ELASTIC SCATTERING AND DISINTEGRATION OF COMPOSITE PARTICLES IN HIGH ENERGY COLLISIONS

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(*Received April 26, 1976*)

A semi-phenomenological analysis of high energy collisions of loosely bound composite particles suggests introducing form factors whose analytic form depends on the reference frames privileged by the structure of the wave functions of such particles. Cross-sections of elastic scattering and disintegration processes of composite particles are discussed as possible tests of the proposed scheme.

1. *Introduction*

Attention will be focused on the description of loosely bound composite particles participating in high energy collisions. In order to avoid several difficulties irrelevant to our purpose, let us assume that a composite particle C interacts weakly with a point particle A. This justifies using the first order Born approximation. Let us further assume that the interaction of A with C is known and additive, much like the electromagnetic interaction of an electron with light nuclei. The recent SLAC experiment [1] on elastic electron-deuteron scattering shows that such a simple model is surprisingly realistic up to very high momentum transfer $t = 6(\text{GeV}/c)^2$, whereas more sofisticated ones, like those based on meson-exchange currents or other mechanisms which enable one to divide the momentum transfer approximately equally between two nucleons [2], turned out to be wrong. Thus the problem consists in properly describing the structure of C for very high energies and momentum transfers. The main point is that regardless of the detailed form of the internal wave functions of C, the corresponding form factors of C become modified when compared to the standard form factors based on the phenomenology of the Feynman diagrams applied to non-point particles. This modification originates in the three-dimensional, or rather canonical symmetry of quantum mechanics as opposed to the four-dimensional relativistic symmetry of field theory. Let us remember that the relationship between these

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symmetries remains even more vague for particles of finite sizes. Besides a number of old papers on this subject, e.g. [3], a great number of contributions have been made, particularly by Soviet physicists (e.g. [4, 5] and others [6]) currently working in this field. They generally treat this problem by establishing relativistic analogues of non-relativistic (NR) wave functions and potentials — called quasi-potentials — which should account for "instantaneous" interaction. Since four-dimensional symmetry introduces additional degrees of freedom (relative times) which are "nonphysical" from the viewpoint of quantum mechanical symmetry, as is well known from the Bethe-Salpeter equation [7], we must impose certain constraints to get rid of them. For example, in the case of the two-body problem, one often makes use of the condition

$$Px = \boldsymbol{P}\boldsymbol{x} - E\Delta t = 0, \tag{1.1}$$

where $P = (\boldsymbol{P}, iE)$ is the total four-momentum, and $x = (\boldsymbol{x}, i\Delta t)$ is the relative four--coordinate of the constitutents. The infinite momentum approximation [8], instead of (1.1), makes use of the vanishing of the Dirac variable x_+. Note that as these constraints do not represent any boundary condition of covariant equations of motion they restrict the very space of solutions. It is shown [5] that Eq. (1.1), which favours the cm-system simultaneity, enables us to regain Schroedinger-like equations, whose solutions are single--time functions generalizing in a smooth way the quantum mechanical nonrelativistic wave functions. Such equations of motion are often called semi-relativistic equations [9], because they violate the relativistic covariance while preserving the relativistic kinematics.

In this situation one is tempted to ask: Does the distinction between the cm-system and all others, implied by condition such as Eq. (1.1), and the quantum mechanical symmetry thus regained involve modification of the standard theory, or does it only provide another presentation of this theory — a distinction without a difference. It seems that the investigation of high energy collisions of composite particles, i.e. at least a three-body problem, can throw some light upon this alternative.

Let us assume that the particle C is composed of two scalar and point particles, "1" and "2", in the internal ground state ψ_0. The latter must be parametrized by three invariants independent of the equation of motion which results in ψ_0, such that

$$\psi_0 = \psi_0(x^2, P_c x, P_c^2), \tag{2.1}$$

where $x = x_2 - x_1$ and P_c is the four-momentum of C. Moreover, the variable $P_c x$ must be present between the arguments of ψ_0, because otherwise ψ_0 would be a form invariant function in space-time continuum: $\psi_0 = \psi_0(x^2)$. This, however, conflicts with any conceivable picture of the particle, because ψ_0 would then *remain constant on the hyperbolae* $\boldsymbol{x}^2 - (\Delta t)^2 = \text{const.}$, and such a "shape" of C would *remain identical in all reference frames*, thus being free of any relativistic distortion due to the motion of C as a whole. In order to deal with some definite model of the wave function we resort to quantum mechanical considerations based on the semi-relativistic equation. For loosely bound C the latter coincides with the NR Schroedinger equation for the relative space coordinates of "1" and "2", which describes perfectly the real situation.

