REGGE BEHAVIOUR AND BJORKEN SCALING FOR DEEP-INELASTIC LEPTON-HADRON SCATTERING PROCESSES

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(Received January 30, 1976)

Within the framework of the Jost-Lehmann-Dyson (JLD) representation and the renormalization-group (RG) equation, it is shown that either the RG technique is not applicable to deep-inelastic phenomena or Regge behaviour and Bjorken scaling for structure functions do not coexist.

1

In recent years the scaling laws for structure functions of deep-inelastic lepton-hadron scatterings have challenged theorists to elucidate adequately these new phenomena. The parton models, based on different assumptions, have provided good fits to experimental data [1]. It is of great importance to consider if these laws can be acceptable within the framework of the quantum field theory (QFT). Up to now, perhaps, three are two trends which are more fascinating than others. The first one is concerned with the asymptotic freedom of gauge fields discovered recently by Politzer [2], and Gross and Wilczek [3]. In particular, the latter two prove that Bjorken scaling, up to logarithmic terms, is possible only in asymptotically free field theories. The second trend is closely connected with the pioneering paper of Bogolubov and his co-workers [4]. Starting from the JLD representation they show that automodel behaviour is compatible with the general principles of QFT.

In addition to Bjorken scaling, the Regge behaviour [5, 6] for structure functions is of great interest, too. Especially, De Rujula and his co-workers [6] suggest that maybe structure functions do not exhibit the desired Regge behaviour.

In this note, combining several results of the general considerations performed for two-body scattering processes [7–9] we obtain some interesting conclusions specified for deep-inelastic phenomena.

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2

To begin with, let us consider the deep-inelastic electron-nucleon scattering. Its cross sections are determined by the Fourier transform of the commutator

$$W_{\mu\nu}(q, p) = \frac{1}{8\pi} \sum_{\sigma} \int \langle p, \sigma | \left[J_\mu\left(\frac{x}{2}\right), J_\nu\left(-\frac{x}{2}\right)\right] | p, \sigma \rangle e^{iqx} dx$$

in which $J_\mu(x)$ are the electromagnetic current components, q is 4-momentum of virtual photon, $q^2 < 0$ and $|p, \sigma\rangle$ is nucleon state with 4-momentum p of mass M and spin σ. The nucleon state is normalized as follows

$$\langle p, \sigma | p', \sigma' \rangle = 2p_0(2\pi)^3 \delta(\vec{p} - \vec{p}')\delta_{\sigma\sigma'}.$$

Following Bogolubov [4], the invariant causal structure functions $F_i(q, p)$ $(i = 1, 2)$ are introduced

$$F_1(q, p) = \frac{1}{M^2} p^\mu p^\nu W_{\mu\nu}(q, p),$$

$$F_2(q, p) = F_1(q, p) - \sum_{\mu} g_{\mu\mu} W_{\mu\mu}(q, p).$$

From the microcausality and spectral conditions one derives that

I. $$\tilde{F}_i(x, p) = \int dq e^{iqx} F_i(q, p) = 0 \quad \text{for} \quad x^2 < 0$$

II. $$F_i(q, p) = 0 \quad \text{if} \quad -q^2/|2qp| > 1.$$

The Lorentz covariance of $F_i(q, p)$ $(i = 1, 2)$ enables us to work only on the rest frame where $p = (M, \vec{0})$. For the sake of simplicity, the index i will be omitted in what follows. Then it is proved [4] that there exists uniquely determined tempered distribution $\psi(\lambda^2, M\vec{\varrho})$ such that the JLD representation is valid

$$F(q) = \int \varepsilon(q_0)\delta[q_0^2 - (\vec{q} - M\vec{\varrho})^2 - \lambda^2]\psi(\lambda^2, M\vec{\varrho}) d\lambda^2 d\vec{\varrho}.$$

The support of $\psi(\lambda^2, M\vec{\varrho})$ is contained in the domain

$$\{\varrho = |\vec{\varrho}| \leqslant 1, \quad \lambda^2 \geqslant (1 - \sqrt{1 - \varrho^2})^2\}.$$

In reality $\psi(\lambda^2, M\vec{\varrho})$ depends upon $\vec{\varrho}$ via ϱ, i.e.,

$$\psi(\lambda^2, M\vec{\varrho}) \equiv \psi(\lambda^2, M\varrho).$$

The asymptotic properties of $F(q)$ are considered respectively in the Regge and Bjorken regions:

$$\nu = 2qp \to +\infty, \; q^2 \text{ fixed},$$

$$\nu = 2qp \to +\infty, \; \xi = -q^2/\nu \text{ fixed}.$$

As it was indicated in [8, 9] we have a one-to-one correspondence between the asymptotic behaviours of structure functions and the conditions imposed on the weight functions. Namely, one obtains:

1) Regge behaviour $F(\nu, q^2) \approx \nu^\alpha(\ln \nu)^\beta f(q^2)$ corresponds to the condition

$$\psi(\lambda^2, kM\varrho) \approx k^{-(\alpha+3)}\left(\ln \frac{1}{k}\right)^\beta \psi_R(\lambda^2, M\varrho) \tag{2.1}$$

as $k \to +0$. (2.1) means that $\psi(\lambda^2, M\varrho)$ possesses a singularity at $\varrho = 0$.

