ON THE ANGULAR DISTRIBUTION OF SPECTATOR NUCLEONS IN HIGH-ENERGY COLLISIONS WITH DEUTERIUM NUCLEI*

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Angular distributions of spectator nucleons in collisions of high-energy particles with deuterium nuclei are discussed in the framework of the impulse model. Comparison with experimental data shows that predictions following from this simple theoretical model are verified by experiment. Some general remarks on the study of angular distributions of spectator nucleons are given.

Collisions of high-energy particles with deuterium nuclei are usually interpreted in terms of the so-called impulse model [1]. According to this model, only one nucleon of the deuteron takes part in the interaction, the other nucleon being only a "spectator". This approximation is justified by the relatively large average distance between the nucleons forming the deuteron[1] and by its low binding energy. Approximate treatment is required in connection with difficulties of the exact calculation within the frame of the strict relativistic theory. According to the impulse model, the spectator nucleon retains its original Fermi-momentum, this being opposite to the Fermi-momentum of the other nucleon which was hit by the incident particle.

In experiments carried out in deuteron-filled bubble chambers one usually considers as "spectators" protons with momenta

$$80 < p_s < 200 \text{ or } 300 \text{ MeV}/c.$$

The lower limit corresponds to the proton track approximately 1 mm long (visual detection limit), and the upper one — to the limit of validity of the Hulthén [2], or similar [3], function $\Phi(p)$ used to describe the proton momentum spectrum (above 200 MeV/c one observes an excess of detected protons over the theoretical distribution). In experiments

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[1] Expectation value of the n–p separation in the deuteron is about 4 fm, thus largely exceeding the range of interaction, and being also much larger than the wave length of incident particles in experiments in the GeV range.

using beams of accelerated deuterons one can observe spectator protons beginning from zero momentum in the deuteron rest frame, spectator being, under these circumstances a fast, well measurable particle [4]. In events with more than one proton in the final state it seems to be a correct procedure to assume that each of them could be a spectator and should be taken as such with an appropriate weight. Often, however, the proton which is the slowest in the deuteron rest frame is simply taken as spectator.

One is sometimes confronted, especially in earlier papers, with a statement that isotropic angular distribution of the spectator nucleon follows from the impulse model, and that observation of such a distribution in an experiment constitutes a confirmation of the applicability of this model. It has been shown by several authors [5–7] that such a statement is, in general, wrong. In connection with the growing interest in the investigations of the interactions with composite systems, of which deuteron being the simplest, it seems worthwhile to discuss this problem more in detail.

One expects the Fermi-momentum distribution in a stationary, unpolarized nucleus to be isotropic. In a case of collision of an incident particle with one of the nucleons in the deuteron one should, however, take into account two factors which can modify this distribution. These factors are: the flux factor and the energy dependence of cross-sections.

The flux factor, which enters the general formula for the collision probability, depends on the relative velocity of the colliding particles, and is given by the formula [8, 9]

$$F = \frac{\varrho_1 \varrho_2}{m_1 m_2} \sqrt{(p_1 p_2)^2 - m_1^2 m_2^2}\,,$$

where ϱ_i are the "static" particle densities, m_i — their masses, and $p_i = (E_i, \vec{p}_i)$ — their four-momenta. Introducing the angle ϑ_n between the direction of the neutron Fermi--momentum in the deuteron and the collision axis, one obtains

$$F = \frac{\varrho_1 \varrho_2}{m_1 m_2} \sqrt{(E_1 E_2 - p_1 p_2 \cos \vartheta_n)^2 - m_1^2 m_2^2}$$

or, in case when $m_1^2 m_2^2$ is much smaller than the first term under the square root,

$$F \approx \frac{\varrho_1 \varrho_2}{m_1 m_2} E_1 E_2 (1 - \beta_1 \beta_2 \cos \vartheta_n),$$

where β_i are the velocities of colliding particles, in units of c. Taking into account that $\beta_1 \approx 1$, $\beta_2 = \beta_p = \beta_n$, and also that Fermi-momenta of the proton and the neutron in the deuteron rest system are opposite to each other, $\cos \vartheta_p = -\cos \vartheta_n$, one finds

$$F \sim C(1 + \beta_p \cos \vartheta_p)$$

where C is a constant. The angular distribution of spectator protons will have this shape, which means an excess of spectators going going forwards in the laboratory frame (in case of deuteron target). The quantity β_p, which plays a role of the asymmetry coefficient, should be averaged over the Fermi-momentum distribution in the deuterium nucleus

$$\beta_p = \int_{p_1}^{p_2} \beta_p p^2 \Phi^2(p) dp.$$

The obtained asymmetry depends only on the Fermi-momentum distribution, and not on the energy or nature of the incident particle. Assuming the Hulthén distribution, one obtains asymmetry of 0.14 for spectators with momenta between 80 and 300 MeV/c, and asymmetry of 0.09 for spectators within the entire momentum range from 0 to 300 MeV/c [9].