In papers [10, 11] we have proposed a geometrical framework, which brings to the utmost the relativistic symmetry breaking of the semi-relativistic equations, and results in internal wave functions which do not depend *a priori* on the relative time variables. Here the relativization of the description of the system takes place only *a posteriori*, i.e. on the level of c-number characteristics of a realized state of this system. The relativistic covariance does not necessarily hold on the level of q-number equations of motion. The semi-relativistic equations provide us with a suitable illustration of such a situation. The c-number argument r of the realized, spherically symmetric state ψ_0 of C obtained from the semi-relativistic equation can be identifield, as a c-number, with the relativistic (c-number) invariant, i.e. it can be relativized according to the equality

$$r^2 = x^2-(P_c x)^2/P_c^2 = x^2|_{S_c}, \tag{3.1}$$

where S_c is the rest-frame of C. Note that the relativization (3.1) of r^2 requires P_c to be the c-number four-momentum of C: $P_c^2 = -m_c^2$, where m_c is the rest-mass of C in the state ψ_0. Consequently, apart from the normalization factor, the relativization of ψ_0 means to replace the absolute distance r between the constituents of C by the invariant quantity from the right hand side of Eq. (3.1):

$$\psi_0(r^2) = \psi_0[x^2+(P_c x)^2/m_c^2]. \tag{4.1}$$

Thus obtained wave function accounts for the internal structure of C in any reference frame, and it is a particular case of the wave function (2.1). We see that ψ_0 depends on the variable $P_c x$, which makes its analytic form dependent on the reference frame; ψ_0 is not a form invariant function in space-time. The so called static approximation provides us with a similar prescription for the boost of the wave function [12]. If the z-axis is taken parallel to the velocity $\boldsymbol{v}_c$ of C, then

$$\psi_0 = \psi_0[x^2+y^2+\gamma_c^2(z-v_c \Delta t)^2], \tag{4.1'}$$

where $x = (x, y, z, i\Delta t)$, and $\gamma_c = (1-v_c^2)^{-1/2}$ is the Lorentz factor of C. Thus ψ_0 accounts for the Lorentz contraction of C, and remains determined for an arbitrary value of Δt as the static shape in S_c expressed in the relative coordinates. In momentum language, Eq. (4.1) means that the fourth component of the relative four-momentum p_c of the constituents of C disappears in S_c, i.e.

$$P_c p_c = 0, \tag{5.1}$$

which is an often used constraint strictly connected with the semi-relativistic approach.

In what follows we will use the wave functions defined in (4.1), but, at least the qualitative results we obtain are independent of the detailed structure of ψ_0. The essential point is that no wave function can ever be a form invariant function in space-time.

For the sake of simplicity we confine ourselves to the spherically symmetric wave function of C in S_c. Nonspherically symmetric functions require a rather involved spin algebra, although their projection onto the Lorentz space-time (relativization) also remains a well defined procedure.

2. Elastic form factors

Let us consider the collision process between A and C, which so far have been assumed to be scalar and point particles. Moreover, we assume that another scalar particle "μ" exchanged between A and C accounts for the "known" interaction between these particles. In the assumed first-order Born approximation the corresponding matrix element is equal to

$$S_{fi} = g_a g_c (2E_a 2E_c 2E'_a 2E'_c)^{-1/2} \int d^4x \int d^4y \Delta^F(x-y;\mu)$$
$$\times \exp[i(P_a - P'_a)y + i(P_c - P'_c)y], \qquad (1.2)$$

where $g_{a,c}$ are the coupling constants, and $E_{a,c}$ and $E'_{a,c}$ are the initial and final energies of the corresponding particles — Fig. 1a.

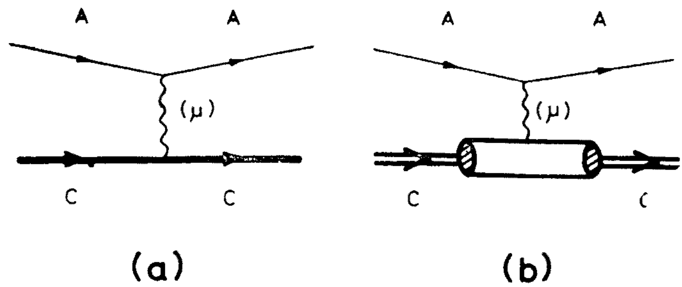

Fig. 1. μ-exchange graph of (a) point particles A, C, and (b) composite particle C scattered from the point particle A

Now let us assume that C becomes an extended particle composed of two scalar and point particles, "1" and "2", of the same masses m in the bound state ψ_0. Denoting the four coordinates of "1" and "2" by $x_{1,2}$, and assuming that ψ_0 is the ground state of mass $m_c = 2m - B$ ($B > 0$ is the binding energy of C), one obtains from (4.1)

$$\psi_0 = \psi_0[x^2 - (P_c x)^2/P_c^2], \quad x = x_2 - x_1, \quad P_c^2 = -m_c^2. \qquad (2.2)$$

