LETTERS TO THE EDITOR

HELICITY ANALYSIS OF DIFFRACTIVELY PRODUCED STATES IN THE RELATIVISTIC QUARK MODEL

By J. Kłosiński*

P.N. Lebedev Physical Institute, Moscow**

(Received July 2, 1975; revised version received October 9, 1975)

The results of an analysis of possible helicity states, based on the relativistic quark model, for diffractively produced mesonic systems are reported here. The model explains approximate t-channel helicity conservation (TCHC) in pion diffractive dissociation.

1. In the series of experiments performed in the past few years, the partial-wave analysis of low-mass (3π)-enhancements produced in the reactions $\pi^{\pm}p \rightarrow \pi^{\pm}\pi^{+}\pi^{-}p$ was done [1]. Results of these analyses show that the simplest description of the decay angular distributions of definite spin-parity (J^P) states are obtained in a Gottfried-Jackson (G–J), t-channel helicity [2] frame. Fitted values of the spin-density matrix elements are consistent with approximate TCHC in pion dissociation. For a qualitative analysis the essentials are: First, violation of the TCHC in production of the 1^+ and 2^- states is less than 10% (ϱ_{00} is always more than 0.9 and independent of momentum transfer to the proton (t')), and second, production of the 2^+ state is small (about 7% of all events) and occurs in pure spin-projection ± 1 in G–J frame.

In this letter we want to point out that the relativistic quark model predicts observed helicity states of various J^P assignments of (3π).

As existing quark models always have various defects, a general one is used here. It gives, of course, only qualitative but consistent with experiment results.

2. The main assumption is that an incoming meson consists of a (valence quark-antiquark ($q\bar{q}$) pair and contains also an arbitrary number of virtual (loop) $q\bar{q}$-pairs [3]. This assumption applies to the produced state, as we are not interested in its specific decay channels.

---

* On leave from Institute of Physics, University of Łódź, Poland.

** Address: P. N. Lebedev Physical Institute, AN SSSR, Leninskij Prospekt 53, Moscow, USSR.

System of valence and loop quarks can be described by coviariant Bethe-Salpeter (B–S) amplitudes [4], in which only the valence quarks variables explicitly are present, and on variables of the loop quarks summations and integrations are performed. These amplitudes can belong to discrete energy eigenvalues (bound states) or to continuum energy eigenvalues [5]. Definite spin-partity-charge parity (J^{PC}) states are then described by the covariant B–S amplitudes [4]. For some low spin states we have

$$0^{-+};\ \chi(q,P) = \gamma_5(A+qP\not{q}B+\not{P}C+(\not{q}\not{P}-\not{P}\not{q})D), \tag{1}$$

$$1^{++};\ \chi_\mu(q,P) = \gamma_5 T_\mu(A+\not{q}B+qP\not{P}C+(\not{q}\not{P}-\not{P}\not{q})D)-\gamma_5 Q_\mu(B-2\not{P}D, \tag{2}$$

$$2^{++};\ \chi_{\mu\nu}(q,P) = M_{\mu\nu}(A+qP\not{q}B+\not{P}C+(\not{q}\not{P}-\not{P}\not{q})D)-N_{\mu\nu}(2qPB-4\not{P}D), \tag{3}$$

$$2^{-+};\ \chi_{\mu\nu}(q,P) = N_{\mu\nu}\gamma_5(A+qP\not{q}B+\not{P}C+(\not{q}\not{P}-\not{P}\not{q})D), \tag{4}$$

where P is the momentum of given J^{PC} state, $P^2 = M^2$, q — the relative momentum of the valence quarks, $ab = a_0b_0-\vec{a}\vec{b}$, $\not{q} = q_0\gamma_0-\vec{q}\vec{\gamma}$, and

$$T_\mu = \gamma_\mu - P_\mu\frac{\not{P}}{M^2},\qquad Q_\mu = q_\mu - P_\mu\frac{qP}{M^2},$$

$$M_{\mu\nu} = T_\mu Q_\nu + Q_\mu T_\nu - \tfrac{2}{3}\left(g_{\mu\nu}-\frac{P_\mu P_\nu}{M^2}\right)\left(\not{q}-\not{P}\frac{qP}{M^2}\right),$$

$$N_{\mu\nu} = Q_\mu Q_\nu - \tfrac{1}{3}\left(g_{\mu\nu}-\frac{P_\mu P_\nu}{M^2}\right)\left(q^2-\frac{(qP)^2}{M^2}\right).$$

Scalar functions A, B, C, D in (1)–(4) depend on q^2 and $(qP)^2$ and contain all effects of loop quarks. States with definite helicity or spin projections (J_3) are obtained in standard manner [4]. The B–S amplitudes are taken as transition amplitudes describing dissociation of a meson into $q\bar{q}$-pairs.

