values of second order and higher by μ/M.

Similarly to [3] we determine the value $F(W^2)$ for each given intermediate state at the threshold point $W^2 = W_{\text{th}}^2$. Taking into consideration all possible (allowed by the conservations laws) intermediate meson-baryon states, and assuming that $SU(3)$ predictions are valid for the constants $g(B, BM)$, we find for the a.m.m. of octet baryons the following system of linear equations

$$\mu_i = B_i + \sum_{j=1}^{9} A_{ij}\mu_j, \qquad i = 1 \div 9, \tag{5}$$

where coefficients B_i, A_{ij} are the function of Λ and of parameter f which determines the ratio of $f/d\,(f+d = 1)$ coupling constants in SU(3)-symmetry.

For nucleons, neglecting the contribution of strange particles, (5) is reduced to the system of two linear equations to determine the a.m.m. of proton and neutron with coefficients depending only upon the magnitude of cutoff Λ. The Table gives the values of the calculated a.m.m. of nucleons according to the magnitude of the cutoff Λ (for comparison similar results calculated in Born's approximation are given). As it is clearly seen from

[1] There is a number of misprints in the formulae of [3].

TABLE I

Born Model			Current algebra model	
Λ	μ_p'	μ_n	μ_p'	μ_n
2.6	1.35	−1.61	1.35	−1.69
2.7	1.40	−1.67	1.40	−1.75
2.8	1.45	−1.72	1.44	−1.81
2.9	1.51	−1.80	1.51	−1.89
3.0	1.56	−1.85	1.56	−1.95

the Table I, the suggested model improves, in a way, the predictions for the a.m.m. of neutron.

Because of the fact that the main contribution in the dispersion integral for the a.m.m. of nucleons is made by the πN-state, one can hardly expect the other possible intermediate

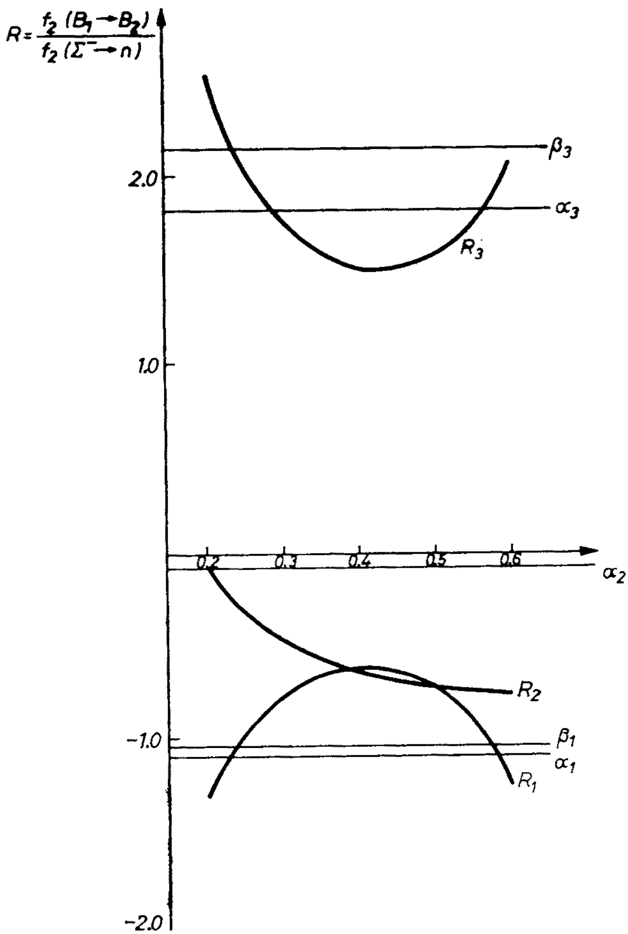

Fig. 3

states to change substantially the found data of the a.m.m. That is why it would be quite natural to assume $\Lambda = 3.0$ (see Table I) when solving the complete system of equations (5).