Let us assume that A interacts with C through the same particle "μ" exchanged between A and the constituent "2", while "1" does not interact directly with A. This corresponds with the assumed additivity of the interaction of A with C. The generalization of (1.2) onto C with an internal structure consists in introducing a suitable form factor of C. Much as in the case of an electron interacting with a nucleon [13], and in accordance with the Feynman diagram from Fig. 1b, the corresponding matrix element takes the following form

$$S_{fi} = g_a g_c (2E_a 2E_c 2E'_a 2E'_c)^{-1/2} \int d^4x_1 \int d^4x_2 \int d^4y \Delta^F(x_2 - y;\mu)$$
$$\times \exp\{i[P_c(x_1+x_2)/2 + P_a y - P'_c(x_1+x_2)/2 - P'_a y]\} F(x_2 - x_1), \qquad (3.2)$$

where $(x_1+x_2)/2$ means the global centre of gravity coordinate of C. If C tends to a point particle then (3.2) must reproduce (1.2). Hence

$$F(x_2-x_1) \xrightarrow[C\to\text{point particle}]{} \delta^{(4)}(x_2-x_1). \tag{4.2}$$

Both the Feynman diagram from Fig. 1b and the non-local Lagrangian require that form factor F be a form invariant function in space-time, i.e.

$$F = F(x^2), \quad x = x_2-x_1. \tag{5.2}$$

Otherwise the Lorentz invariance of the laws of motion would be violated. On the other hand, let us remember that the violation of the covariance of laws of motion does not involve violation of the covariance of the S-matrix. For example, the semi-relativistic equation, although violating the covariance, results in an elastic scattering amplitude which can be parametrized in terms of the momentum invariants, and thus fulfils the requirements of relativity. The form invariance of F in space-time means that in momentum space, F depends *a priori* on the momen tumtransfer t only. Thus we encounter the following dilemma: on one hand the internal structure of C is described by a form invariant function $F(x^2)$, while on the other — as is known from Section 1 — the internal wave function ψ_0 of C can never be a form invariant function. Thus two functions F and ψ_0, of entirely different analytic forms, would describe the internal structure of C in the same space-time continuum. This conclusion is quite general, i.e. independent of whether one deals with the Bethe–Salpeter functions [14], the static approximation wave functions [12], or any others because none of them is ever form invariant in space-time.

Let us call the form invariant form factors the "hyperbolic (H) type" form factors, and denote them by F_H, i.e. $F_H = F_H(x^2)$. The consequence of the indefinite space-time metric is that *a priori* (that is in the empty space whose symmetry governs covariant laws of motion) the vicinity of two four-points x_1 and x_2 in space and time separately is meaningless. It gains meaning only *a posteriori*, when e.g. a particle exists in a definite state of its four-momentum P ($P^2 = -m^2$) which distinguishes between different reference frames by the shape of the representation of P. The only exception is when x_1 and x_2 coincide, for then $\delta^{(4)}(x) = (2\pi)^{-4}\int d^4p \exp(ipx)$ provides us with the covariant expression of the form factor of a point particle — cf. Eq. (4.2). This "discountinuity" between the coincidence and vicinity of two four-points, alien to Galilean geometry dealing with two invariant intervals, is the basis of the proposed modification of the form factor structure. As an example of difficulties characteristic of the H-type form factors let us consider the conventionally defined charge distribution of the nucleon [13]

$$\varrho(r) = (2\pi)^{-3}e\int d^3q \exp(i\boldsymbol{q}\boldsymbol{x}^*)F_H(\boldsymbol{q}^2 = t), \quad (F_H(0) = 1), \tag{6.2}$$

where $\boldsymbol{q}$ is the momentum transfer in the cm-system of an electron colliding with a nucleon, and $r = |\boldsymbol{x}^*|$ is the distance between the bare nucleon and the meson cloud in the same cm-system. However, if ϱ is to describe the charge distribution of the nucleon, then its argument r should denote this distance in the rest-frame S_n of the nucleon, not in S^*. If, instead of the nucleon, the deuteron were the extended particle C, then the internal structure of C is relatively well known, and the continuity argument, when going to the NR limit, requires the form factor of C to behave quite differently from F_H. Following this argument, and following quite a general argument compatible with the first one, namely that for a point particle one should regain its world line in the relative four-coordinate x — cf. Fig. 2a, b — we assume the form factor F to be proportional to

$$\psi_0^*[x^2+(P_c'x)^2/m_c^2]\psi_0[x^2+(P_cx)^2/m_c^2]. \tag{7.2}$$

However, in order to regain the four-dimensional δ-function in the limit of a point particle, as required by (4.2), we unavoidably must cross the world-line $\delta^{(3)}(\boldsymbol{x}-\boldsymbol{v}\Delta t)$ with a space-

