OVERLAP FUNCTION IN HIGH ENERGY COLLISIONS OF COMPOSITE OBJECTS

By A. Białas

Institute of Physics, Jagellonian University, Cracow**
Institute of Nuclear Physics, Cracow

and W. Czyż

Institute of Nuclear Physics, Cracow*

(*Received June 22, 1976*)

It is shown that incoherent geometrical composition of probabilities gives a good approximation for the overlap function in collisions of two composite objects. It follows that a detailed understanding of the incident hadron fragmentation is crucial for the correct estimate of the overlap slope in hadron-hadron collisions.

In this paper we discuss the relation between elastic and inelastic collisions in high energy interactions of two composite objects in the limit of infinitely many constituents. We argue that the elastic amplitudes and total cross-sections are given with good accuracy by Glauber model[1]. As shown below this assumption uniquely implies that the inelastic processes are given by incoherent composition of probabilities for interactions of independent constituents.

The Glauber model expression for the elastic amplitude of two composite systems with A and B constituents is, for a given impact parameter $\boldsymbol{b}$ [1, 2]:

$$\mathcal{M}(\boldsymbol{b}) = 1-e^{-\frac{1}{2}\sigma_{\mathrm{T}}AB\int d^2 s D_A(\boldsymbol{b}-\boldsymbol{s})D_B(\boldsymbol{s})}, \tag{1}$$

where σ_{T} is the total cross section for scattering of one constituent from A with one constituent from B (elementary collision). A and B are assumed very large. D_A and D_B are single constituent densities projected on the plane perpendicular to the direction of motion:

$$D_{A,B}(\boldsymbol{s}) = \int_{-\infty}^{+\infty} dz \varrho_{A,B}(\boldsymbol{s}, z). \tag{2}$$

---

* Address: Instytut Fizyki UJ, Reymonta 4, 30-059 Kraków, Poland.

** Address: Instytut Fizyki Jądrowej, Radzikowskiego 152, 31-342 Kraków, Poland.

[1] The inelastic shadowing contributes — in e. g. nuclear collisions — corrections to the integral cross sections of the order of magnitude of 10% only.

In (1) we assumed that the elastic amplitude of the elementary collision is purely imaginary. All the results will be obtained under this assumption.

As shown by Van Hove [3] the information about inelastic collisions relevant to the elastic collisions is contained in the overlap function, related to the elastic amplitude by the formula

$$\sigma_{in}(b) = 2\,\mathrm{Re}\,\mathcal{M}(b) - |\mathcal{M}(b)|^2. \tag{3}$$

From the formulae (1) and (3) we thus obtain

$$\sigma_{in}(b) = 1 - e^{-\sigma_T AB \int d^2 s D_A(b-s) D_B(s)}. \tag{4}$$

We shall show now that Eq. (4) follows from incoherent composition of probabilities [4] for interactions of independent constituents. Indeed the probability calculus gives the following expression for

$$\sigma_{in}(b) = \int \prod_{l=1}^{A} \prod_{k=1}^{B} d^2 s_l^A d^2 s_k^B D_A(s_l^A) D_B(s_k^B) \left[1 - \prod_{i=1}^{A} \prod_{j=1}^{B} (1 - \sigma_T(b - s_i^A + s_j^B))\right]. \tag{5}$$

An analogous reasoning as given in Ref. [3] shows that the leading contribution to (5) for $A, B \to \infty$ (with the constraint $\sigma_T AB = \mathrm{const}$) is (we approximate $\sigma_T(b) \approx \sigma_T \delta^{(2)}(b)$)

$$\sigma_{in}(b) = 1 - \left[1 - \sigma_T \int d^2 s D_A(b-s) D_B(s)\right]^{AB}, \tag{6}$$

hence it coincides with Eq. (4) for $A, B \gg 1$. The consequence of this theorem is that the incoherent composition of independent inelastic interactions is justified if Glauber model is accepted for elastic amplitudes.

The above considerations indicate that in the case of nuclear collisions the overlap function calculated from formula (5) gives a good description of the elastic scattering. On the other hand it is known that in hadron-hadron collisions the overlap function calculated directly from inelastic cross-sections gives the slope of the elastic scattering much smaller than found in experiment [5, 6, 7]. In the following we wish to discuss the possible source of this discrepancy which is suggested by the above discussion.

First we observe that the particles produced in collisions of two composite objects may come from two sources: (*a*) particles produced in collisions of different constituents, and (*b*) spectators arising from break-up of the incident objects. In calculations of the overlap slope the role of spectators is of primary importance, since they are very strongly peaked forward and backward in c. m. system — thus giving a large contribution to the slope. Moreover, the spectrum of spectators is determined uniquely by the wave functions of the composite objects. Consequently, at least in the limit of infinitely many constituents, it is the contribution from spectators which guarantees that the spatial extensions of the colliding objects are properly taken into account and thus slope of the overlap correctly estimated.

It follows from this argument that if the distribution of spectators is not treated carefully, the resulting slope fails to describe the spatial extension of the system and thus differs from the one observed in experiment. This may happen in particular if only single particle inclusive (or semi-inclusive) distributions are used to fit the data, as is the case in Refs

[5, 6, 7]. In such calculations most of the structure of the production amplitude is averaged over and the spectator spectrum is distorted by: (*a*) a background of particles produced in collisions of different constituents, (*b*) a possible mixing of spectators from two colliding objects, and finally (*c*) complicated transitions from constituent states to the observed final states which destroy the possible factorization of the wave function[2].

To summarize, the presented geometrical picture of hadron-hadron collisions implies a rather rich structure of the production amplitudes. This structure is missing in existing simple calculations of the overlap function. Our analysis suggests that a possible improvement can be obtained if the process of the fragmentation of the incident hadrons is better understood.

REFERENCES

[1] T. T. Chou, C. N. Yang, *Phys. Rev.* **170**, 1591 (1968).
[2] W. Czyż, L. C. Maximon, *Ann. Phys.* (USA) 52, 59 (1969).
[3] L. Van Hove, *Rev. Mod. Phys.* **36**, 655 (1964).
[4] A. Białas, M. Błeszyński, W. Czyż, *Nucl. Phys.* **B. 111**, 461 (1976).
[5] L. Michejda, *Nucl. Phys.* **B4**, 13 (1968), L. Michejda, *Fortschr. Phys.* **16**, 707 (1968).
[6] L. Michejda, J. Turnau, A. Białas, *Nuovo Cimento* **A56**, 241 (1968),
[7] S. Jadach, J. Turnau, *Phys. Lett.* **50B**, 369 (1974), S. Jadach, J. Turnau, *Acta Phys. Pol.* **B5**, 677 (1974).
[8] Z. Ajduk, R. Stroynowski, *Phys. Lett.* **30B**, 179 (1969).
[9] Z. Ajduk, *Nuovo Cimento* **16A**, 111 (1973).
[10] S. Pokorski, L. Van Hove, *Acta Phys. Pol.* **B5**, 229 (1974).
[11] A. Białas, N. Sakai, *Phys. Lett.* **55B**, 81 (1975).

---

[2] The importance of a careful treatment of the fragmentation region is supported by the analysis of Refs [8, 9, 10, 11].