

DYNAMICAL SYMMETRY RESTORATION IN THE INTERACTIONS OF MESONS

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Effective potentials of the quartic meson-meson interaction and of the large N $O(N)$ σ -model are examined. In both models the first order radiative corrections restore the symmetry broken in the tree approximation. The potential of the $O(N)$ σ -model has a minimum which is lower than the one found previously by Coleman, Jackiw and Politzer.

1. Introduction

Evaluation of the effective potential is a particularly useful tool for field-theoretical investigations, especially for problems of the ground state and symmetry breaking. The effective potential was introduced by Goldstone, Salam, Weinberg and Jona-Lasinio [1], [2]. The recent revival of interest in this problem is due mainly to Coleman, Weinberg and Jackiw [3], [4]. They were the first to evaluate the radiative corrections to the potential by summing up infinite sets of Feynman diagrams in a loop expansion. Coleman and Weinberg have also shown that the radiative corrections may be the origin of spontaneous symmetry breaking.

In this paper we give two examples of models exhibiting an inverse phenomenon namely the dynamical restoration of symmetry. These models are the $\lambda\Phi^4$ theory and the large N $O(N)$ generalization of the σ -model of Coleman, Jackiw and Politzer [5]. Both these models have a common feature: One encounters tachyon poles while studying spontaneous symmetry breaking in four dimensions.

In our previous paper [6] we have suggested that we should probably expand Green's functions around the symmetric vacuum, since then tachyons do not appear. In the present article we give more details which can, hopefully, elucidate this problem. We demonstrate that the potential actually has two minima — one corresponding to the symmetric vacuum and another one, leading to the asymmetric ground state. For the symmetric minimum the potential has a lower value than for the asymmetric one. Thus we must expand around the global minimum; expanding around the local minimum leads to the tachyon disaster.

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2. The model and the combinatorial trick

Let us consider the usual massive Φ^4 theory defined by the Lagrangian:

$$\mathcal{L} = \frac{1}{2} (\partial\Phi)^2 - \frac{1}{2} m^2 \Phi^2 - \frac{1}{4!} g \Phi^4. \quad (2.1)$$

We find particularly useful to replace (2.1) by

$$\mathcal{L} = \frac{1}{2} (\partial\Phi)^2 - \frac{1}{4!} (g - 6\lambda) \Phi^4 - \frac{1}{2} \Phi^2 \Psi + \frac{1}{4\lambda} \Psi^2 - \frac{1}{2} \frac{m^2}{\lambda} \Psi. \quad (2.2)$$

The Euler-Lagrange equations of motion for (2.2) are

$$\square\Phi + m^2\Phi^2 + \frac{1}{3!} g\Phi^3 = 0, \quad (2.3)$$

$$\Psi = \lambda\Phi^2 + m^2. \quad (2.4)$$

(2.3) is just the same equation that one obtains from (2.1) and is λ -independent, (2.4) contains no derivatives and is only the equation of constraints. From (2.4) the Lagrangian (2.2) is seen to be equivalent to that in (2.1). Thus both Lagrangians (2.1) and (2.2) are equivalent and lead to the same dynamical equations of motion but they generate different Feynman rules.

While calculating Green's functions we encounter the Φ^4 vertices as the $\Phi^2\Psi$ vertices connected by a Ψ -line. After some combinatorics we find that the same result would be obtained if we would use only the Φ^4 vertices with the coupling constant $g/4!$ (see Fig. 1).

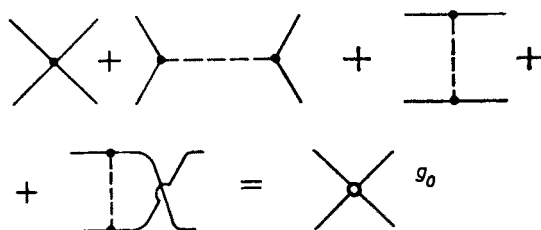


Fig. 1. Feynman rules generated by the Lagrangian (2.2). Green's functions are independent of λ

