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THE SIX QUARK MODEL WITHOUT RIGHT HANDED CURRENTS*

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It is shown that a "minimal" six quark model based on the Weinberg-Salam theory of unified weak and electromagnetic interactions may be constructed if a small (consistent with experimental uncertainties) deviation from e-μ universality is assumed. This minimal model offers a solution to the difficulties of the standard charm scheme associated with $R = \sigma(e^+ + e^- \to \text{hadrons})/\sigma(e^+ + e^- \to \mu^+ + \mu^+)$ and the abundance of $\mu^-\mu^+$ and $\mu^-\mu^-$ pairs in neutrino reactions. It also explains in a natural way the character and smallness of CP violation.

1. Introduction

In spite of the successes and popularity of the standard charm model [1-4], there are some reasons to consider its modification. They are connected with the following difficulties of the four quark model:

1. The ratio $\sigma(e^+ + e^- \to \text{hadrons})/\sigma(e^+ + e^- \to \mu^+ + \mu^-)$ predicted by the charm model is $3\frac{1}{3}$ while the measured values is ≈ 5.
2. The roughly estimated cross sections for the dimuon events are too small to account for the observation [5]. The same concerns the $\mu^-\mu^-$ and $\mu^+\mu^+$ events respectively.
3. CP violation is not accounted for in this scheme.

Recently there were several attempts to solve these difficulties by extending the standard model and including some additional quarks and leptons into the theory [6-9]. Some of the proposed versions of the theory contain right handed, charged weak currents coupling with a full strength neutron and one of the heavy quarks. Such coupling should change drastically the slope of $\sigma^{\nu}_{\text{tot}}$ versus E and the ratio $\sigma^{\bar{\nu}}_{\text{tot}}/\sigma^{\nu}_{\text{tot}}$ above the appropriate threshold. Since we do not yet observe such effects, it seems worthwhile to try first to construct a satisfactory 6 quark model which contains only V-A currents and is as close to the standard

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charm model as possible (a "minimal" model). In a previous paper [9] by S. Tatur and the present author, the 6 quark model with Harari's interpretation of ψ (3100), ψ (3700) and ψ (4200) as bound states of three heavy quark-antiquark pairs [10] was examined from this "minimality" point of view. It turned out that none of the difficulties of the 4 quark model (except R ratio) could be solved, in fact some new troubles appeared.

In this paper we want to show that by combining a 6 quark minimal model with Wilczek's idea of very well satisfied X spin symmetry [6] it is possible to construct a satisfactory model solving the problems enumerated above.

2. *Model*

The gauge group of the model is the same as in the standard charm model, i. e. $SU_2 \times U_1 \times SU_3^{\text{colour}}$. There are 6 quarks p, c, t, n, λ, b grouped in the following multiplets

$$\begin{pmatrix} \mathrm{p} \\ c_{\mathrm{pn}}\mathrm{n}+c_{\mathrm{p}\lambda}\lambda+c_{\mathrm{pb}}\mathrm{b} \end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix} \mathrm{c} \\ c_{\mathrm{cn}}\mathrm{n}+c_{\mathrm{c}\lambda}\lambda+c_{\mathrm{cb}}\mathrm{b} \end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix} \mathrm{t} \\ c_{\mathrm{tn}}\mathrm{n}+c_{\mathrm{t}\lambda}\lambda+c_{\mathrm{tb}}\mathrm{b} \end{pmatrix}_{\mathrm{L}},$$

$$\mathrm{p_R}, \mathrm{c_R}, \mathrm{t_R}, \mathrm{n_R}, \lambda_{\mathrm{R}}, \mathrm{b_R}. \tag{1}$$

Following Wilczek we assume that c and t quarks are nearly degenerate and therefore the physical 1^- states are $1/\sqrt{2}(\bar{\mathrm{c}}\mathrm{c}+\bar{\mathrm{t}}\mathrm{t})$, $1/\sqrt{2}(\bar{\mathrm{c}}\mathrm{c}-\bar{\mathrm{t}}\mathrm{t})$, and $\bar{\mathrm{b}}\mathrm{b}$. In this ideal case (later on we shall consider some small corrections) the first state couples to the photon with double strength of that for ψ (3100) in the standard charm model, while the second one (the X-spin vector) is decoupled from the photon and therefore cannot be observed in the experiments in which ψ (3100) has been found. Using this assumption, the whole family ψ' (3700), ψ'' (4200), ... is interpreted as radial excitation of ψ (3100) — exactly as in the charm model. The quark b is assumed to be much heavier that c or t quarks. (If the recent rumour about a narrow vector meson with mass $\sim$ 6 Gev is confirmed it will fit this scheme perfectly).

The above possibility (including the structure of multiplets (1)) was briefly considered by Wilczek and rejected on the grounds that Cabibbo universality $c_{\mathrm{pn}}^2+c_{\mathrm{p}\lambda}^2 = 1$ implies $c_{\mathrm{pb}} = 0$ (or extremely small) which leads to some serious problems. We propose the following way out of this difficulty.

As it is well known, the condition for cancelling anomalies in the $SU_2 \times U_1$ group demands the existence of equal numbers of leptons and quark doublets. We have, therefore, a heavy charged lepton L and a new neutrino ν_{L} which we assume to be massive. With degeneracy of the neutrinos broken it is meaningful to introduce some mixing between them, thus we may assume the following form for the lepton doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix} \nu_\mu \cos\alpha+\nu_{\mathrm{L}}\sin\alpha \\ \mu \end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix} -\nu_\mu \sin\alpha+\nu_{\mathrm{L}}\cos\alpha \\ \mathrm{L} \end{pmatrix}_{\mathrm{L}}. \tag{2}$$

If $\sin\alpha \neq 0$ then the $e-\mu$ universality is violated. Experimentally we have [11]

$$\frac{f_\pi(e)}{f_\pi(\mu)} = \frac{1}{\cos\alpha} = 1.007 \pm 0.014; \tag{3}$$

therefore, at the level of one standard deviation

$$\sin^2\alpha \leqslant 0.042. \tag{4}$$

Testing Cabibbo universality, the $e-\mu$ universality was always taken for granted. With $\sin\alpha \neq 0$ the conclusions change in the following way.

