Vol. B7 (1976) ACTA PHYSICA POLONICA No 11

DIFFRACTIVE DISSOCIATION IN UNCORRELATED JET MODEL

By J. Stern*

Institute of Physics, Jagellonian University, Cracow

(Received July 13, 1976)

Numerical estimates of diffractive dissociation in uncorrelated jet model are performed. The model describes fairly well the absolute magnitude and the general behaviour of diffractive cross-section. It fails badly, however, for the multiplicity distribution. This is mainly due to the fact that only one coherent state has been used as the input.

1. *Introduction*

As is generally known (see e. g. Ref. [1]), the diffractive production can be calculated from amplitudes of non-diffractive production by using the unitarity condition. Assuming that the non-diffractive production is described by a hadronic bremsstrahlung model [2, 3], the high-energy limit of diffractive cross-sections were obtained [4], taking into account formulae of Ref. [1] and using the method of de Groot [5].

In the present paper we continue this study of diffractive dissociation in uncorrelated jet model. We perform numerical estimates and show in some more detail how diffractive and also non-diffractive cross-sections and multiplicities behave and to what extent the model agrees with experimental data for proton-proton collisions. Two versions of the model are considered. In the first one it is assumed that the produced objects are pions. In the second version the produced objects are clusters. In both versions we find a qualitative agreement with data on diffractive production although the version with clusters is better. However, the model is not good for description of non-diffractive processes: discrepancies occur when in the pion version the average total multiplicity and in both versions the KNO scaling function are compared with experiment.

The origin of the discrepancy can be traced to the fact that only one coherent state has been used as the input instead of a superposition of coherent states.

---

* Fellow of the I. I. K. W., Belgium. On leave from the Instituut voor Theoretische Fysica, Katholieke Universiteit Leuven, Belgium. Mailing address: Celestijnenlaan 200 D, B-3030 Heverlee, Belgium.

In spite of this drawback, and maybe because of it, we think that these results show clearly which restrictions should be taken into account when building up a model for multiparticle production.

In Sections 2 and 3 we describe diffractive and non-diffractive production when the pions are produced directly and we compare the results with the data. We do the same for the average total multiplicity and the KNO scaling function in Section 4. In Section 5 we confront with the data the version of the model where clusters are produced. Section 6 contains an explanation of the discrepancies we have found.

2. *Diffractive production*

It was shown in Ref. [4], taking into account formulae of Ref. [1], that the diffractive cross-sections for production of N mesons are given by

$$\sigma_N^{\mathrm{D}} = \sigma_{\mathrm{el}} \frac{1}{N+1} \left(\frac{\lambda \langle q_\perp^2 \rangle \ln s/\bar{\mu}^2}{\Omega_1^2} \right)^N, \tag{2.1}$$

with

$$\Omega_1^2 = \lambda \langle q_\perp^2 \rangle (\ln s/\bar{\bar{\mu}}^2 - 2\psi(\lambda+1) + 1/\lambda\alpha \langle q_\perp^2 \rangle),$$

for N even, and by

$$\sigma_N^{\mathrm{D}} = 0$$

for N odd. In formula (2.1), s is the squared total c. m. energy of the system and $\psi(z)$ the digamma function. Parameter λ is the meson coupling constant, $\langle q_\perp^2 \rangle$ the average squared transverse momentum of a meson and α equals to half of the slope of the elastic cross-section. Parameter $\bar{\bar{\mu}}$ is defined by

$$\langle q_\perp^2 \rangle \ln \bar{\bar{\mu}} = \int d^2 q_\perp \ln \left(\sqrt{q_\perp^2 + \mu^2}\, e^\gamma\right) q_\perp^2 f(q_\perp), \tag{2.2}$$

where $f(q_\perp)$ is the normalized transverse momentum distribution of a meson, μ its mass and γ Euler's constant equal to 0.5772.