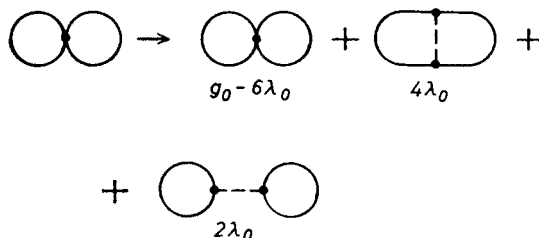


Fig. 2. Certain contributions to the proper vertices are proportional to the constants occurring in the combination $g - 2\lambda$

We must be more careful while calculating the proper vertices. If the 1pI diagram can be made disconnected by removing one Φ^4 vertex then replacing this vertex by a pair of $\Phi^2\Psi$ vertices may yield the 1pI as well as the 1pR (one-particle-reducible) diagram. Fig. 2 shows that in this case the contribution to the proper vertex carries a factor $g - 2\lambda$.

Making use of the freedom of the choice of λ we set

$$\lambda = \frac{1}{2}g. \quad (2.5)$$

This reduces considerably the number of diagrams contributing to the 1pI functions. In particular, all the "bubble diagrams" like those of Fig. 2 as well as all but one of the contributions from diagrams containing self-closing loops cancel [6]. More to the point, only one self-closing loop contribution survives — namely the one-loop Ψ -tadpole. As we shall see later this tadpole also cancels, but the reasons are not of the combinatorical nature.

3. Calculation of the effective potential

Referring the reader to the original papers¹ we briefly recall that the effective potential V_{eff} is a generating functional of the 1pI Green's functions taken for external momenta equal to zero. Thus we have:

$$\int d^4x V(\varphi_c(x)) = \sum_n \frac{1}{n!} \int d^4x_1 \dots d^4x_n \Gamma^{(n)}(x_1, \dots, x_n) \varphi_c(x_1) \dots \varphi_c(x_n), \quad (3.1)$$

where $\Gamma^{(n)}$'s are the 1pI functions evaluated at zero momenta, φ_c 's are the classical fields (defined as the vacuum expectation values of the quantum fields Φ). The definition (3.1) can be immediately generalized to the case of an arbitrary number of different classical fields.

Substituting (2.5) into (2.2) we obtain

$$\mathcal{L} = \frac{1}{2}(\partial\Phi)^2 + \frac{1}{12}g\Phi^4 - \frac{1}{2}\Phi^2\Psi + \frac{1}{2g}\Psi^2 - \frac{m^2}{g}\Psi. \quad (3.2)$$

The tree approximation gives for the effective potential:

$$V_{\text{tree}} = -\frac{1}{12}g\varphi_c^4 + \frac{1}{2}\varphi_c^2\psi_c - \frac{1}{2g}\psi_c^2 + \frac{m^2}{g}\psi_c. \quad (3.3)$$

One-loop contributions to V_{eff} are presented in Fig. 3. Assuming (2.5) we have reduced the number of the contributions to $V_{1\text{-loop}}$ since one-loop diagrams with external φ -lines cancel. We have

$$V_{1\text{-loop}} = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{\psi_c}{-k^2} \right)^n, \quad (3.4)$$

¹ Ref. [7] offers an extensive introduction and contains many references.

where k denotes Euclidean momenta. The summation in expressions of this type can be performed explicitly [3] and we obtain

$$V_{\text{eff}} = -\frac{1}{12} g \varphi_c^4 + \frac{1}{2} \varphi_c^2 \psi_c - \frac{1}{2g} \psi_c^2 + \frac{m^2}{g} \psi_c + \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \log(k^2 + \psi_c). \quad (3.5)$$

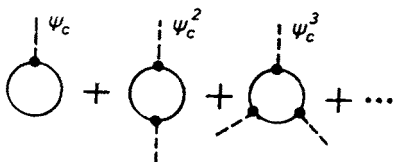


Fig. 3. One-loop contribution to the effective potential

In order to determine the ground state we look for the minima of (3.5). The conditions

$$\partial V / \partial \varphi_c = 0, \quad \partial V / \partial \psi_c = 0 \quad (3.6)$$

yield

$$-\frac{1}{3} g \varphi_c^2 + \psi_c = 0 \quad \text{or} \quad \varphi_c = 0 \quad (3.7)$$

and

$$\frac{1}{2} \varphi_c^2 - \frac{1}{g} \psi_c + \frac{m^2}{g} + \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 + \psi_c} = 0 \quad (3.8)$$

respectively.