3. To determine the amplitudes of the diffractive process one should have, according to the Good–Walker hypothesis [6], the amplitudes of elastic scattering of dissociated system. This, in our case, consists of $q\bar{q}$-pairs. Due to the additivity of the quark model, we need to know the elastic scattering amplitudes of the quarks. We parametrize the quark elastic scattering amplitudes as follows:

$$F(qX \to q'X') = \bar{u}(p')[\mathscr{A}(s_q, t)+\not{K}^{+}\mathscr{B}(s_q, t)]u(p) \tag{5}$$

(for antiquark scattering: $u(p) \to v(\bar{p})$, $\mathscr{A} \to \bar{\mathscr{A}}$, $\mathscr{B} \to \bar{\mathscr{B}}$, $K^+_\mu \to K^-_\mu$), where quark (p) and antiquark ($\bar{p}$) momenta are $p = \frac{1}{2}P+q$ and $\bar{p} = \frac{1}{2}P-q$, respectively, (P and q appear in (1)–(4) above) and

$$K^{\pm}_\mu = \frac{(\frac{1}{2}P\pm qP_X)_\mu}{\sqrt{(\frac{1}{2}P\pm q+P_X)^2}} \equiv \frac{(\frac{1}{2}P\pm q+P_X)_\mu}{\sqrt{s_q}}.$$

P_X is the target particle momentum, $\mathscr{A}(s_q, t)$ ($\mathscr{B}(s_q, t)$) is the invariant amplitude of quark elastic scattering without (with) helicity flip averaged (summed) on all possible spin projections of the particle $X(X')$, t is the four-momentum transfer squared and $s_q = \frac{1}{4}s$ for a high energy collision.

The amplitude of a diffractively produced $|J^{PC}, J_3\rangle$ state by πX-scattering is then [7]

$$T(\pi X \to (JJ_3)X')$$

$$= \int \frac{d^4q}{(2\pi)^4} \operatorname{Tr} \{\bar{\chi}_{JJ_3}(q', P')[\mathscr{A}+k^+\mathscr{B}]\chi_\pi(q, P)(-\bar{p}\bar{f}(\bar{p}^2)-m\bar{g}(\bar{p}^2))$$

$$+\bar{\chi}_{JJ_3}(q', P')(pf(p^2)-mg(p^2))\chi_\pi(q, P)[\bar{\mathscr{A}}+k^-\bar{\mathscr{B}}]\}, \quad (6)$$

where $\bar{\chi} = \gamma_0\chi^+\gamma_0$, $q' = q+\Delta/2$, $P' = P+\Delta$ and Δ — four momentum transfer. Two terms in (6) account for quark and antiquark (valence or loop) scattering. The scalar functions $f(p^2)$, $\bar{f}(\bar{p}^2)$, $g(p^2)$, $\bar{g}(\bar{p}^2)$ take into account the effects of additional quarks present apart from scattered one.

4. After substitution of B–S amplitudes into (6) and calculation of the traces we go to the G–J frame, where dependence on φ_q (φ_q — azimuthal angle of the relative momentum of the valence quarks) factorizes. This allows us to analyze the possible values of the helicites (J_3) of the produced J^{PC} states. (Details of these calculations are given elsewhere [8]. Here only the results are given.)
The general expression for the production amplitude of a given $|J^{PC}J_3\rangle$ state is [8]

$$T(\pi X) \to (JJ_3)X')$$

$$= \alpha(J^{PC}, J_3)[F^+(\mathscr{A}+\bar{\mathscr{A}})+F^+(\mathscr{A}-\bar{\mathscr{A}})]$$

$$+\beta(J^{PC}, J_3)[F_1^+(\mathscr{B}+\bar{\mathscr{B}})+F_1^-(\mathscr{B}-\bar{\mathscr{B}})], \quad (7)$$

where $F^\pm$ and $F_1^\pm$ are integrals over the rest of the variables in (6) and depend on J^{PC} and t. Their relative values, for a given J^{PC}, can be estimated by analysis of their algebraic structure. $\alpha(J^{PC}, J_3)$ and $\beta(J^{PC}, J_3)$ are discussed below.

The results can be summarized as follows:
(i) For given J^{PC} states:

— $J^{PC} = 1^{+-}$; $\alpha = 1$ for $J_3 = 0$, $\alpha = 0$ for $J_3 = \pm 1$ and $\beta = 1$ for $J_3 = 0, \pm 1$, with $F_1^- \ll F_1^+$ is obtained. It means that a state with $J_3 = \pm 1$ is produced due to quark helicity flip amplitudes ($\mathscr{B}+\bar{\mathscr{B}}$).

— $J^{PC} = 2^{++}$; $\alpha = 0$ for $J_3 = 0, \pm 1, \pm 2$ and $\beta = 1$ for $J_3 = \pm 1$, $\beta = 0$ for $J_3 = 0, \pm 2$ with $F_1^+ \approx F_1^-$. So, 2^{++} state is produced exclusively in $J_3 = \pm 1$ state and this is entirely due to the quark helicity-flip amplitudes[1].