The results of the calculations are given in Fig. 3. The calculations show that the a.m.m. of proton and neutron in the considered model depends not much upon the parameter f. In the region $f \sim 0.4$ the a. m. m. of neutron approximates to its experimental value. Thus for $f = 0.4$ we find: $\mu'_p = 1.61$ (1.57); $\mu_n = -1.93\,(-1.78)$; $\mu_\Lambda = -0.76$ (-0.68); $\mu_{\Sigma^0} = 0.11\,(0.15)$; $\mu'_{\Sigma^-} = -0.82\,(-0.75)$; $\mu'_{\Sigma^+} = 1.04\,(1.05)$; $\mu'_{\Xi^-} = -0.55\,(-0.55)$; $\mu_{\Xi^0} = -1.10\ (-1.08)$; $\mu_{\Sigma\Lambda} = 0.99$ (1.14), where figures in parenthesis represent the results of calculation with Born's amplitude for the same cutoff Λ. As it is clearly seen from calculated data the a. m. m.'s of Λ and Σ'^{+} hyperons are not far from their experimental values $\mu_\Lambda^{\text{exp}} = (-0.80 \pm 0.07)\, e\hbar/2m_\Lambda c$, $\mu_{\Sigma^+}'^{\text{exp}} = (2.02 \pm 0.58)\, e\hbar/2m_\Sigma c$ [11], the obtained data for a. m. m.'s of Ξ^--and Σ^--hyperons also correspond to the measurements that have been recently performed: $\mu_{\Xi^-} = -(2.2 \pm 0.8)$ [12], $\mu_{\Xi^-} = (0.1 \pm 2.1)$ [13] and $-1 < \mu'_{\Sigma^-} < 0$ [14], $\mu'_{\Sigma^-} = (-0.89 \pm 0.47)\, e\hbar/2m_\Sigma c$ [15]. The data given above show that the calculated values of a.m.m. Σ^- and Ξ^--hyperons differ radically from SU(3)-symmetry predictions. The existing experimental data [12–15] are in agreement with our values.

3. Constants of weak magnetism in baryon-baryon transitions

The formalism of the sidewise dispersion relations method can be applied for the investigation of form factors of weak baryon-baryon transitions. The vector part of the vertex of weak baryon-baryon transition $\tilde{B}_1 \to B_2$ ($\tilde{B}$ is a virtual baryon) can be represented as the sum of the following six summonds:

$$\bar{u}(p)\Gamma_\mu^V(p, p+l) = \bar{u}(p)\left\{\gamma_\mu f_1^+ - if_2^+ \frac{\sigma_{\mu\nu} l_\nu}{m_1+m_2} + f_3^+ \frac{l_\mu}{m_1+m_2}\right\} \frac{\hat{p}+\hat{l}+m_1}{2m_1}$$

$$+\bar{u}(p)\left\{\gamma_\mu f_1^- - if_2^- \frac{\sigma_{\mu\nu} l_\nu}{m_1+m_2} + f_3^- \frac{l_\mu}{m_1+m_2}\right\} \frac{-\hat{p}-\hat{l}+m_1}{2m_1},$$

where $f_j^\pm \equiv f_j^\pm(l^2, W^2)$ are the form factors of the transition, $m_{1(2)}$ are masses of the initial (final) baryons. The form factor $f_2^+(W^2)$ goes to the "weak magnetism" constant in the limit of $W \to m_1, l^2 \to 0$.

Let us suppose (similarly to the electromagnetic transition), that for $l^2 = 0$ the form factor $f_2^+(0, W^2) \equiv f_2(W^2)$ satisfies the unsubtracted dispersion relation in W^2. Then according to the assumptions made in the previous section, the "weak magnetism" constant may be represented as (I), the imaginary part may have the expression coinciding with (2) (substituting everywhere $M_2 \to m_3$ the mass of the intermediate baryon). The projection operator $v_\mu^{(2)}$ for the investigated vertex has the form

$$v_\mu^{(2)}(W^2, l^2 = 0) = -\frac{m_2}{2(W^2-m_2^2)^2}(\hat{p}+\hat{l}+m_1)\{al_\mu - \gamma_\mu \hat{l}\}u(p), \tag{6}$$

where

$$a = \frac{4(W^2 - m_1 m_2 + m_1^2)}{W^2 + 2m_1 m_2 + m_1^2} \,.$$

In the limit $l^2 = 0$ the form factor f_3 does not make a contribution in Im $f_2(W^2)$ and the method under consideration permits us to obtain linear relations connecting the constants f_1 and f_2 of different transitions of octet baryons. We shall consider the transitions with the violation of strangeness. Further, we shall take the values f_1 predicted by Cabibbo's model when calculating the values f_2. Therefore, taking into accout the results obtained in the previous paragraph, Born's model may serve quite applicable for the calculation of the amplitude of process $MB \to BV_\mu$ near the threshold (V_μ is a weak vector current).