In four dimensions both m and g require renormalization. We shall use the same method of subtraction as in Ref. [5]:

$$\frac{m^2}{g} = \frac{m_R^2}{g_R} - \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2}, \quad (3.9)$$

$$\frac{1}{g} = \frac{1}{g_R} - \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{k^2 + M^2}, \quad (3.10)$$

where the subtraction point M is arbitrary. Substituting (3.9) and (3.10) into (3.8) and (3.7) we obtain:

$$\varphi_c = 0 \quad \text{or} \quad \psi_c = 0 \quad (3.11)$$

and

$$\frac{\partial V}{\partial \psi_c} = 0 = \frac{1}{2} \varphi_c^2 - \frac{1}{g_R} \psi_c + \frac{m_R^2}{g_R} + \frac{1}{32\pi^2} \psi_c \log \psi_c / M^2, \quad (3.12)$$

which are the conditions for minima of the renormalized potential. The renormalized effective potential in one-loop approximation is

$$V_{\text{eff}} = \frac{1}{2} \varphi_c^2 \psi_c + \frac{m_R^2}{g_R} \psi_c - \frac{1}{2g_R} \psi_c^2 - \frac{1}{128\pi^2} \psi_c^2 + \frac{1}{64\pi^2} \psi_c^2 \log \frac{\psi_c}{M^2}. \quad (3.13)$$

This result bears a striking resemblance to that which Coleman, Jackiw and Politzer obtained in the leading $1/N$ approximation to the $O(N)$ generalization of the σ -model [5]. This model is defined by:

$$\mathcal{L} = \frac{1}{2} \sum_{a=1}^N (\partial \Phi_a)^2 - \frac{1}{2} \mu^2 \sum_a \Phi_a^2 - \frac{\lambda}{8N} \left(\sum_a \Phi_a^2 \right)^2 \quad (3.14)$$

or, equivalently by

$$\mathcal{L} = \frac{1}{2} \sum_{a=1}^N (\partial \Phi_a)^2 + \frac{1}{2} \frac{N}{\lambda} \chi^2 - \frac{1}{2} \chi \sum_a \Phi_a^2 - \frac{N \mu^2}{\lambda} \chi. \quad (3.15)$$

Exactly as in our case, the only contributions to V_{eff} stem from the diagrams of Fig. 3 with dashed lines interpreted as the $O(N)$ singlet χ_c .

In this approach it turns out that

$$V_{\text{eff}} = \frac{1}{2} \sum_{a=1}^N (\varphi_c^a)^2 \chi_c + \frac{N}{\lambda_R} \mu_R^2 \chi_c - \frac{N}{2\lambda_R} \chi_c^2 - \frac{N}{128\pi^2} \chi_c^2 + \frac{N}{64\pi^2} \chi_c^2 \log \frac{\chi_c}{M^2}. \quad (3.16)$$

The formal similarity between (3.13) and (3.16) will allow us to study both models simultaneously. We must remember that the large N limit was essential for the derivation of the latter expression while the former one was obtained after the complete one-loop calculations. It is a fortuity of no significance that it was possible to redefine both Lagrangians in such a manner that the first orders of two different expansion schemes gave similar results. However, this will spare us some unnecessary calculations.

4. Green's functions

Combining (3.11) and (3.12) we obtain the two following sets of necessary conditions for minima of the potential (3.13):

$$\varphi_c^2 = -\frac{2m_R^2}{g_R}, \quad \psi_c = 0 \quad (4.1)$$

or

$$\varphi_c = 0, \quad \psi_c = m_R^2 + \frac{g_R}{32\pi^2} \psi_c \log \frac{\psi_c}{M^2}. \quad (4.2a, b)$$

For negative ψ_c V_{eff} becomes complex, therefore only the points $\psi_c \geq 0$ may be regarded as possible candidates for the stable ground state of our model.