In order to study the behaviour of σ_N^{D} with the energy s and the number of produced mesons N, we have first to assume some definite transverse momentum distribution for the mesons. We follow the suggestion of Barshay and Chao [6] and take $f(q_\perp)$ in the form

$$f(q_\perp) = \frac{\beta^2}{2\pi\, \Gamma(2, \beta\mu)}\, e^{-\beta\sqrt{q_\perp^2 + \mu^2}}, \tag{2.3}$$

where $\Gamma(a, z)$ is the incomplete gamma function and β a parameter which can be easily determined once $\langle q_\perp^2 \rangle$ and μ are known. Furthermore, we take λ equal to 1 in order to obtain a flat leading particle spectrum [2, 3], and α equal to 6. We also assume that the produced mesons are pions. Thus we take μ equal to the pion mass and $\langle q_\perp^2 \rangle$ equal to 0.16 $(\mathrm{GeV}/c)^2$. This allows us to determine β and $\bar{\bar{\mu}}$, for which we find respectively the values 6.5433 and 1.0395. The value of β is in fairly good agreement with the fitted value of Barshay and Chao which is 6.75.

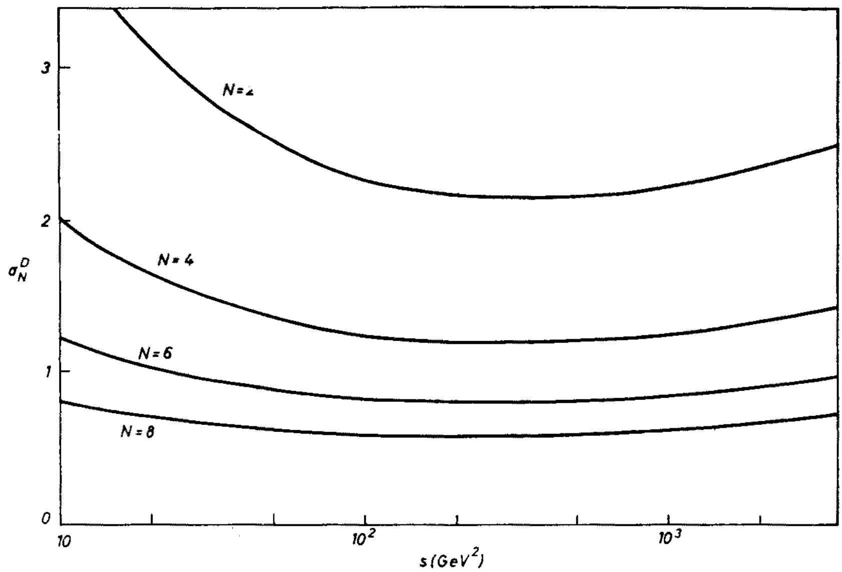

Fig. 1. Cross-sections for diffractive production of N pions versus energy s in case of pion production

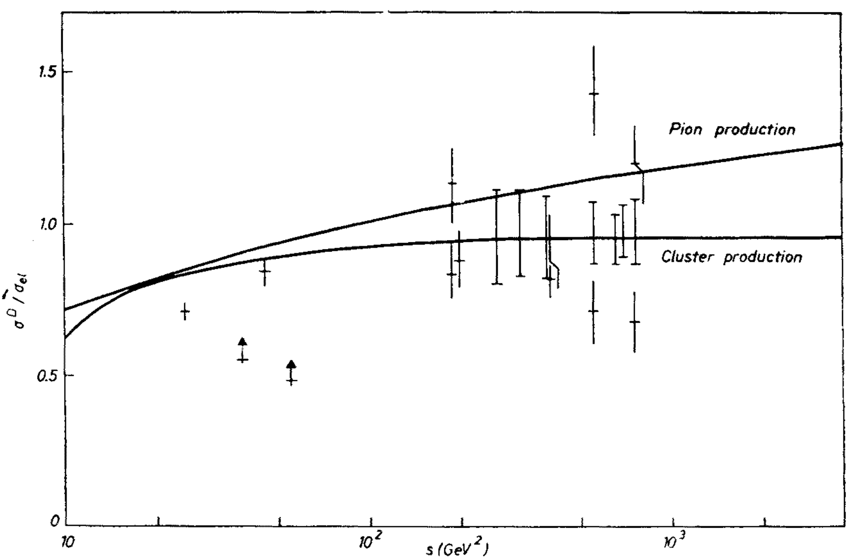

Fig. 2. Ratio of the total diffractive cross-section to the elastic cross-section versus energy s. Experimental points are taken from a compilation of Fiałkowski and Miettinen [9]

Now, all parameters being determined, we are able to represent the diffractive cross--section (2.1) for some values of N as a function of energy s. This is done in Fig. 1. One secs that the cross-sections first decrease with growing energy up to an energy between 100 and 350 GeV^2 and then increase with it. The decrease and increase happen faster for small N. Moreover the overall magnitudes of the cross-sections decrease rather rapidly with growing N, which is in marked difference with Regge-pole models which predict σ_N^{D} to be approximately independent of N [7,8].