In Born's approximation for Im $f_2(W^2)$ of weak $B_1 \to B_2$ transition we find the following expression:

$$\mathrm{Im}\, f_2^{(1\to 2)}(W^2) = \frac{m_1+m_2}{32\pi}\,\frac{q^*}{W}\,\frac{g(B_1,\, B_3 M)}{W^2 - m_2^2}\left\{\frac{2g(B_s,\, B_3 M)}{W^2 - m_s^2}\,[f_1(s \to 2)A_1^s\right.$$

$$\left. + f_2(S \to 2)A_2^s] + \frac{g(B_2,\, B_u M)}{u(W^2)}\,[f_1(3 \to u)A_1^u + f_2(3 \to u)A_2^u] + \frac{2g(B_3,\, B_2 M)}{t(W^2)}\, f_M(i \to t)A^t\right\}, \tag{7}$$

where m_3 is the mass of intermediate baryon, m_s, m_u are the masses of the pole baryons in the s and u channels. Matrix element of the $u_i \to u_t$ weak transition was chosen as follows:

$$\langle M_i(q)|J_\mu^V|M_t(p)\rangle = f_M(l^2)\,(q+p)_\mu + f_M^-(l^2)l_\mu .$$

The analytic expressins for functions $A_{1,2}(W^2)$ and $u(W^2)$, $t(W^2)$ are given in Appendix 2.

In the limit $W \to m$, $\mu \to 0$ we get the following expression for the "weak magnetism" constant

$$f_2^{(1\to 2)} = \lim_{W \to m} \int\limits_{W^2}^{\Lambda W^2} dW^2\, \frac{\mathrm{Im}\, f_2^{(1\to 2)}(W^2)}{W^2 - m^2} = Kg(B_1,\, B_3 M)$$

$$\times \{(-\tfrac{3}{2})\,[g(B_s,\, B_3 M) f_1(s \to 2) - g(B_2,\, B_u M) f_1(3 \to u)] + \tfrac{1}{2}\, g(B_3,\, B_2 M_t) f_M(i \to t)\},$$

where

$$K = -\ln \Lambda / 16\pi^2$$

As it is generally known there exist four independent weak transitions with a change of strangeness at the level of isotopic symmetry between the octet baryons

$$\Lambda \to \mathrm{p}V_\mu^-, \quad \Sigma^- \to \mathrm{n}V_\mu^-, \quad \Xi^- \to \Lambda V_\mu^-, \quad \Xi^0 \to \Sigma^+ V_\mu^- .$$

Taking into consideration all possible (allowed by the conservation laws) meson-baryon intermediate states (Fig. 1) and using for the constants of weak and strong interactions the predictions of SU(3)-symmetry, one can easily find, in the present limit, the following expressions for the "weak magnetism" constants of the considered transitions:

$$f_2(\Lambda \to p) = \frac{K}{\sqrt{6}}\{-f_1(\Sigma^- \to n)(4f-1)^2 + f_1(\Xi^0 \to \Sigma^+)(8f^2-4f+14) + f_M(-8f^2+4f-5)\},$$

$$f_2(\Sigma^- \to n) = \sqrt{2}\, f_2(\Sigma^0 \to p) = K\{f_1(\Sigma^- \to n)(-32f^2+32f-9) + 4f_1(\Xi^0 \to \Sigma^+)(f-1) + f_M[(1-f)(1+2f)/3-2+9f-10f^2]\},$$

