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CONSTITUENT QUARKS AND DEVIATIONS FROM BJORKEN SCALING*

By W. Królikowski

Institute of Theoretical Physics, Warsaw University, Warsaw**

AND

M. Święcki

Nicolaus Copernicus Astronomical Centre, Polish Academy of Sciences, Warsaw***

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We interpret the observed departure from Bjorken scaling as a coherent effect of the internal structure of constituent quarks in the nucleon. Then the elastic form-factor of the constituent quark might be determined by precise measurements of this departure, giving crucial information on the nature of quarks. The presented model is orthogonal to that proposed by Chanowitz and Drell.

In this note we propose to interpret the recently observed departure from Bjorken scaling in deep inelastic electron [1] and muon [2] scattering as a coherent effect of the internal structure of constituent quarks in the nucleon. These are presumed to manifest themselves as some clusters of partons, i. e. of current quarks and gluons, providing coherent corrections to incoherent scattering described by the parton model. It is rather plausible that in the limit of infinite momentum transfer (or, practically, for Q^2 big enough) these parton clusters should be considered as completely dissociated and, consequently, the coherent corrections to the parton model should vanish (cf. [3]). Then the "true" Bjorken scaling should set in if we assume that the possible log Q^2-terms of field-theoretical origin can be neglected in the region under consideration, i. e. that the respective mass scale is very large.

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** Address: Instytut Fizyki Teoretycznej, Uniwersytet Warszawski, Hoża 69, 00-681 Warszawa, Poland.

*** Address: Centrum Astronomiczne PAN, Al. Ujazdowskie 4, 00-478 Warszawa, Poland.

The above picture of deep inelastic scattering is not necessarily inconsistent with the quark-gluon field theory. In particular, the zeroth moment of the structure function $\nu W_2(\nu, Q^2)$ with respect to $x = Q^2/2M\nu$ may be constant already in the present deep inelastic region. We will assume in fact this constancy. This assumption will enable us to connect the increase in $\nu W_2(\nu, Q^2)$ for small x with the decrease in $\nu W_2(\nu, Q^2)$ for large x (when Q^2 is growing) if we apply an adequate scaling variable $\bar{x} = \bar{x}(x, Q^2)$ which increases to x (with growing Q^2).

In order to connect the nucleon structure function $\nu W_2(\nu, Q^2)$ with the constituent-quark elastic form-factor inside the nucleon let us assume that the hadron production in deep inelastic lepton-nucleon scattering can be neglected if not accompanied by the desintegration of the constituent quark which is actually struck by the virtual photon. The probability of this desintegration is equal to $1-G_q^2(Q^2)$, where $G_q^2(Q^2)$ is the probability that the constituent quark survives the collision with the virtual photon. Thus, under our assumption we get

$$\nu W_2(\nu, Q^2) = [1-G_q^2(Q^2)]\, F_2[\bar{x}(x, Q^2)], \tag{1}$$

where $G_q^2(Q^2) \to 0$ and $\bar{x}(x, Q^2) \to x$ for $Q^2 \to \infty$. Here, the function $G_q(Q^2)$ has the interpretation of the constituent-quark elastic form factor inside the nucleon. It is related to the lepton-constituent-quark elastic cross-section inside the nucleon by formula

$$\frac{d\sigma_{lq}}{d\Omega} = \left(\frac{d\sigma_{lq}}{d\Omega}\right)_{\text{Mott}} G_q^2(Q^2). \tag{2}$$

When the constituent quark survives the collision, the nucleon as a whole may *(i)* also survive (then we observe elastic scattering on the nucleon) or *(ii)* be excited into an isobar whose subsequent decay contributes to coherent hadron production off the nucleon, or finally *(iii)* desintegrate into pieces contributing to incoherent hadron production. Since, due to our assumption, we can neglect hadron production in the deep inelastic region unless the constituent quark desintegrates, we in fact assume that contributions from the processes *(ii)* and *(iii)* are negligible in comparison with the correction term $G_q^2(Q^2)F_2[\bar{x}(x, Q^2)]$ which is taken into account in formula (1). It means that the possible additional term $G_q^2(Q^2)F_2'(x, Q^2)$ in formula (1), connected with processes *(ii)* and *(iii)*, is assumed to be negligible in the deep inelastic region i. e., $F_2 \gg F_2'$ there.