In view of confrontation with experiment we now sum the diffractive cross-sections (2.1) over all N's different from zero and obtain for the total diffractive cross-section something of the order of the elastic one. The ratio $\sigma^{\mathrm{D}}/\sigma_{\mathrm{el}}$ is plotted in Fig. 2 and compared to experimental values [9]; the agreement is reasonable.

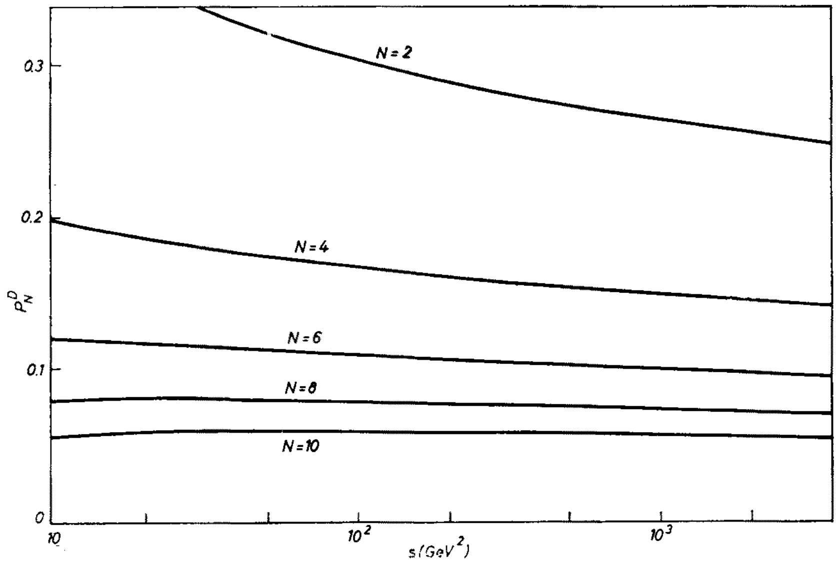

Fig. 3. Probability of production of N pions diffractively versus energy s for pion production

We can also determine the probability of producing N pions diffractively. Fig. 3 shows its behaviour for some values of N versus energy s. It appears to be almost Furry's distribution with parameter proportional to ln s, i. e. to the average diffractive multiplicity.

Indeed, the average diffractive multiplicity grows approximately logarithmically with energy. At high energies it is fairly well given by

$$\langle N^D \rangle = 0.7 \ln s/\bar{\mu}^2 + 1.2. \tag{2.4}$$

As expected, the coefficient in front of the ln s term appears to be smaller than the one obtained in non-diffractive production, as we shall see in next section, where we give a short description of the non-diffractive production.

3. *Non-diffractive production*

In essentially the same way as has been described in Ref. [4] to obtain the diffractive cross-sections for production of N mesons, we can calculate the non-diffractive ones, using formulas of Ref. [1]. Expressed in terms of σ_{el}, the result is

$$\sigma_N^{\mathrm{ND}} = 8\pi^2[\Gamma(\lambda+1)]^2\, \Omega_1^4 \sigma_{\mathrm{el}} \left(\frac{s}{\bar{\mu}^2}\right)^{-\lambda} \frac{(\lambda \ln s/\bar{\mu}^2)^N}{(N+2)\, N!}, \tag{3.1}$$

where $\Gamma(z)$ is the gamma function and $\bar{\mu}$ is defined by

$$\ln \bar{\mu} = \int d^2q_\perp \ln (\sqrt{q_\perp^2 + \mu^2 e^\gamma}) f(q_\perp). \tag{3.2}$$

For pion production we find $\bar{\mu}$ equal to 0.5804.

From equation (3.1) we obtain easily the probability of producing N mesons non--diffractively, which is

$$P_N^{\mathrm{ND}} = e^{-(\langle N^{\mathrm{ND}}\rangle + 1)} \frac{(\langle N^{\mathrm{ND}}\rangle + 1)^{N+1}}{(N+2)\, N!}, \tag{3.3}$$

where the average non-diffractive multiplicity is given by

$$\langle N^{\mathrm{ND}}\rangle = \lambda \ln s/\bar{\mu}^2 - 1. \tag{3.4}$$

We see that the distribution (3.3) is similar to the Poisson distribution.