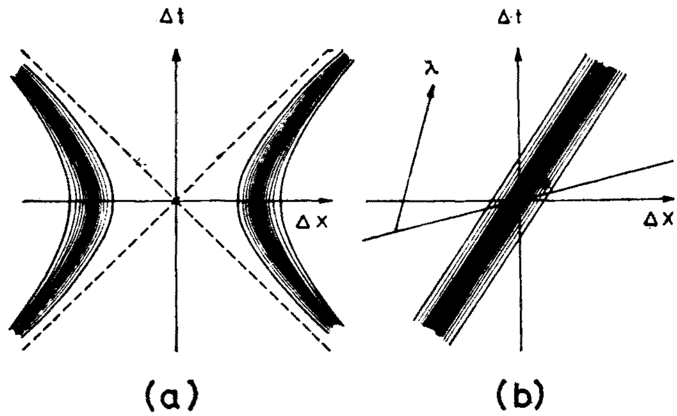

Fig. 2. Topology of (a) "hyperbolic", and (b) "eliptic" form factors of C

-like surface or, in other words, we must introduce a time-like vector λ — cf. Fig. 2b. — perpendicular to this surface at the cross point. Thus we put the form factor of C in the form

$$F_E(x) = N\psi_0^*[x^2+(P_c'x)^2/m_c^2]\psi_0[x^2+(P_cx)^2/m_c^2]\delta(\lambda x), \tag{8.2}$$

where the normalization factor N ensures that $\int d^4xF_E(x) = 1$. The general question then arises, what reality determines λ, as there is no λ given *a priori*. Form factors such as that defined in (8.2), let us say of "elliptic (E) type", will be denoted by F_E. *In contrast to the* H-*type, the* E-*type form factors must resort somewhat to reality in determining* λ. Similarly the time-like vector P_c/m_c indicates the distinguished position of the rest-frame of C through the analytic form of the internal wave function of C which depends on P_c/m_c, and therefore is not form invariant in space-time. Due to this, one can expect that λ will play a similar role for the wave function which describes the relative motion of the colliding particles A and C — cf. Eq. (10.2). Since the localization of λ influences, through F_E, the measurable cross sections, a suitable experiment could, in principle, determine λ.

Inserting (7.2) into (3.2) one obtains

$$S_{fi} = g_ag_c(2\pi)^4(2E_a2E_c2E_a'2E_c')^{-1/2}\delta^{(4)}(P_a+P_c-P_a'-P_c')\Delta^F(t;\mu)F_E(t,u,u'),$$

$$F_E(t,u,u') = N\int d^4x \exp\left[\frac{i}{2}(P_c-P_c')x\right]\psi_0^*[x^2+(P_c'x)^2/m_c^2]$$

$$\times\psi_0[x^2+(P_cx)^2/m_c^2]\delta(\lambda x), \tag{9.2}$$

$$t = (P_c-P_c')^2 \geqslant 0, \quad u = -(\lambda P_c)/m_c, \quad u' = -(\lambda P_c')/m_c.$$

The variables t, u, u' are three invariants which parametrize F_{E} in momentum space. When C becomes a point particle λ ceases to play any role, because independently of the localization of λ, $F_{\mathrm{E}}(t, u, u') \equiv 1$, as required by (3.2). Thus, as has been said before, the point particle does not introduce any modification of the standard Feynman graphs. Also in the NR limit ($c \to \infty$) where no room exists for H-type form factors, and each time-like vector becomes parallel to $(0, 0, 0, i)$ in any reference frame, one finds that $F_{\mathrm{E}}^{(\mathrm{NR})}$ depends *a priori* on t only, much like the H-type form factor in the relativistic case,

$$F_{\mathrm{E}}^{(\mathrm{NR})}(t) = \int d^3x \exp\left[\frac{i}{2}(\boldsymbol{P}_c - \boldsymbol{P}'_c)\boldsymbol{x}\right] |\psi_0(\boldsymbol{x}^2)|^2, \qquad t = (\boldsymbol{P}_c - \boldsymbol{P}'_c)^2. \tag{9'.2}$$

Since, within the E-type form factors, we are forced to distinguish between different reference frames via λ, it seems reasonable to suppose that, as the internal wave function ψ_0 of C distiguishes the rest-frame S_c of C, the whole isolated system $A+C$ distinguishes its overall cm-system S^*. This is automatically realized by the semi-relativistic picture, while the manifestly covariant amplitude forces us to assume explicitly

$$\lambda = (P_a + P_c)/W, \qquad s = -(P_a + P_c)^2 = W^2. \tag{10.2}$$

Then

$$u = u' = (s + m_c^2 - m_a^2)/(2m_c\sqrt{s}) = \gamma_c^*, \qquad F_{\mathrm{E}} = F_{\mathrm{E}}(t, s), \tag{11.2}$$

where $\gamma_c^* = (1 - v_c^{*2})^{-1/2}$ is the Lorentz factor of C in the cm-system S^* of $A+C$. The s-dependence of F_{E} conflicts with the phenomenology of the Feynman graphs applied to particles with internal structure, although the quantitative effects are very weak and depend on v_c^* in a non-singular way, i.e. $F_{\mathrm{E}}(t, s) - F_{\mathrm{E}}(t, \infty)$ remains finite. Moreover, this dependence on s vanishes in the following particular cases: 1° in the NR limit, as then $v_c^*/c \to 0$ — cf. (9'.2); 2° for infinitely heavy, external centre C, or more precisely, for $\sqrt{s}/m_c \to 1$, because then $v_c^* \to 0$; 3° in the infinite momentum limit, when $v_c^* = 1$ as then one obtains $F_{\mathrm{E}}(t, \infty)$, which is well defined according to the non-singular behaviour of F_{E} on v_c^*, and as such, independent of s.