It was argued [4] that for smaller values of Q^2, where the "precocious" Bjorken scaling is being turned on and the zeroth moment of $\nu W_2(\nu, Q^2)$ is tending to its constant asymptotic value, the structure function $\nu W_2(\nu, Q^2)$ should contain the factor $1-G_N^2(Q^2)$ describing coherent corrections due to the nucleon structure as a whole. In this case $G_N(Q^2)$ is the nucleon elastic form-factor defined by formula

$$\frac{d\sigma_{lN}}{d\Omega} = \left(\frac{d\sigma_{lN}}{d\Omega}\right)_{\text{Mott}} G_N^2(Q^2), \tag{3}$$

where the left-hand side is the Rosenbluth lepton-nucleon elastic cross-section. In the present note we apply the idea of Ref. [4] to the constituent quark rather than to the nucleon as a whole.

Following the suggestion of the quark-gluon field theory let us assume in addition that for the form (1) we have

$$I_2 \equiv \int_0^1 dx\nu W_2(\nu, Q^2) = \text{const.} \tag{4}$$

Then, taking $Q^2 \to \infty$ we get

$$I_2 = \int_0^1 dx F_2(x). \tag{5}$$

From formulae (1) and (4) we obtain

$$\frac{1}{1-G_q^2(Q^2)} = \frac{1}{I_2}\int_0^1 dx F_2[\bar{x}(x, Q^2)]. \tag{6}$$

Formula (6), if true, expresses $G_q(Q^2)$ in terms of the asymptotic nucleon structure function $F_2(x)$ and the adequate scaling variable $\bar{x}(x, Q^2)$. Thus (6) may be used to measure $G_q(Q^2)$. If the asymptotic form $\bar{x}(x, Q^2)$ is

$$\bar{x}(x, Q^2) \underset{Q^2\to\infty}{\simeq} \left[1-\left(\frac{\overline{M}^2}{Q^2}\right)^{\kappa}\alpha(x)\right]x, \tag{7}$$

then we conclude from (6) that

$$G_q^2(Q^2) \underset{Q^2\to\infty}{\simeq} \left(\frac{\overline{M}^2}{Q^2}\right)^{\kappa}\frac{1}{I_2}\int_0^1 dx\,\frac{d}{dx}[x\alpha(x)]F_2(x). \tag{8}$$

For instance, if

$$\frac{1}{\bar{x}} = \frac{1}{x} + \left(\frac{\overline{M}^2}{Q^2}\right)^{\kappa}, \tag{9}$$

we have $\alpha(x) = x$ and get

$$G_q^2(Q^2) \underset{Q^2\to\infty}{\simeq} \left(\frac{\overline{M}^2}{Q^2}\right)^{\kappa}\frac{1}{I_2}\,2\int_0^1 dx\,xF_2(x). \tag{10}$$

Thus, if the adequate scaling variable $\bar{x}$ is the Bloom-Gilman variable x' then $\overline{M} = M$ and $\kappa = 1$ and we conclude that

$$G_q^2(Q^2) \underset{Q^2\to\infty}{\simeq} \frac{M^2}{Q^2}\,\frac{1}{I_2}\,2\int_0^1 dx\,xF_2(x) \tag{11}$$

which gives certainly a non-conventional asymptotic behaviour for an elastic form-factor. In order to get for $G_q(Q^2)$ the more conventional hadron-like behaviour $1/Q^2$ or $1/Q^4$ we should take $\kappa = 2$ or $\kappa = 4$, respectively. We should keep in mind, however, that quarks are presumably non-conventional particles, having a simpler structure than hadrons and appearing so far only in zero-triality bound states. The square-root behaviour $1/\sqrt{Q^2}$ of $G_q(Q^2)$ given by (11), if true, would mean that the constituent quarks are more localized (and peaked) structures in space than hadrons. This would explain why the coherent correction due to the constituent-quark structure, $1-G_q^2(Q^2)$, are significant with growing Q^2 longer than those due to the nucleon structure as a whole, $1-G_N^2(Q^2)$, even if the respective characteristic masses are of the same order of magnitude.