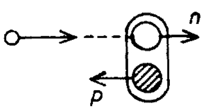
The second factor which influences the angular distribution of spectators is the energy dependence of the cross-section of the investigated process. In a collision of an incident particle with a moving nucleon, the effective collision energy depends on the magnitude and direction of the target nucleon Fermi-momentum [10]

$$E_{\mathrm{CM}} = \sqrt{m_1^2+m_2^2+2m_1m_2\gamma_r},$$

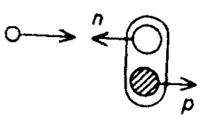
where

$$\gamma_r = \gamma_1\gamma_2-\sqrt{\gamma_1^2-1}\,\sqrt{\gamma_2^2-1}\cos\vartheta.$$

This energy varies within rather wide limits, corresponding to the two limiting situations



$$\gamma_r = \gamma_r^{\min} = \gamma_1\gamma_2-\sqrt{\gamma_1^2-1}\,\sqrt{\gamma_2^2-1} \quad \text{(spectator backwards)}$$



$$\gamma_r = \gamma_r^{\max} = \gamma_1\gamma_2+\sqrt{\gamma_1^2-1}\,\sqrt{\gamma_2^2-1} \quad \text{(spectator forwards)}$$

For target nucleons with 200 MeV/c momentum the spread in collision energy attains 10% of the nominal energy, E^0_{CM}, corresponding to the collision of the incident particle with a nucleon at rest. This spread can be cut down to about 4% by selecting only events with $p_s < 80$ MeV/c, but then the spectator angular distribution can be fully investigated only in an experiment with accelerated deuterons (in a standard set-up such spectators are not detected). This situation is well illustrated in Fig. 1, taken from the review paper [11].

In many cases the cross-section of the investigated process varies significantly over the energy interval and this will influence the angular distribution of spectators. Results of calculations which take into account both factors (i.e. flux of colliding particles and energy dependence of cross-sections) are shown in Fig. 2, taken from Ref. [12]. Using the Hulthén function, these calculations were performed separately for spectator momenta in the interval from 80 to 200 MeV/c (Fig. 2a) and from 0 to 80 MeV/c (Fig. 2b). The parameter, n, given with the curves, is the exponent in the parametrization of the energy dependence of cross-sections in the form

$$\sigma = \sigma_0 p_{\mathrm{LAB}}^{-n}.$$

It can be seen from the figure that the angular distribution of spectators is expected to be isotropic only for $n = 1$ i.e. for cross-sections falling with energy as p_{LAB}^{-1} (in this case both considered effects compensate each other). For all values of n different from one, asymmetry of the angular distribution of spectator nucleons is expected to increase with spectator momentum.

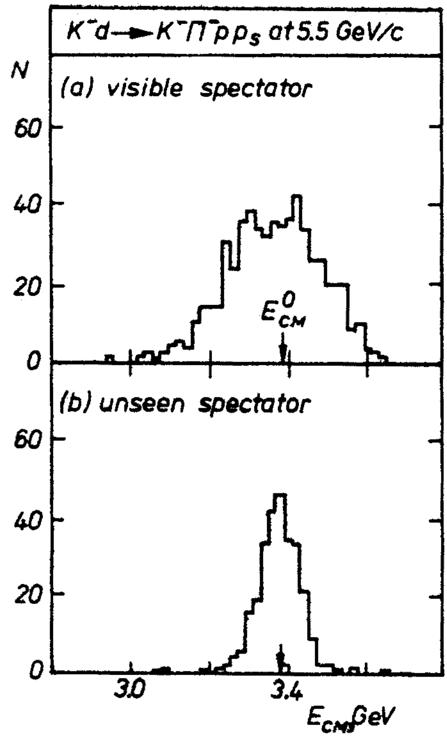


Fig. 1. CMS energy distribution in the reaction $K^-n \to K^-\pi^-p$ at 5.5 GeV/*c* [11]. (a) — events with visible spectators, (b) — events with unvisible spectators