Besides these three limiting cases, F_{E} also remains independent of s for Gaussian wave functions $\psi_0 \sim \exp(-r^2/2\sigma^2)$, when one obtains from (9.2)

$$F_{\mathrm{E}}(t, s) = F_{\mathrm{E}}(t) = \exp\left[-\frac{\sigma^2 t}{16(1 + t/4m_c^2)}\right]. \tag{12.2}$$

The same shape of F_{E} is obtained from Gaussian functions by privileging, e.g. the Breit system of C, where instead of (10.2) we should take

$$\lambda = (P_c + P'_c)/(2m_c\sqrt{1 + t/4m_c^2}), \qquad u = u(t) = u'(t) = \sqrt{1 + t/4m_c^2}\,. \tag{13.2}$$

This can be generalized for the case 4°, where F_{E} depends on t only, when one distinguishes the class of reference frames where $u = u(t)$, and $u' = u'(t)$, e.g. the Breit system of C, or the lab-system of C, as then

$$F_{\mathrm{E}}(t, u, u') = F_{\mathrm{E}}(t, u(t), u'(t)) = \mathscr{F}_{\mathrm{E}}(t), \tag{14.2}$$

Here the form factor dependence on t only, as required by the Feynman diagram phenomenology, is realized, but *a posteriori*, i.e. by privileging some reference frames where the wave function of C is presented. This does not take place *a priori*, i.e. without indicating any reference frame as is the case for H-type form factors. It is remarkable that several authors, e.g. Gross [15], construct $\mathscr{F}_E(t)$ in the Breit system, without clearly stating that this is an arbitrary hypothesis which favours some reference frame, just as in our case where the overal cm-system becomes distinguished through (10.2). Besides the aforementioned reasons based on quantum mechanical symmetry, the privileged role of the cm-system is also supported by considering collisions of two composite particles C and $\tilde{C}$. Then, according to Gross, the determination of the corresponding form factors $\mathscr{F}_E^{(C)}(t)$ and $\tilde{\mathscr{F}}_E^{(\tilde{C})}(t)$ should demand two entirely different Breit systems of C and $\tilde{C}$, respectively, whereas the cm-system of $C+\tilde{C}$ is the only one which is symmetric for the whole process.

Apart from the above mentioned dependence of F_E on the localization of λ, there is a difference between all form factors of the E- and H-types which reveals itself in momentum space as well. One can show that independently of the localization of λ, all E-type form factors depend on t through a variable such as in Eq. (12.2), namely

$$\tau = t/(1+t/n^2m_c^2), \tag{15.2}$$

where n is the number of constituents of C (in our case $n = 2$). When $t \to \infty$, then τ tends to a finite value $\tau_{\max} = n^2m_c^2$, hence F_E — neglecting its weak dependence on s — tends to a finite asymptotic value. The relation (15.2) provides us with a prescription for obtaining the relativistic form factor F_E from the, let us call it so, static form factor F_E^0 which coincides with the NR one,

$$F_E^0(t) = \int d^3x|\psi_0(r)|^2 \exp(i\boldsymbol{q}\boldsymbol{x}) = F_E^{(NR)}(t), \quad (t = \boldsymbol{q}^2). \tag{16.2}$$

Then, apart from the possible s-dependence of F_E, its t-dependence takes the form

$$F_E(t) = F_E^0\left(\frac{t}{1+t/n^2m_c^2}\right). \tag{17.2}$$

For the sake of illustration, let us suppose that the electron is an extended particle of classical radius $r_c = 1/(137m_e)$, and let the static form factor of the electron be of the Gaussian form

$$F_E^0(t) = \exp[-t/(137m_e)^2] \quad (n = 1). \tag{18.2}$$

According to (17.2) the electron form factor revealed in elastic collisions will be equal to

$$F_E(t) = \exp\left[-\frac{t/m_e^2}{137^2(1+t/m_e^2)}\right], \tag{19.2}$$

which never falls below the value $\exp[-1/(137^2)] \cong 1-1/137^2 \cong 1$. Thus, because $r_c \ll 1/m_e$, the internal structure of the electron, if it exists, can never be observed in elastic collisions, and the electron behaves like a point particle, which is consistent with all experiments.

On the other hand, for electron-deuteron elastic collisions we have $\tau = t/(1+t/16m^2)$, where m is the nucleon mass, and up to $t \ll 16m^2$ the static (NR) form factor $F_E^0(t)$ almost coincides with F_E. This is in agreement with [1] where the elastic cross-section has been measured for t up to 6 $(\mathrm{GeV}/c)^2$.