First, let us assume that $m_R^2 < 0$ and $g_R < 0$. In the tree approximation, as it is well known, the state of lowest energy is not $\langle |\Phi| \rangle = \varphi_c = 0$, but $\varphi_c = \pm (-2m_R^2/g_R)^{1/2}$.

Whichever of these we choose to be the definition of the ground state, the symmetry $\Phi \leftrightarrow -\Phi$ is broken. Shifting the fields in order to make them have zero expectation values in the vacuum state we find that now the new squared mass is positive.

Unfortunately the attempt to include the radiative corrections into this nice scheme is halted by serious difficulties — one encounters tachyon poles in the propagators. In order to see this, let us allow φ_c and ψ_c to depend on space-time and let us derive the effective action Γ (generating functional of the 1pI Green's functions). In analogy to (3.13) we find

$$\Gamma = \int d^4x \left[\frac{1}{2} \varphi_c \square^2 \varphi_c + \frac{1}{12} g \varphi_c^4 + \frac{1}{2g} \psi_c^2 - \frac{1}{2} \varphi_c^2 \psi_c - \frac{m^2}{4g} \psi_c \right] - \frac{1}{2} \text{tr} \log (-\square^2 + \psi_c). \quad (4.3)$$

If we expand around (4.1) and shift the field precisely as in the quasiclassical approximation:

$$\varphi \rightarrow \varphi + \sqrt{-2m^2/g}, \quad \psi \rightarrow \psi, \quad (4.4)$$

the propagators are found to be [6]:

$$D_{\Phi\Phi} = \frac{1/g_R + (1/32\pi^2) [1 - \log(p^2/M^2)]}{-2m_R^2/g_R + p^2[1/g_R + (1 - \log(p^2/M^2))/32\pi^2]}, \quad (4.5)$$

$$D_{\Psi\Psi} = \frac{p^2}{-2m_R^2/g_R + p^2[1/g_R + (1 - \log(p^2/M^2))/32\pi^2]}. \quad (4.6)$$

We have only quoted the result since the calculations go exactly like the derivation of the formulae (3.14) and (3.15) of Ref. [5]. Since both propagators exhibit tachyon poles, we see that conditions (4.1) do not determine the ground state.

Now let us investigate the consequences of the assumption that the vacuum is determined by the conditions (4.2).

First we notice that if $m_R^2/g_R < 0$ then the equation (4.2b) has a unique (real and positive) solution $\psi_c \equiv \tilde{\psi}$. Now, instead of (4.4) we must perform the following shift:

$$\varphi \rightarrow \varphi, \quad \psi \rightarrow \psi + \tilde{\psi}. \quad (4.7)$$

This yields for the effective action:

$$\Gamma = \int d^4x \left[\frac{1}{2} \varphi_c \square^2 \varphi_c + \frac{1}{12} g \varphi_c^4 - \frac{1}{2} \varphi_c^2 \psi_c - \frac{1}{2} \tilde{\psi} \varphi_c^2 + \frac{1}{2g} \psi_c^2 + \frac{1}{g} \tilde{\psi} \psi_c - \frac{m^2}{g} \psi_c \right] - \frac{1}{2} \text{tr} \log (-\square^2 + \psi_c + \tilde{\psi}). \quad (4.8)$$

φ -tadpoles vanish since the linear terms of the expansion of (4.8) cancel. Indeed, differentiating (4.8) with respect to ψ_c and setting $\psi_c = \tilde{\psi}$ we obtain:

$$\frac{1}{g} \tilde{\psi} - \frac{m^2}{g} - \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + \tilde{\psi}} = \frac{1}{g_R} \tilde{\psi} - \frac{m_R^2}{g_R} - \frac{1}{2} \tilde{\psi} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \left(\frac{1}{k^2 + M^2} - \frac{1}{k^2 + \tilde{\psi}} \right),$$

where use was made of the subtraction prescriptions (3.10) and (3.11). Integrating on the right hand side we obtain:

$$\frac{1}{g_R} \tilde{\psi} - \frac{m_R^2}{g_R} + \frac{1}{32\pi^2} \tilde{\psi} \log \tilde{\psi}/M^2;$$

this is equal to zero in view of (3.12). For the propagator of the field Φ we get

$$D_{\Phi\Phi} = \frac{1}{p^2 + \tilde{\psi}}. \quad (4.9)$$

Now let us evaluate the propagator of ψ . Using the renormalization prescription (3.11) we find:

$$D_{\psi\psi} = - \left[\frac{1}{g_R} - \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \left(\frac{1}{k^2(k^2 + M^2)} - \frac{1}{(k^2 + \tilde{\psi})[(k+p)^2 + \tilde{\psi}]} \right) \right]^{-1}.$$