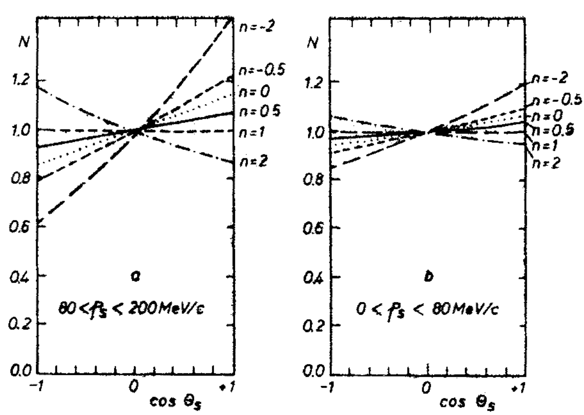


Fig. 2. Angular distribution of spectator nucleons as function of the energy dependence of cross-sections (see text) for $80 < p_s < 200$ MeV/*c* (a), and $0 < p_s < 80$ MeV/*c* (b) [12]

We shall now give some examples of the comparison of the above predictions with experimental data.

1. "diffractive" processes — $\sigma \approx$ const. — spectator angular distribution with forward excess

   examples: $\pi^+ d \to pp_s\pi^+\pi^-\pi^0$ at 1.5 GeV/c [7],

   $dp \to (np)p$ at 3.3 GeV/c [4],

2. processes with cross-section falling with energy as p_{LAB}^{-1}, spectators emitted isotropically

   examples: $K^+d \to K^+\pi^- pp_s$ at 8.2 GeV/c [13],

   $\bar{p}d \to \bar{p}np_s\pi^-$ at 14.6 GeV/c [14],

3. processes with cross-section falling with energy faster than p_{LAB}^{-1}, spectator angular distribution with backward excess

   example: $\pi^+ d \to pp_s\pi^+\pi^-$ at 2.7 GeV/c [15].

Wider analysis of existing experimental data turns out to be difficult because of the scarcity of information about energy dependence of cross-sections on the neutron. It seems, however, that the simple theory considered reproduces basic features of the experimental distributions.

One can therefore formulate the following conclusions:

1. Angular distributions of spectator nucleons contain interesting information and one should therefore try to obtain them with the maximum possible care (we shall call attention to losses of slow spectator tracks at small forward angles, near 180°, and also of steep tracks).

2. Inclusive angular distribution of spectators in high-energy interactions in deuterium should exhibit an asymmetry with the forward excess, common to all inclusive reactions with total cross-sections which are approximately constant. Finding such a shape may be considered as a check of the experimental procedure used, and of the lack of appreciable biases in the detection of spectators.

3. Because of generally different energy behaviour of cross-sections for various reaction channels (diffractive, meson exchange, etc.) one should study the corresponding spectator angular distributions separately. This has been, to our knowledge, pointed out for the first time in Ref. [16].

Finally, one should note that the simple picture of the impulse approximation discussed here does not take into account the fact that the target nucleon in deuterium is not on its mass-shell which, strictly speaking, leads to non-conservation of energy in the entire system. Taking into account the virtuality of the target nucleon should not, however, have an appreciable influence on the angular distributions.

Evaluation of final-state interactions which might lead to the appearance of high--momentum spectators (non-Hulthén tail in the momentum distribution) is beyond the scope of the present report.

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Note

The author's stay in Aachen (RWTH, III Phys. Institute) has permitted him to become acquainted with an extensive study of the spectator angular distribution performed in Ref. [17]. The reaction studied was $\gamma d \rightarrow pp_s\pi^-$ in the energy range from 0.15 to 2 GeV (continuous bremsstrahlung spectrum from the DESY electron synchrotron). Over this energy range the reaction cross-section first rises steeply, then falls down, and finally levels-off. The spectator angular distribution was studied in three intervals of incident energy and also in five intervals of spectator momentum. The experimental histograms show highly varying behaviour. Theoretical distributions were obtained assuming deuteron wave function with 7% admixture of the D-state, and taking into account the virtuality of the target neutron which modifies the effective collision energy. Overall agreement with model predictions was found for spectator momenta up to about 300 MeV/*c*, and after introducing the spectator weighting — up to 400 MeV/*c* or even higher.

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