Note that the saturation effect of $F_E(t)$ for $t \to \infty$ is due to the Lorentz contraction of the particle C. With increasing t the recoil of C increases generating the Lorentz contraction, which prevents the penetration of C at its proper distances less than $1/nm_c$. In contrast to the E-type form factors there is no room for Lorentz contraction or any other relativistic distortion of C within the H-type form factors. The form invariant functions in space-time, such as F_H, are insensitive to the motion (recoil) of the interacting particles. Although the nonrelativistic (or static) form factors are always of the E-type, in this (NR) limit τ equals t, and the absence of the Lorentz contraction makes $F_E^{(NR)}(t)$ analogous to the relativistic H-type form factors. Large momentum transfers ($t \gtrsim \tau_{max}$) should then distinguish between the modification of the cross-sections due to the internal structure of the colliding particles (F_E), and that due to the interaction between them described by the H-type form factors (F_H).

3. *Disintegration process*

Within a simple model similar to that discussed in the previous section let us now analyse the disintegration of C due to its collision with the point particle A. Let us start with the production process of the particles "1" and "2" in the lower vertex of the diagram

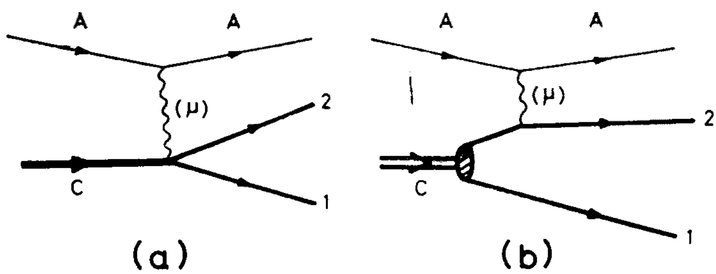

Fig. 3. Disintegration graph of (a) point particle C into "1" and "2", and (b) composite particle C

in Fig. 3a. In the assumed Born approximation, the corresponding matrix element is equal to

$$S_{fi} = g_a g(2E_a 2E_c 2E_a' 2E_1' 2E_2')^{-1/2} \int d^4x \int d^4y \exp[i(P_c - P_1' - P_2')x + i(P_a - P_a')y] \Delta^F(x-y;\mu), \tag{1.3}$$

where

$$P_c^2 = -m_c^2, \quad P_a^2 = -m_a^2, \quad P_1'^2 = P_2'^2 = -m^2.$$

If the particle C is composed of two scalar and point particles "1" and "2", then let the production of "1" and "2" mean the disintegration of C. In this case $m_c = 2m - \Delta m$, $\Delta m > 0$ is the binding energy of C. According to this, the amplitude (1.3) will be modified by a suitable form factor G. Moreover, let us assume that an interaction takes place between A and "2", leaving "1" as a "spectator". This corresponds to the additivity of the interaction which works very well in the electromagnetic interaction of an electron with light nuclei. The matrix element then takes the following form

$$S_{fi} = g_a g(2E_a 2E_c 2E'_a 2E'_1 2E'_2)^{-1/2} \int d^4x_1 \int d^4x_2 \int d^4y \Delta^F(x_2-y;\mu) \times \exp\{i[P_c(x_1+x_2)/2+P_a y - P'_1 x_1 - P'_2 x_2 - P'_a y]\} G(x_2-x_1). \tag{2.3}$$

In order to regain (1.3) when C becomes a point particle it is again necessary that

$$G(x_2-x_1) \xrightarrow[C\to\text{point particle}]{} \delta^{(4)}(x_2-x_1). \tag{3.3}$$

If one assumes the Feynman diagram factorization of two vertices — cf. Fig. 3 — and the spherically symmetric state of C, then the disintegration amplitude in the momentum representation can depend on the variable $(P_c-P'_1)^2$ only. Consequently, apart from kinematics of the phase space, this implies the same spherical symmetry of spectators in the lab-system. However, arguments like these raised by elastic scattering make us suppose that the response of C can be more involved, and it can violate the factorization of the Feynman diagram. Smooth transition to the experimentally well established NR amplitude inclines to make an Ansatz, and to put

$$G(x) = -(\lambda P_c)/m_c \kappa^{-3/2} \psi_0[x^2+(P_c x)^2/m_c^2]\delta(\lambda x),$$
$$\kappa^{3/2} = \int d^3x \psi_0(\boldsymbol{x}^2), \tag{4.3}$$

which, independently of λ, fulfils the condition $\int d^4x G(x) = 1$, and in the limit of the point particle C, $G(x) \to \delta^{(4)}(x)$, as required by (3.3). The presence of a so-far arbitrary time-like unit vector λ is unavoidable if one tries to maintain both, the Feynman amplitude (1.3) for the production of point particles "1" and "2" from a point vertex, and the well established in the NR physics disintegration amplitude expressed by the corresponding NR wave function ψ_0. Note again that in the NR limit the localization of λ is irrelevant, because

$$G(x) \xrightarrow[c\to\infty]{} \kappa^{-3/2}\psi_0(\boldsymbol{x}^2)\delta(\Delta t)$$

independently of λ.