Integration yields:

$$D_{\psi\psi} = - \left\{ \frac{1}{g_R} - \frac{1}{32\pi^2} \log \frac{\tilde{\psi}}{M^2} - \frac{1}{32\pi^2} \left[\left(\frac{p^2 + 4\tilde{\psi}}{p^2} \right)^{1/2} \log \frac{(p^2 + 4\tilde{\psi})^{1/2} + (p^2)^{1/2}}{(p^2 + 4\tilde{\psi})^{1/2} - (p^2)^{1/2}} - 1 \right] \right\}^{-1}. \quad (4.10)$$

The expression in the square bracket is a monotonically increasing function of p^2 which is equal to one for $p^2 = 0$. Therefore, the propagator (4.10) has no tachyon poles provided that

$$\frac{1}{g_R} - \frac{1}{32\pi^2} \log \frac{\tilde{\psi}}{M^2} - \frac{1}{32\pi^2} < 0. \quad (4.11)$$

Returning to (3.14) we find that this is just the condition for minimum of V_{eff} at $\psi_c = \tilde{\psi}$:

$$\partial^2 V / \partial \psi_c^2 |_{\psi_c = \tilde{\psi}} > 0. \quad (4.12)$$

Fortunately we are able to prove that inequality (4.12) holds. From (3.12) we have:

$$\frac{1}{2} \varphi_c^2 = \frac{1}{g_R} \psi_c - \frac{m_R^2}{g_R} - \frac{1}{32\pi^2} \psi_c \log \frac{\psi_c}{M^2}. \quad (4.13)$$

The potential constrained by the requirement (4.13) is:

$$V = - \frac{1}{2g_R} \psi_c^2 + \frac{1}{128\pi^2} \psi_c^2 - \frac{1}{64\pi^2} \psi_c^2 \log \frac{\psi_c}{M^2}; \quad (4.14)$$

thus

$$\left. \frac{\partial^2 V}{\partial \psi_c^2} \right|_{\tilde{\psi}} = - \frac{1}{g_R} + \frac{1}{32\pi^2} \log \frac{\tilde{\psi}}{M^2} + \frac{1}{32\pi^2} = - \frac{\partial}{\partial \psi_c} \left(\frac{1}{2} \varphi_c^2 \right) |_{\psi_c = \tilde{\psi}} \quad (4.15)$$

Examining $\frac{1}{2}\varphi_c^2$ as a function of ψ_c defined by the equation of constraints (4.13) we find that for $\psi_c = 0$ $\frac{1}{2}\varphi_c^2 = -m_R^2/g_R > 0$. Then as ψ_c grows, $\frac{1}{2}\varphi_c^2$ increases and assumes its maximum value at $\psi_c = M^2 \exp(32\pi^2/g_R - 1)$ and then begins to decrease. At $\psi_c = \tilde{\psi}$, φ_c^2 is equal to zero and is still decreasing, so

$$\left. \frac{\partial^2 V}{\partial \psi_c^2} \right|_{\psi_c = \tilde{\psi}} = - \frac{\partial}{\partial \psi_c} (\tfrac{1}{2} \varphi_c^2) |_{\psi_c = \tilde{\psi}} > 0 \quad (4.16)$$

and the point (3.7) is a true minimum of V_{eff} .

The potential is non-analytic at $\psi_c = 0$. Therefore we cannot repeat the analogous reasonings in order to check whether or not the point (4.1) is a minimum of V . However, it is not necessary. Substituting the conditions (4.1) and (4.2) in (3.14) we find that the potential is equal to zero at the former point and assumes the negative value at the latter one. Thus at (4.1) the potential has at most a local minimum. The origin of the tachyon poles we have found previously resulted from expanding around the wrong vacuum as expected.