In case the produced mesons are pions, the ratios of the total diffractive cross--section to the total non-diffractive cross-section and to the inelastic one decrease fast with energy and are at 3500 GeV^2, respectively, equal to 33.8% and 25.3%. Let us now look at the total average multiplicity and the KNO scaling function.

4. *Average total multiplicity and KNO scaling function*

Since we have now the explicit formulae for diffractive and non-diffractive production it is useful to look at the average total charged multiplicity $\langle N_c\rangle$ and at the KNO scaling function [10] in order to compare them with experiment. Therefore we use a two--component scheme as was suggested e. g. by Fiałkowski and Miettinen [11]. We do it here for the pion production.

The average multiplicity is given by

$$\langle N\rangle = \varrho\langle N^{\mathrm{D}}\rangle + (1-\varrho)\langle N^{\mathrm{ND}}\rangle, \tag{4.1}$$

where

$$\varrho = \sigma^{\mathrm{D}}/\sigma_{\mathrm{in}}.$$

This gives us

$$\langle N_c\rangle = \tfrac{2}{3}\langle N\rangle + 2. \tag{4.2}$$

In Fig. 4 ln $\langle N_c\rangle$ is plotted versus ln s. Experimental data points are taken from Ref. [12]. We see here that $\langle N_c\rangle$ is somewhat too small at high energies where the model is supposed to give a correct description. That discrepancy will be removed, however, in case of cluster production.

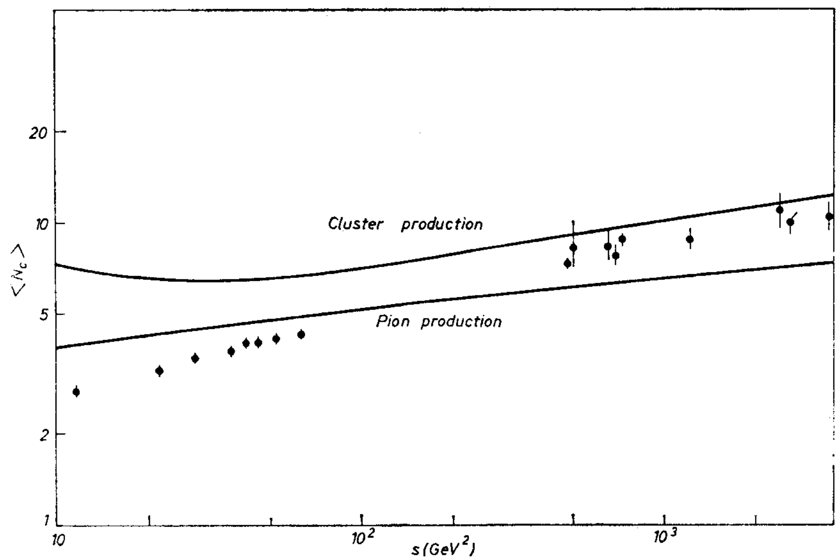

Fig. 4. Average total charged multiplicity versus energy s compared to experimental data taken from Ref. [12]

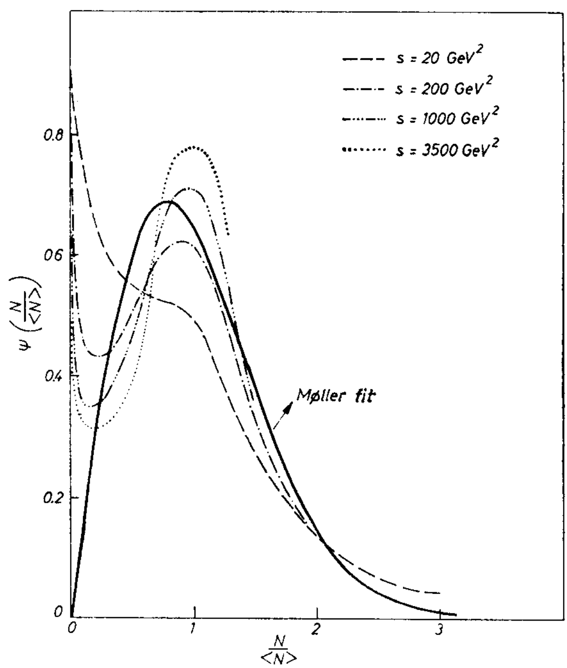

Fig. 5. KNO scaling function $\psi\left(\frac{N}{\langle N\rangle}\right)$ versus $N/\langle N\rangle$ for pion production compared to the experimental fit of Møller [13]