$$f_2(\Xi^- \to \Lambda) = \frac{K}{\sqrt{6}}\{f_1(\Sigma^- \to n)(56f^2-52f-14) - f_1(\Xi^0 \to \Sigma^+)(1+2f)^2 + f_M(20f^2-16f+5)\},$$

$$f_2(\Xi^0 \to \Sigma^+) = \sqrt{2}\, f_2(\Xi^- \to \Sigma^0) = K\{f_1(\Sigma^- \to n)(-8f^2+12f-4) + f_1(\Xi^0 \to \Sigma^+)(-4f^2+4f-9) + f_M[(1-f)(4f-1)/3+f+2]\}, \quad (8)$$

where the constant f_M has the following unitary structure

$$\langle P|V^\alpha_\beta|P\rangle = f_M(\bar{P}^\alpha_\gamma P^\gamma_\beta - \bar{P}^\gamma_\beta P^\alpha_\gamma)$$

P is an octet of pseudoscalar mesons.

According to the expectations in the present limit the derived "weak magnetism" constants (8) satisfy the correlations predicted by SU(3)-symmetry:

$$f_2(\Lambda \to p) = -\frac{1}{\sqrt{6}}\{2f_2(\Xi^0 \to \Sigma^+) - f_2(\Sigma^- \to n)\},$$

$$f_2(\Xi^- \to \Lambda) = \frac{1}{\sqrt{6}}\{f_2(\Xi^0 \to \Sigma^+) - 2f_2(\Sigma^- \to n)\}.$$

While calculating in the case of experimental masses of baryons and mesons, we also assume that the unitary symmetry predictions are valid for the strong interaction constants.

It is easy to find expressions for constants $f_1(B_1 \to B_2)$ and f_M using Cabibbo's model predictions. Finally, we obtain for the "weak magnetism" constants a system of four linear equations with coefficients representing the functions of parameters f and Λ.

As one can see from Fig. 4, in the range of parameter $f \sim 0.2 \div 0.6$ (with $\Lambda = 3.0$) the signs of the "weak magnetism" constants of β-decays of hyperons agree with the predictions of Cabibbo's model but their magnitudes differ. Thus for $f = 0.4$ we find: $f_2(\Lambda \to p) = -0.091$ (-0.52); $f_2(\Sigma^- \to n) = 0.146$ (0.48); $f_2(\Xi^- \to \Lambda) = -0.090$ (-0.036); $f_2)\Xi^0 \to \Sigma^+) = 0.228$ (0.89), where figures in parenthesis represent the values of constants derived in Cabibbo's model.

Hence the calculated values of constants depend logarithmically upon the unknown generally cutoff parameter Λ, the analysis of ratios of the constants which are not sensitive to the change of parameter Λ in large limits presents a great interest.

Fig. 5 shows the ratios of the "weak magnetism" constants of the hyperon β-decays as functions of the parameter f and also similar values derived in Cabibbo's model (taking