2) The asymptotic form $F(\nu, \xi) \approx \nu^\alpha(\ln \nu)^\beta h(\xi)$ corresponds to

$$\psi(r\lambda^2, M\varrho) \approx r^\alpha(\ln r)^\beta \psi_{Bj}(\lambda^2, M\varrho). \tag{2.2}$$

In the case when $\alpha = 0$, $\beta = 0$ we have exact Bjorken scaling and when $\alpha = 0$, $\beta \neq 0$, Bjorken scaling, up to logarithmic term, holds. Remember that (2.1) and (2.2) should be understood in the sense of the distribution theory, i.e., for arbitrary test functions $\varphi_1(\lambda^2)$ and $\varphi_2(\varrho)$ we have, respectively, the limit relations

$$\frac{1}{k^{-(\alpha+3)}\left(\ln \frac{1}{k}\right)^\beta} \int d\lambda^2 d\varrho\varrho^2 \psi(\lambda^2, kM\varrho)\varphi_1(\lambda^2)\varphi_2(\varrho) \xrightarrow[k\to+0]{} \int d\lambda^2 d\varrho\varrho^2 \psi_R \times (\lambda^2, M\varrho)\varphi_1(\lambda^2)\varphi_2(\varrho), \tag{2.3}$$

$$\frac{1}{r^\alpha(\ln r)^\beta} \int d\lambda^2 d\varrho\varrho^2 \psi(r\lambda^2, M\varrho)\varphi_1(\lambda^2)\varphi_2(\varrho) \xrightarrow{r\to+\infty} \int d\lambda^2 d\varrho\varrho^2 \psi_{Bj}(\lambda^2, M\varrho)\varphi_1(\lambda^2)\varphi_2(\varrho). \tag{2.4}$$

Next let us make use of the RG approach to investigate the ultraviolet structure of $F(q)$. For simplicity, suppose we are dealing with the one-charge case. Let $\mathscr{Z}_2$ be the renormalization constant of the nucleon wave-function, then the RG equation is established [7] to be

$$\left(\mu^2 \frac{\partial}{\partial\mu^2} + \beta(g)\frac{\partial}{\partial g} - 2\gamma(g)\right)\chi(\lambda^2, M; g, \mu^2) = 0, \tag{2.5}$$

where μ^2 is the subtraction point, g is the coupling constant,

$$\beta(g) = \mu^2 \frac{\partial g}{\partial \mu^2}, \qquad \gamma(g) = \tfrac{1}{2}\mu^2 \frac{\partial \ln \mathscr{Z}_2}{\partial \mu^2}$$

and

$$\chi(\lambda^2, M; g, \mu^2) = \int dv d\varrho\varrho^2 \varphi_1(v)\psi(\lambda^2 v, M\varrho)\varphi_2(\varrho). \tag{2.6}$$

Note that due to (2.3) and (2.4) the RG equation should not be established directly for the weight function.

The dimensional analysis provides

$$\chi(\lambda^2, M; g, \mu^2) = \chi\left(\frac{\lambda^2}{\mu^2}, \frac{M^2}{\mu^2}; g\right).$$

Let us put $x = \lambda^2/\mu^2$, $y = M^2/\mu^2$; Eq. (2.5) turns out to be

$$\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} - \beta(g)\frac{\partial}{\partial g} + 2\gamma(g)\right)\chi(x, y; g) = 0. \tag{2.7}$$

With the aid of (2.7) the asymptotic behaviour of $\chi(x, y; g)$ is investigated in the ultraviolet domain

$$\lambda^2 \gg \mu^2 \gg M^2.$$

Then the variable y drops out, this implies that $\chi(x, y; g)$ behaves like

$$\chi(x, y; g) \approx \chi(x, 0; g). \tag{2.8}$$

However, owing to (2.1) the limit relation (2.8) is impossible as long as $\psi(\lambda^2, M\varrho)$ possesses at $\varrho = 0$ singularity, i.e., Regge behaviour occurs. In the opposite case when $\psi(\lambda^2, M\varrho)$ is regular at $\varrho = 0$, (2.8) is correct and then Eq. (2.7) is powerful enough to give the solutions in the ultraviolet region and all that has been found in [7]. For convenience,