Let us look at Fig. 5. There we have plotted the KNO scaling function defined by

$$\psi\left(\frac{N}{\langle N\rangle}\right) = \langle N\rangle \frac{\sigma_N^{\mathrm{D}}+\sigma_N^{\mathrm{ND}}}{\sigma_{\mathrm{in}}} \tag{4.3}$$

versus $N/\langle N\rangle$ and compared to the experimental fit of Møller [10]. Because of the peculiar fact that for N odd σ_N^{D} is equal to zero, we took the arithmetic average of the separate curves that we obtained for N odd and for N even at a given energy. We immediately see that there is no scaling and that at high energies the diffraction peak develops while the non-diffractive bump grows. This feature shows that the model fails badly in the description of multiplicities and distributions.

5. *Cluster production*

What we did in previous sections for the pion production is now repeated for the case where the produced objects are clusters which we assume to decay isotropically into 3 pions. Formulae given above remain valid for meson cluster production, however the parameters β, $\bar{\mu}$ and $\bar{\bar{\mu}}$ are now functions of the cluster mass $\mathscr{M}$ and its average squared c. m. transverse momentum $\langle P_\perp^2\rangle$, rather than the pion mass and the pion transverse momentum.

We shall first determine the parameters. Next, we shall point out the differences between the behaviour of cross-sections and multiplicities of pions produced directly and of pions produced via clusters.

A) Parameters

To determine β, $\bar{\mu}$ and $\bar{\bar{\mu}}$ we have to know $\mathscr{M}$ and $\langle P_\perp^2\rangle$. We can get an estimate of them by using the assumption about the cluster decay. Indeed, as done in Ref. [14], this assumption allows us to average over the decay distribution and cluster production and to write the following relation between $\mathscr{M}$ and $\langle P_\perp^2\rangle$:

$$\langle q_\perp^2\rangle = \frac{2}{3}\left(\frac{\mathscr{M}^2}{9}-\mu^2\right)\left(1+2\frac{\langle P_\perp^2\rangle}{\mathscr{M}^2}\right)+\mu^2\,\frac{\langle P^2\rangle}{\mathscr{M}^2}, \tag{5.1}$$

where μ is the decay pion mass and $\langle q_\perp^2\rangle$ its average squared c. m. transverse momentum which we take equal to 0.16 $(\mathrm{GeV}/c)^2$. To obtain a total diffractive cross-section which does not diverge we have moreover to satisfy the inequality

$$\langle P_\perp^2\rangle < \frac{1}{2\lambda\alpha\psi(\lambda+1)} \tag{5.2}$$

as implied by equation (2.1). Thus for λ equal to 1 and α equal to 6 we have to take

$$\langle P_\perp^2\rangle < 0.1971\ (\mathrm{GeV}/c)^2 \tag{5.3}$$

and, thus with relation (5.1),

$$\mathscr{M} > 1.3941\ \mathrm{GeV}. \tag{5.4}$$

In Ref. [2] it was estimated that $\langle \mathcal{M} \rangle$ is equal to 1.3 GeV when the clusters decay into 3 pions on the average. Therefore we assume $\mathcal{M}$ to be close to its lower bound and take it equal to 1.4 GeV. With this value, we obtain from relation (5.1) that $\langle P_\perp^2 \rangle = 0.1887$ $(\text{GeV}/c)^2$.

This gives us a value of 16.1765 for the slope β of the transverse momentum distributions of the clusters. We find also $\bar{\mu}$ equal to 2.6059 and $\bar{\bar{\mu}}$ equal to 2.7229. We note immediately that this value of β is much bigger than the one we found in case of pion production. In reality it is so big that the transverse momentum distribution of the clusters approximately becomes

$$f(P_\perp) = \delta(P_\perp^2 - \langle P_\perp^2 \rangle). \tag{5.5}$$

This can be seen from the value of $\bar{\mu}$ (equal to $\bar{\bar{\mu}}$) which is here 2.6108 and thus very close to the values of $\bar{\mu}$ and $\bar{\bar{\mu}}$ that we obtained here above.

B) Cross-sections and multiplicities

Using the assumption that each produced cluster decays into 3 pions, we can represent similarly, as we did in previous sections, cross-sections, multiplicities and KNO scaling function for the production of the decay pions.