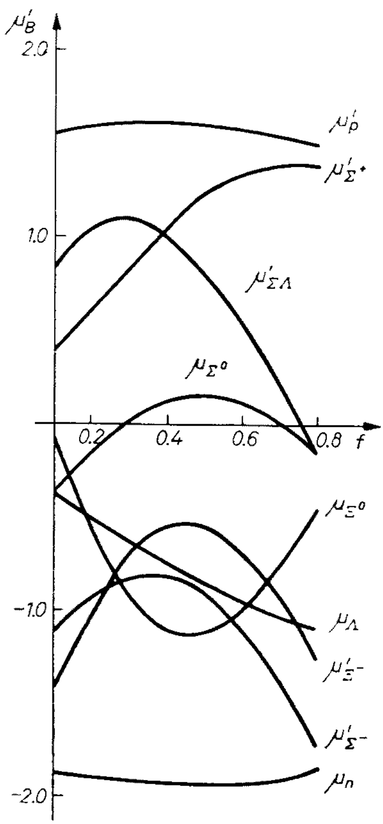

Fig. 4

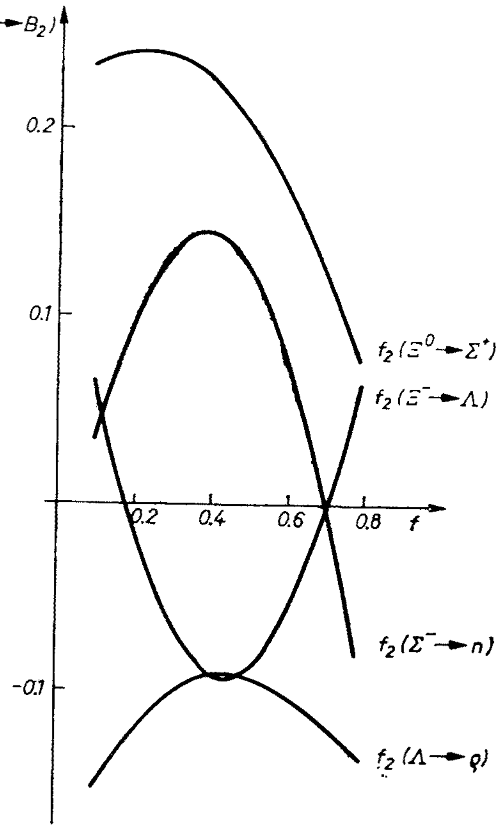

Fig. 5

Fig. 5. $R_1 = f_2(\Lambda \to p)/f_2(\Sigma^- \to n)$, $R_2 = f_2(\Xi^- \to \Lambda)/f_2(\Sigma^- \to n)$, $R_3 = f_2(\Xi^0 \to \Sigma^+)/f_2(\Sigma^- \to n)$; diagrams $a_j(\beta_j)$ are corresponding predictions of Cabibbo's model taking into account corrections for the mass differences of baryons

into consideration (β) and without taking into account (α) the difference of masses of the hyperons). In the region $f = 0.2 + 0.3$ the considered values (except the value of ratio $f_2(\Xi^- \to \Lambda)/f_2(\Sigma^- \to n)$) do not contradict Cabibbo's model predictions (both with taking into account the mentioned corrections and without them).

Thus, the analysis has shown that the sidewise dispersion relations in the limits of plausible assumptions permit one to obtain reasonable predictions for the electromagnetic and weak characteristics of baryons. The calculated values of a.m.m.'s of baryons and "weak magnetism" constants are found to be in disagreement with unitary symmetry predictions.

In conclusion, the author expresses a deep gratitude to Professor M. Rekalo for numerous useful discussions of a number of problems that have been tackled in the present study.

APPENDIX 1

Analytic expressions for the functions have the form

$$E_1(W^2) = \frac{M_1}{W^2 A_1^-}\{2M_2B^- - M_1A_2^- + \mu^2 M_1\},$$

$$E_2(W^2) = \frac{2M_1}{\Delta^+(A_1^-)^2(A_2^+ - \mu^2)}\{\Delta^+ A_1^-(M_1A_2^- - 2M_2B^-) - \mu^2 B^- A_1^+ + \mu^2 A_1^- \Delta^- \Delta^+$$

$$+\mu^4 A_1^+ + 2W^2(-M_2\Delta^+ A_1^- - \mu^2 B^- + \mu^4)Q(z)\},$$

$$K_1(W^2) = -[2\Delta^+ W^2 A_1^-]^{-1}\{-2M_1A_2^- B^- + M_2A_2^- A_1^- + \mu^2 M_2 A_1^+\},$$

$$K_2(W^2) = -[2\Delta^+ A_1^-(A_2^+ - \mu^2)W^2]^{-1}\{M_1A_1^-(A_2^-)^2 + M_1^2\Delta^+(A_2^-)^2$$

$$-2M_2W^2A_1^-A_2^- + 4M_1M_2^2A_1^-A_2^- - 4M_2^3A_1^-B^- + 4M_2^2\Delta^+(B^-)^2$$

$$-4M_1M_2\Delta^+A_2^-B^- + \mu^2[2W^2M_1^2M_2 + B^+(2W^2\Delta^- - 2M_2^2M_1$$

$$+\mu^2M_1)] + 4W^4\Delta^+M_2^2Q(z)\},$$

where

$$\Delta^\pm = M_2 \pm M_1, \qquad A_{1,2}^\pm = W^2 \pm M_{1,2}^2,$$

$$B^\pm = W^2 \pm M_1M_2, \qquad Q(z) = \tfrac{1}{2}\, z \ln\left(\frac{z+1}{z-1}\right) - 1,$$