Besides arguments similar to those from Section 2 let us mention another one which also supports the hypothesis of the distinguished role of the overall cm-system i.e. the localization of λ as given in (10.2). Suppose namely that A represents the source of an external, hence infinitely heavy field of force. Thus the overal cm-system S^* coincides with the rest-system S_a of A, and the simultaneity of $S_a = S^*$ is privileged by this external field being static in S^* only. Therefore one can expect that the equal-time wave function of C in S^* will be responsible for the structure of C revealed in the interaction of C with A.

Inserting (4.3) into (2.3) one obtains the cross-section for the disintegration of C in the following form

$$d\sigma = I(s)\, |\Delta^F(t;\mu)|^2 |G|^2 \delta^{(4)}(P_a+P_c-P'_a-P'_1-P'_2) \frac{d^3P'_a}{2E'_a}\frac{d^3P'_1}{2E'_1}\frac{d^3P'_2}{2E'_2}. \tag{5.3}$$

Here $I(s)$ is some function of s and coupling constants which are of no importance for our purposes, the propagator Δ^F accounts for the "known interaction" of A with C, and the invariant function G is the four-dimensional Fourier transform of $G(x)$, namely

$$G = (\lambda P_c)/m_c \int d^4x \psi_0[x^2+(P_c x)^2/m_c^2]\delta(\lambda x) \exp[i(P_1'-P_c/2)x]. \tag{6.3}$$

In the lab-system, where C before the disintegration is at rest, the function G takes a simple form

$$G = \int d^3x \psi_0(\boldsymbol{x}^2) \exp(i\boldsymbol{q}\boldsymbol{x}),$$
$$\boldsymbol{q} = \boldsymbol{p}_s - \boldsymbol{V}[(m^2+\boldsymbol{p}_s^2/c^2)^{1/2}+\Delta m/2-m], \tag{7.3}$$

where $\boldsymbol{p}_s = \boldsymbol{P}_1'|_{\text{lab}}$ is the spectator three-momentum in lab-system, $\Delta m = 2m-m_c > 0$ is the mass defect of C, and $\boldsymbol{V}$ is the velocity of the "rest-frame" of λ (i.e., where $\lambda = (\boldsymbol{0}, i)$) in the lab-system.

The effect which focuses our attention is due to the difference between $\boldsymbol{q}$ and $\boldsymbol{p}_s$, which implies the shift of the argument of ψ_0, and hence the unisotropy of spectators in the lab-system. One easily sees that the $\boldsymbol{V}$-dependence of this difference has its origin in different simultaneities of different reference frames, while the second factor $(m^2+\boldsymbol{p}_s^2/c)^{1/2}+\Delta m/2-m$ corresponds with the off-shell mass (energy) of the spectator inside C. Remember that ψ_0 is the eigenstate of the invariant mass of C to the eigenvalue m_c, and thereby $m-\Delta m/2 = m_c/2$ is the mean energy of each constituent of C in the lab-system. On the other hand, $(m^2+\boldsymbol{p}_s^2/c^2)^{1/2}$ is the on-shell mass of the spectator in the same lab-system after the disintegration.

By taking λ from (10.2) $\boldsymbol{V}$ becomes the velocity of the overall cm-system S^* in the lab-system S_L. Note that the difference between $\boldsymbol{q}$ and $\boldsymbol{p}_s$ vanishes in the following particular cases, whence the spectator distributions — apart from the phase space factor — regain spherical symmetry in S_L. (*i*) In the NR limit, as both aforementioned effects are relativistic. (*ii*) For infinitely heavy C, as then $S^* = S_L$, hence $\boldsymbol{V} = 0$. (*iii*) If, instead of favouring S^* as given by (10.2), one takes $\lambda = P_c/m_c$ favouring the lab-system S_L, as then $\boldsymbol{V} = 0$. Finally, (*iv*) when C becomes an infinitely loosely bound system. Then $\Delta m \to 0$, and the Fermi momentum $\boldsymbol{p}_s$ also vanishes, thus $\boldsymbol{q} = \boldsymbol{p}_s = 0 = \Delta m$, and the bracket in (7.3), vanishes. The case (*iv*) must take place for selfconsistency reasons. Otherwise, any free particle could be incorporated into C thus changing $\boldsymbol{V}$, and consequently the measurable cross-section.