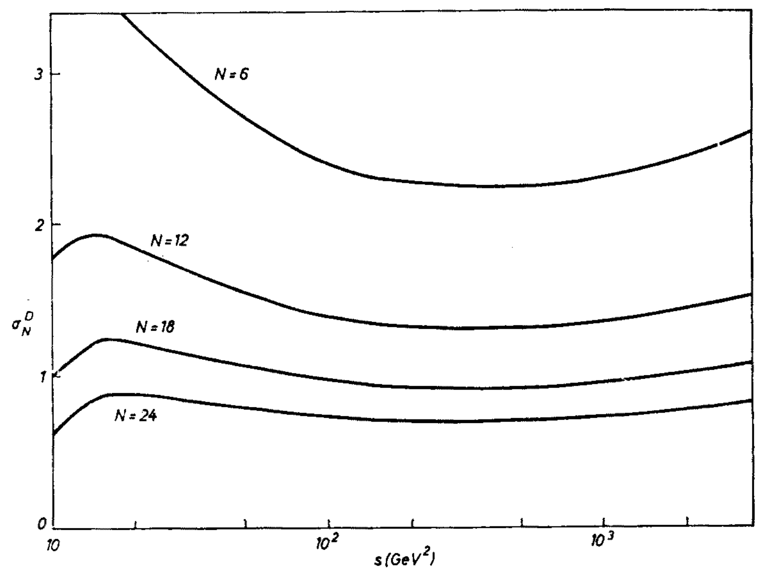

Fig. 6. Same as Fig. 1 for cluster production

In Fig. 6 we have plotted the diffractive cross-section (2.1) versus energy s for some values of N, the number of produced pions. Comparing to Fig. 1 we note that the overall magnitudes of the cross-sections have strongly increased.

When we sum the diffractive cross-sections (2.1) over the even N's up to 10, we obtain the total diffractive cross-section. We compare it to experimental data on σ^D/σ_{el}. This is done in Fig. 2. The agreement now is quite good.