It may be intriguing to notice that the behaviour $1/\sqrt{Q^2}$ for $Q^2 \to \infty$ of the elastic form-factor would be consistent with the Drell-Yan-West rule [5] if the inelastic structure function of the constituent quark behaved as $(1-x)^0 = 1$ for $x \to 1$. The latter behaviour in turn would be consistent with statistical equipartition of the longitudinal momentum among two partons within the constituent quark. In fact, in the case of statistical equipartition of the longitudinal momentum among n partons the one-parton correlation functions are [6]

$$f_\alpha^{(n)}(x) = \int_0^1 \dots \int_0^1 dx_1 \dots dx_n (n-1)! \delta(x_1 + \dots + x_n - 1)\delta(x - x_\alpha) = (n-1)(1-x)^{n-2}$$

$$(\alpha = 1, 2, \dots, n; n \geqslant 2), \tag{12}$$

thus $f_\alpha^{(2)}(x) = 1$. So, we should gain in this case an argument for the composite model of coloured quarks discussed previously ([7], cf. also [8, 9]), where the structure of the constituent quark was dominated for $x \to 1$ by a two-parton configuration of a current quark and a gluon (= "coquark") forming together a current coloured quark. Let us mention that in such a composite model of quarks, if we used the statistical approximation (12), we should have the behaviours $1/(Q^2)^{2.5}$ and $(1-x)^4$ for the nucleon [7] and $1/(Q^2)^{1.5}$ and $(1-x)^2$ for the pion. These predictions are, perhaps, not inconsistent with the newest nucleon data [1].

Finally, let us remark that the argument in the present note leading to the form-factor $1-G_q^2(Q^2)$ in formula (7) is orthogonal to the idea of Chanowitz and Drell [10] who introduced instead a dressed-quark elastic form-factor, assuming thereby that the dressed quark survives the collision with the virtual photon without desintegrating into pieces. With growing Q^2 the Chanowitz-Drell form-factor decreases to zero, whereas $1-G_q^2(Q^2)$ increases to unity.

In conclusion, we can see that formulae (1) and (4) are consistent with the qualitative features of the observed scaling violation. In fact, for growing Q^2 the structure function $\nu W_2(\nu, Q^2)$ given by (1) and (4) increases for small x and decreases for large x. More precise measurements of the scaling violation are necessary in order to check whether the functions $G_q(Q^2)$ and $\bar{x}(x, Q^2)$ (as well as $F_2(x)$) can be determined in such a way that the relations (1) and (4) be quantitatively consistent with the data. If it was true, we should be tempted to say the elastic form-factor of the constituent quark inside the nucleon has been measured. If, in this case, it turned out that $G_q(Q^2) \sim 1/\sqrt{Q^2}$ for $Q^2 \to \infty$ (or equivalently, if the adequate scaling variable $\bar{x}$ was asymptotically of the Bloom-Gilman type), then the intriguing possibility would arise that the constituent quarks are composite quark-coquark structures as it was suggested previously ([7] cf. also [8, 9]). We hope to come back to this discussion elsewhere.

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Note added in proof: The square-root behaviour $1/\sqrt{Q^2}$ of the quark form-factor $G_q(Q^2)$ might be consistent with the observed $1/p_\perp^8$ dependence of the large p_T inclusive cross-section for the process $p+p \to \pi +$anything, if the acting mechanism was the hard qq collision. In fact, we should have in this case the factor $G_q^4(Q^2)/Q^4 \sim 1/Q^8$ leading to the $1/p_\perp^8$ dependence. This remark we owe to Dr. S. Pokorski.

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