— $J^{PC} = 2^{-+}$; $\alpha = 1$ for $J_3 = 0$, $\alpha = 0$ for $J_3 = \pm 1, \pm 2$ and $\beta = 1$ for $J_3 = 0, \pm 1$, $\beta = 0$ for $J_3 = \pm 2$ with $F_1^- \gg F_1^+$. Thus, state 2^{-+} is produced only in the states with $J_3 = 0, \pm 1$ and state with $J_3 = \pm 1$ is produced almost entirely due to ($\mathscr{B}-\bar{\mathscr{B}}$) combination of the quark helicity-flip amplitudes.

---

[1] Diffractive production of the 2^+ (and 1^-) states with polarization $J_3 = 0$ is forbidden when natural parity is exchanged in t-channel as here, because we treat the target merely as a spectator [9]. General analysis done in Ref. [9] does not exclude $J_3 = \pm 2$ nor does it give any restrictions on the polarization of other states.

(*ii*) These results generalize to the series of states: 1^{++}, 3^{++}, 5^{++}, ... ,2^{++}, 4^{++}, 6^{++}, ..., 2^{-+}, 4^{-+} 6^{-+}, ...
(*iii*) Obtained results are true for diffraction dissociation of other 0^{-+} mesons, e.g. K^+, K^-.
(*iv*) The photoproduction of vector mesons according to the scheme of the vector dominance model was also considered — a photon goes into the vector meson and then the vector meson scatters elastically. Each transition was calculated by means of relativistic quark model. The results show that there is no TCHC in this process for given polarization of an incoming photon. The final vector meson can have two values of polarization (in G–J frame) with equal weights. Transition amplitudes contain $\mathscr{A}$ and $\bar{\mathscr{A}}$ as well as $\mathscr{B}$ and $\bar{\mathscr{B}}$.

5. From cited results it is seen that in the limit $\mathscr{B} = \bar{\mathscr{B}} = 0$, e.g. for absence of helicity--flip in quark elastic scattering, exact TCHC will be observed (F_1^+ and F_1^- cannot be zero simultaneously). The degree of violation of TCHC as well as the production rate of the 2^+ state crucially depend on the ratio $\mathscr{B}/\mathscr{A}$-helicity flip to non flip quark scattering amplitudes. From experiments with polarized protons [10] it may be inffered (through additivity of quark model), that the ratio $|\mathscr{B}/\mathscr{A}|$ is always a few percent, So, the effects of TCHC violation in diffraction dissociation of 0^- mesons as well as production rate of 2^+ state should be small, as observed by experiment [1], where the contribution of 2^+ states in all (3π) diffractive events and $\pm$ 1 helicity in 1^+ and 2^- states are comparable and small (a few percent). One can conclude that the relativistic quark model can explain the observed helicities in diffractive production of mesonic systems. More quantitative analysis is in progress.

The author wishes to thank Professor N. A. Dobrotin, Professor E. L. Feinberg and all members of their laboratories in FIAN for their hospitality. He also wishes to acknowledge, with pleasure, the stimulating discussions with I. W. Andriejev.

REFERENCES

[1] G. Ascoli et al., *Phys. Rev. Lett.* **26**, 929 (1971); *Phys. Rev.* **D7**, 669 (1973); G. Ascoli, *Proc. of the XVII Int. Conf. on High Energy Physics*, London 1974, p. 11–8; Yu. M. Antipov et al., CERN-IHEP Collab., *Nucl. Phys.* **B63**, 153 (1973); G. Thompson, et al., *Phys. Rev.* **D9**, 560 (1974); G. Otter et al., *Nucl. Phys.* **B80**, 1 (1974).

[2] K. Gottfried, J. O. Jackson, *Nuovo Cimento*, **33**, 309 (1964).

[3] H. J. Lubatti, K. Miyasu, *Nucl. Phys.* **B59**, 525 (1973).

[4] C. H. Llewellyn-Smith, *Ann. Phys.* (USA) **53**, 521 (1969); M. Böhm et al., *Nucl. Phys.* **B51**, 397 (1973); **B69**, 349 (1974); *Phys. Lett.* **50B**, 457 (1974).

[5] N. Nakanishi, *Prog. Theor. Phys. Suppl.* **43**, 1 (1969).

[6] M. L. Good, W. O. Walker, *Phys. Rev.* **120**, 1857 (1960).

[7] N. Byers, *Acta Phys. Pol.* **B3**, 889 (1972).

[8] J. Kłosiński, to be published in *Sov. Yad. Phys.*

[9] P. Dennery, A. Krzywicki, *Phys. Rev.* **136**, B839 (1964).

[10] *Proc. of the XVII-th Int. Conf. on High Energy Physics*, London 1974: M. B. Wicklund, p. I–176; A. Yokosava, p. I–21; S. Narushev, p. I–25.