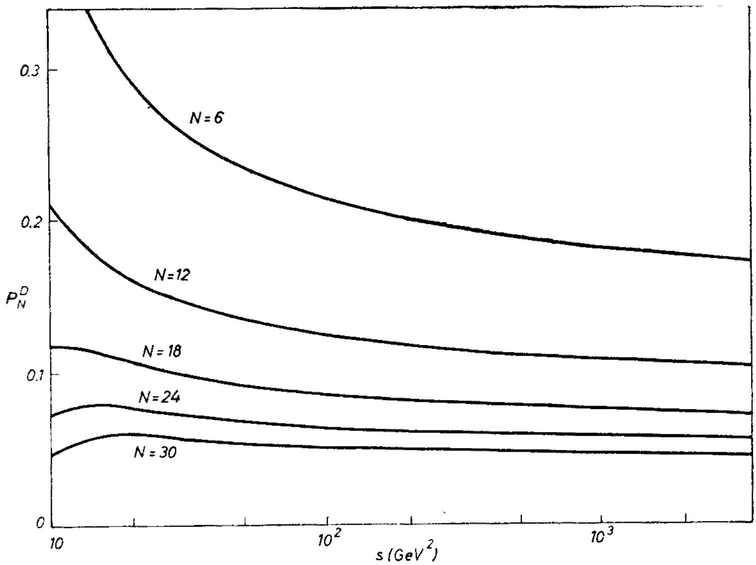

Fig. 7. Same as Fig. 3 for cluster production

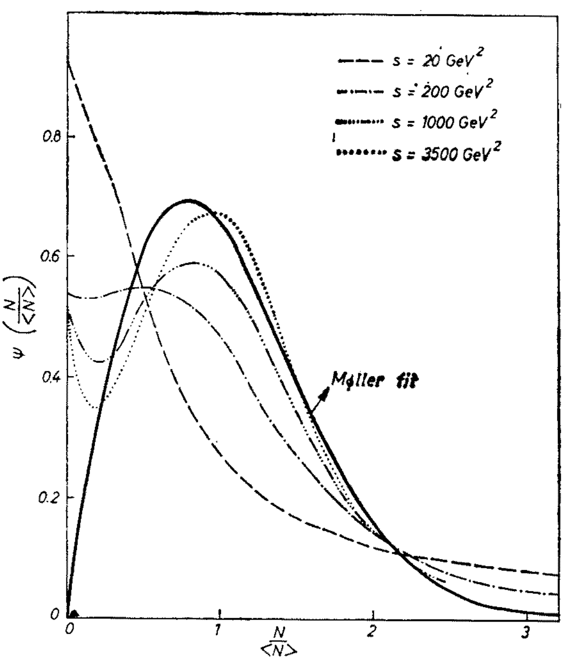

Fig. 8. Same as Fig. 5 for cluster production

In Fig. 7 the probability of producing N pions diffractively is represented. We see that the bumps which are typical to Furry's distribution have now appeared at low energies (as compared to these observed in Fig. 3). The parameter of this distribution is again proportional to the average diffractive multiplicity which is here however approximately energy independent. Indeed for $s \geqslant 50$ GeV^2, it is very well given by

$$\langle N^D \rangle = 0.06 \ln s/\bar{\bar{\mu}}^2 + 13.5. \tag{5.6}$$

The ratios of the total diffractive cross-section to the total non-diffractive cross--section and to the inelastic one for cluster production at 3500 GeV^2 are reduced by approximately 30% when compared to the one obtained for pion production. They are respectively equal to 22% and 18%.

In Fig. 4 where we represented the average total number of charged pions produced directly, we now plot the same quantity for charged pions which are produced via clusters. The discrepancy we noted at high energies has disappeared and the agreement with data is reasonable.

Let us finally look at Fig. 8 to the KNO scaling function, which with our assumptions is the same for the production of the pions and of the clusters. We see that the discrepancy with Møller's fit is still very big, although it is reduced when compared to what was obtained in Fig. 5. Thus the model still fails badly for multiplicity distribution. In the next section we discuss the possible origin of this discrepancy.

6. *Discussion*

We shall try to see how the approximations which have been done in the model can explain the discrepancies we found in previous Sections 4 and 5B.

The important approximation made in Ref. [1] was that the authors have represented the diffractive amplitude by the overlap matrix element. This is correct only to the second order in the overlap matrix. Although it is presumably rather difficult to estimate the error induced by this approximation, it may be quite substantial, particularly at small impact parameters, where σ_{el}/σ_{tot} is large. It would be certainly very interesting to estimate these higher-order effects.

If one sticks to the approximation of Ref. [1], however, then there exists another way of removing the discrepancy we found for the KNO function. This is to use as the input a superposition of the coherent states and not just one of them. If we assume for non--diffractive production that the distribution we found is valid for only one component, to get the observed distribution we have to take a weighted average over the distributions describing the different components. This will essentially broaden the pseudo-Poisson distribution and give a better behaviour for the KNO function in Figs. 5 and 8.

Thus we can say that the disagreements we found with experimental data clearly indicate that a superposition of coherent states has to be used as the input in multiparticle production models. This points out the direction of further work in this field.

I want to express my gratitude to Dr A. Białas for helpful discussions. I thank also Dr A. Kotański for reading the manuscript and the Jagellonian University for financial support.

REFERENCES

[1] A. Białas, A. Kotański, *Acta Phys. Pol.* **B4**, 659 (1973).
[2] S. Pokorski, L. Van Hove, *Acta Phys. Pol.* **B5**, 229 (1974).
[3] L. Stodolsky, *Phys. Rev. Lett.* **28**, 60 (1972).
[4] A. Białas, J. Stern, K. Zalewski, *Acta Phys. Pol.* **B7**, 197 (1976).
[5] E. H. de Groot, *Nucl. Phys.* **B48**, 295 (1972).
[6] S. Barshay, Y.-A. Chao, *Phys. Lett.* **38B**, 225 (1972).
[7] W. R. Frazer, D. R. Snider, *Phys. Lett.* **45B**, 136 (1973).
[8] R. Roberts, D. P. Roy, *Phys. Lett.* **46B**, 201 (1973).
[9] K. Fiałkowski, H. I. Miettinen, *preprint* TH-2062-CERN (1975).
[10] Z. Koba, H. B. Nielsen, P. Olesen, *Nucl. Phys.* **B40**, 317 (1972).
[11] K. Fiałkowski, H. I. Miettinen, *Phys. Lett.* **43B**, 61 (1973).
[12] D. R. O. Morrison, *Proc. R. Soc. London* **A335**, 461 (1973).
[13] R. Møller, *Nucl. Phys.* **B74**, 145 (1974).
[14] F. Hayot, F. S. Henyey, M. Le Bellac, *Nucl. Phys.* **B80**, 77 (1974).