The electron-deuteron collision resulting in the disintegration of the latter should be the best suited process to prove or disprove the unisotropy effect due to $\boldsymbol{q} \neq \boldsymbol{p}_s$. In this case the interaction is relatively well known and it fulfils the additivity assumption which is crucial when speaking of true spectators. Neglecting spins which do not contribute to the unisotropy if particles are unpolarized, let us identify A with the electron, and C with the deuteron of mass $M = m_c = 2m-\Delta m$. Here m is the nucleon mass and Δm is the binding energy of the deuteron. Moreover, let us denote by Θ_s the lab-angle between the lab-momentum of the spectator and primary electron, and by $d\Omega_s$ the element of spherical angle of the spectator in the same lab-system S_L. Neglecting the electron mass one obtains that

$$\frac{d^3\sigma}{dt\,dp_s\,d\Omega_s} = J(s)\,|D^F(t)|^2 \frac{p_s^2/E_s}{1-E_s/M+(p_s/M)\cos\theta_s} |G(q^2)|^2, \tag{8.3}$$

where $J(s)$ is some s-dependent kinematic factor, D^{F} is the photon propagator, and $E_s = (m^2+p_s^2)^{1/2}$ 'is the spectator energy in S_L.

Within this simplified model, two factors are responsible for unisotropy of spectators. First due to the phase space, equal to $[1-E_s/M+(p_s/M)\cos\Theta_s]^{-1}$, and the second, which we are interested in, due to the difference between $\boldsymbol{p}_s$ and $\boldsymbol{q}$. The essential point is that only the second one depends on s (via $\boldsymbol{V}$). The contamination of the double scattering [16], as well as the final state interaction [17], or the phase space factor, all of these also disturb the angular distribution of spectators, but these modifications are independent of s. Therefore, the investigation of the *s-dependent anisotropy of spectators seems to be a suitable test* of the proposed hypothesis (10.2).

The quantitative discussion of the unisotropy effect of the spectators requires analysing different effects already mentioned. However, in order to get some idea of the magnitude

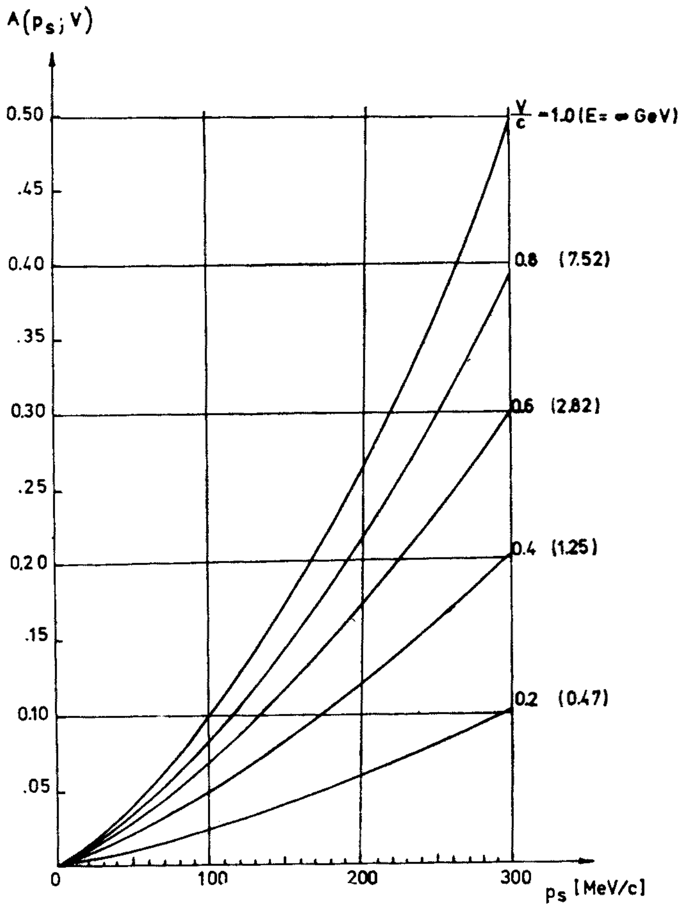

Fig. 4. Asymmetry coefficient $A(p_s;\, V)$ of "spectators" as function of momentum p_s of the spectator and lab-energy E of the impinging electron $\left(V = \dfrac{E}{E+M},\ M\text{—deuteron mass}\right)$

of the s-dependence of unisotropy, let us confine ourselves to the analysis of the factor $|G(\boldsymbol{q}^2)|^2$. Taking the Hulthén function for ψ_0 with parameters from [18] the forward--backward asymmetry coefficient of the spectators of a given momentum $p_s = |\boldsymbol{p}_s|$ is plotted in Fig. 4, as a function of p_s for different values of $V = V(s)$. This asymmetry cannot be directly compared with experiment for reasons explained before, but numbers which can be realistic are the relative unisotropies of spectators for different values of s (or V), and these can be read off from Fig. 4 [19]. We see that the effect is rather small for mean Fermi momenta (about 50 MeV/c for the deuteron), but it increases with increasing p_s. For infinite s it increases to a finite, maximum value (due to the non-singular character of this effect in the V variable) equal to about 20% for the spectator momentum p_s of 150 MeV/c.

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