TABLE I

$\lim_{\tau\to+\infty} \bar{g}(\tau) = g_\infty < +\infty$	$I_0 < +\infty$	$I_0 = \infty, I_1 < +\infty$	$I_0 = \infty, I_1 = \infty, I_2 < +\infty$
$g_\infty \neq 0$	const.	$r^{-2\gamma(g_\infty)}$	$r^{-2\gamma(g_\infty)}\varphi(\ln r)$
$g_\infty = 0$			$\varphi(\ln r)$

TABLE II

$g_\infty = +\infty$	Normal case $\gamma(\bar{g}) \approx -v\beta(\bar{g})/2\bar{g}$	Abnormal case $\gamma(\bar{g}) \approx -v\beta(\bar{g})/2\bar{g} - w\beta(\bar{g})/2$
$\bar{g}(\tau) \sim (\ln\tau)^{1/n}, n > 1$	$(\ln\ln r)^{v/n}$	$(\ln\ln r)^{v/n}\exp(w(\ln\ln r)^{1/n})$
$\bar{g}(\tau) \sim \tau^{1/n}, n > 0$	$(\ln r)^{v/n}$	$(\ln r)^{v/n}\exp(w(\ln r)^{1/n})$
$\bar{g}(\tau) \sim \exp\tau^{1/n}, n > 1$	$\exp(v(\ln r)^{1/n})$	$\exp[v(\ln r)^{1/n}]\exp[w\exp((\ln r)^{1/n})]$
$\bar{g}(\tau) \sim \exp(\varphi_0\tau)$	$r^{v\varphi_0}$	$r^v\exp(wr^{\varphi_0})$

the corresponding forms for $\psi(\lambda^2, M_\varrho)$ are listed in Tables I and II[1]. We see that exact Bjorken scaling is possible only when $I_0 < +\infty$ and Bjorken scaling, up to the logarithmic factor, can occur in several cases.

3

From the foregoing discussions we arrive at the following conclusions:

1. If the existence of Bjorken scaling is not derived from the RG equation, then Regge behaviour and Bjorken scaling can possibly coexist.
2. If the RG equation is applicable, then Regge behaviour and Bjorken scaling cannot coexist. And in this case, in contrast with Gross and Wilczek [3], it is indicated that an exact scaling law as well as a scaling law within possible logarithmic factors is possible in the RG point of view.

It is obvious that the generalization of classes of deep-inelastic lepton-hadron scattering processes is straightforward.

REFERENCES

[1] W. Królikowski, J. Bartelski, *Nuovo Cimento* **21A**, 265 (1974); K. Kitani, H. Yoshii, *Nuovo Cimento Lett.* **4**, 431 (1970).
[2] H. D. Politzer, *Phys. Rev. Lett.* **30**, 1346 (1973).
[3] D. J. Gross, F. Wilczek, *Phys. Rev.* **D8**, 3633 (1973); **9**, 980 (1973).
[4] N. N. Bogolubov, A. N. Tavkhelidze, V. S. Vladimirov, JINR, P2-6342, E2-6490, Dubna 1972.
[5] C. A. Orzalesi, *Phys. Rev.* **D6**, 3702 (1972).
[6] A. De Rujula, *Phys. Rev.* **D10**, 1649 (1974).
[7] Tran huu Phat, *Nuovo Cimento* **28A**, 337 (1975).
[8] Tran huu Phat, *Nuovo Cimento* **28A**, 364 (1975); **30A**, 47 (1975); *Acta Phys. Austriaca* (in press).
[9] Tran huu Phat, *Acta Phys. Hung.* (in press); *Acta Phys. Austriaca* (to be published).

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[1] In Tables I and II $\bar{g}(\tau)$ is the invariant coupling constant fulfilling the equation

$$\frac{\partial}{\partial\tau}\bar{g}(\tau, g) = \beta(\bar{g}), \bar{g}(0, g) = g; \tau = \ln r$$

and I_0, I_1, L_2 are the following quantities:

$$I_0 = \int_0^{+\infty} d\tau\gamma(\bar{g}(\tau)), I_1 = \int_0^{+\infty} d\tau(\gamma(\bar{g}(\tau)) - \gamma(g_\infty)), I_2 = \int_0^{+\infty} d\tau[\gamma(\bar{g}(\tau)) - \gamma(g_\infty) - (\bar{g} - g_\infty)\gamma'(g)].$$