The same analysis can be performed in the $O(N)$ σ -model (3.15). Then, in analogy to (4.1) and (4.2) the potential (3.16) has two possible minima:

$$\frac{1}{N} \sum_{a=1}^N \varphi_c^a \varphi_c^a = -\frac{2\mu_R^2}{\lambda_R}, \quad \chi_c = 0 \quad (4.17)$$

and

$$\sum_{a=1}^N \varphi_c^a \varphi_c^a = 0, \quad \chi_c = \mu_R^2 + \frac{\lambda_R}{32\pi^2} \chi_c \log \frac{\chi_c}{M^2}. \quad (4.18)$$

The use of (4.17) alone would undoubtedly lead us to the tachyon-like inconsistencies indicated in Ref. [5]. Now we can see that it was caused by the presence of another (lower) minimum of V_{eff} .

Expanding around (4.18) and shifting the fields:

$$\varphi_c^a \rightarrow \varphi_c^a, \quad \chi_c \rightarrow \chi_c + \tilde{\chi}, \quad (4.19)$$

where

$$\tilde{\chi} = \mu_R^2 + \frac{\lambda_R}{32\pi^2} \tilde{\chi} \log \frac{\tilde{\chi}}{M^2},$$

we obtain

$$\begin{aligned} \Gamma = \int d^4x & \left[\frac{1}{2} \sum_{a=1}^N \varphi_c^a \square^2 \varphi_c^a - \frac{1}{2} \sum_{a=1}^N \varphi_c^a \varphi_c^a \chi_c - \frac{1}{2} \tilde{\chi} \sum_{a=1}^N \varphi_c^a \varphi_c^a \right. \\ & \left. + \frac{1}{2} \frac{N}{\lambda} \chi_c^2 + \frac{N}{\lambda} \tilde{\chi} \chi_c - \frac{\mu^2 N}{\lambda} \chi_c \right] - \frac{1}{2} \text{tr} \log [-\square^2 + \chi_c + \tilde{\chi}]. \end{aligned} \quad (4.20)$$

Now the propagators are free of the tachyon poles:

$$D_{\varphi^a\varphi^b} = \delta_{ab}/(p^2 + \tilde{\chi}) \quad (4.21)$$

and

$$D_{\chi\chi} = -\frac{1}{N} \left\{ \frac{1}{\lambda_R} - \frac{1}{32\pi^2} \log \frac{\tilde{\chi}}{M^2} - \frac{1}{32\pi^2} \left[\left(\frac{p^2 + 4\tilde{\chi}}{p^2} \right)^{1/2} \log \frac{(p^2 + 4\tilde{\chi})^{1/2} + (p^2)^{1/2}}{(p^2 + 4\tilde{\chi})^{1/2} - (p^2)^{1/2}} - 1 \right] \right\}^{-1}, \quad (4.22)$$

which suggests that the model of Coleman, Jackiw and Politzer is still consistent in four dimensions.

5. Conclusions

We have investigated the ground state in two models: in a $\lambda\Phi^4$ theory and in the $O(N)$ generalization of the σ -model. We have demonstrated that the radiative corrections may restore the symmetry of the vacuum even if it was assumed to be broken in the tree approximation. To the present author's knowledge the radiative corrections were known to be a possible origin of the symmetry breaking only. It is interesting that the same mechanism may cause the inverse effect, namely dynamical symmetry restoration. This confirms that the classical arguments for symmetry breaking must be applied with great care.

It is a well-known fact that in the single parameter models no reliable conclusions can be drawn from the study of the effective potential in any approximation concerning the finite number of loops. Our calculations confirm this fact. Namely, higher loops will add higher powers of $g_R \log \psi_c/M^2$ to the potential (3.13). It may happen that these corrections will turn our global minimum into a local minimum, or even a maximum, so that the above restored symmetry can be dynamically broken again. We are not able to do more than raise this possibility. This is in contradistinction to the $O(N)\sigma$ -model which is actually a two-parameter model ($1/N$ and λ). This time calculations performed in the leading order of the large N limit can, hopefully, be trusted.

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