$$z = (A_2^+ - \mu^2)/\sqrt{(A_2^+ - \mu^2)^2 - 4W^2M_2^2},$$

in the limit $\mu = 0$, $W = M_1 = M_2$:

$$E_1 = -E_2 = 1, \qquad K_1 = K_2 = 0.$$

APPENDIX 2

Analytic expressions for functions $A_{1,2}^{s,t,u}(W^2)$ at the threshold point $W = W_{th} = m_3 + \mu$ are as follows:

1. s-channel

$$A_1^s = -aR_1^s + 2R_2^s,$$

$$R_1^s = m_3(A_2^- + \Delta_{12}^+\Delta_{2s}^+) - \frac{A_3^+ - \mu^2}{2W^2}(B_{2s}\Delta_{12}^+ + m_sA_2^-),$$

$$R_2^s = m_3(2B_{1s} - m_2\Delta_{1s}^+) + \frac{A_3^+ - \mu^2}{2W^2}(m_2B_{1s} - 2W^2\Delta_{1s}^+),$$

$$A_2^s = -\frac{2A_2^-}{\Delta_{2s}^+}\left\{m_3\Delta_{1s}^+ - \frac{B_{1s}}{2W^2}(A_3^+ - \mu^2)\right\}.$$

2. u-channel

$$A_1^u = aR_1^u - 2R_2^u,$$

$$R_1^u = -m_3\Delta_{12}^+\Delta_{3u}^- + \frac{A_3^+ - \mu^2}{2W^2}\left\{-m_uA_2^- - m_2\Delta_{12}^+\Delta_{3u}^+ + \frac{A_3^+ - \mu^2}{2W^2}(2m_2W^2 + m_1A_2^+)\right\},$$

$$R_2^u = m_3[m_u(\Delta_{12}^- + m_1) - m_1m_3 - A_2^-] + \frac{A_3^+ - \mu^2}{2W^2}\{W^2(\Delta_{13}^+ - 2m_u)$$

$$+ m_2[-m_1(\Delta_{3u}^- + \Delta_{23}^+) + A_3^+ - \mu^2]\},$$

$$A_2^u = \frac{2A_2^-}{\Delta_{3u}^+}\left\{m_3m_u + \frac{A_3^+ - \mu^2}{2W^2}\left[\frac{m_1m_2}{2W^2}(A_3^+ - \mu^2) - m_2m_3 - m_1m_u\right]\right\}.$$

3. t-channel

$$A^t = aA_1^t + A_2^t,$$

$$A_1^t = -\frac{A_3^- + \mu^2}{4W^2}\left\{-\frac{A_3^+ - \mu^2}{2W^2}[m_1A_2^+ + 2m_2W^2] + m_3(A_2^+ + 2m_1m_2)\right\},$$

$$A_2^t = m_3(B_{12} + m_3\Delta_{12}^-) - \frac{A_3^+ - \mu^2}{2W^2}(W^2\Delta_{13}^+ + m_2m_3\Delta_{13}^- + m_2\mu^2).$$

Functions $u(W^2)$ and $t(W^2)$ are as follows:

$$u(W^2) = \tfrac{1}{2}\left\{\Delta_{3u}^+\Delta_{3u}^- - \frac{A_3^+ - \mu^2}{2W^2}A_2^-\right\},$$

$$t(W^2) = \tfrac{1}{2}\left\{\mu^2 - \mu_t^2 - \frac{A_3^- + \mu^2}{2W^2}A_2^-\right\}.$$

Where m_1, m_2, m_3, m_s, m_u are the masses of initial, final, intermediate and pole (in s- and u-channels) baryons.

The following designations are introduced:

$$A_j^{\pm} = W^2 \pm m_j^2, \quad B_{ij} = W^2 \pm m_im_j,$$

$$\Delta_{ij}^{\pm} = m_i \pm m_j.$$

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