Denoting by G_0, the standard Fermi coupling constant relevant for purely electronic interactions (mediated by $W^{\pm}$), we have

$$G_\mu = G_0\cos\alpha, \quad G_\beta = G_0|c_{\rm pn}|,$$

$$G_{\Delta S=1} = G_0|c_{\rm p\lambda}|\cdot\begin{cases}1 & \text{for an electron in the final state}\\ \cos\alpha & \text{for a muon in the final state.}\end{cases} \tag{5}$$

Since $|c_{\rm p\lambda}|$ is small, it will not be important for further considerations which the expression for $G_{\Delta S=1}$ is used. From (5), we have

$$c_{\rm pn}^2+c_{\rm p\lambda}^2 = \frac{G_\beta^2+G_{\Delta S=1}^2}{G_0^2} = \cos^2\alpha\,\frac{G_\beta^2+G_{\Delta S=1}^2}{G_\mu^2} \tag{6}$$

and

$$c_{\rm pb}^2 = 1-(c_{\rm pn}^2+c_{\rm p\lambda}^2) \approx \sin^2\alpha+\frac{G_\mu^2-G_\beta^2-G_{\Delta S=1}^2}{G_\mu^2}. \tag{7}$$

As far as the Cabibbo universality in the conventional form: $G_\mu^2 = G_\beta^2+G_{\Delta S=1}$ is concerned, the following number is quoted [12]: $(G_\beta^2+G_{\Delta S=1}^2)/G_\mu^2 = 1.001\pm0.004$ hence again at the level of one standard deviation, we have

$$\frac{G_\mu^2-G_\beta^2-G_{\Delta S=1}^2}{G_\mu^2} \leqslant 0.003. \tag{8}$$

Combining (7) and (8) we obtain

$$|c_{\rm pl}| \leqslant \sqrt{0.045} = 0.21. \tag{9}$$

This bound on $c_{\rm pb}$ is not small — it is of the order of the Cabibbo angle. We assume in our model that $|c_{\rm pb}|$ is close to this bound. Now we can study the problems mentioned in the introduction.

3. *The R ratio*

Above the highest threshold for $\bar{\rm b}$b production the value of R given by a simple counting of quark charges is 5 — the same as in the model of Harari. Below that but above the charm threshold, the value is slightly smaller: $R = 4\frac{2}{3}$. The measurement in this region gives $R \approx 5$ which is not in bad agreement if we remember that in the asymptotically free theory, the limiting value should be approached from above [13] and that below the charm threshold the experimental value of $R \sim 2.5$ is really somewhat higher than predicted by the parton-quark model ($R = 2$). If this picture is correct, it means that the threshold for L^+L^- production has not yet been reached in the e^+e^- experiment. This gives us the first clue concerning the mass of the heavy charged lepton $m_L \gtrsim 4$ Gev.

4. The dimuon events

The lepton mixing we have previously introduced now provides the possibility for a better understanding of the neutrino reactions with two charged leptons in the final state [14–17]. In contrast to the 4 quark model, in the present scheme there are two additional sources of $\mu^-\mu^+$ events and one efficient source of dimuons of the same sign.

The first additional source of $\mu^-\mu^+$ is simply the reflection of the existence of two similar quarks: c and t. The total number of particles with quantum numbers c or t produced by neutrinos from the valence neutron quarks is proportional not to $\sin^2\theta_C$ as in the 4 quark model, but to the sum

$$|c_{cn}|^2 + |c_{tn}|^2 = 1 - |c_{pn}|^2 = |c_{p\lambda}|^2 + |c_{pb}|^2. \tag{10}$$

We have argued that the assumption $0 \neq |c_{pb}| \approx |c_{p\lambda}| \approx \sin\theta_C$ is compatible with the present experimental situation and in fact it is essential to the very existence of our model. With this assumption the sum to which the total number of dimuons is proportional becomes

$$|c_{cn}|^2 + |c_{tn}|^2 \approx 2\sin^2\theta_C.$$

The second source of dimuons is connected with heavy lepton production. The lepton current in our model has the form which implies the new phenomena leading both to $\mu^-\mu^-$ (or $\mu^+\mu^+$) and $\mu^-\mu^+$ events. The charged lepton current which follows from the assumed form of the lepton doublets is

$$\begin{aligned} J^l_\varrho = {} & \bar{e}\gamma_\varrho(1-\gamma_5)\nu_e + \cos\alpha\,\bar{\mu}\gamma_\varrho(1-\gamma_5)\nu_\mu + \cos\alpha\,\bar{L}\gamma_\varrho(1-\gamma_5)\nu_L \\ & + \sin\alpha\,\bar{\mu}\gamma_\varrho(1-\gamma_5)\nu_L - \sin\alpha\,\bar{L}\gamma_\varrho(1-\gamma_5)\nu_\mu. \end{aligned} \tag{11}$$

With such a leptonic weak current, the ν_μ interacting with nucleons via the exchange of W has two options now: in most cases ($\sim \cos^2\alpha$) it will transform into μ^-, but from time to time (with probability $\sin^2\alpha\, \times$ mass dependent threshold factor) instead of μ^- the heavy lepton L^- will be produced. It decays by one of the following four sequences:

$$\begin{aligned} \text{A)}\quad & \nu_\mu + N \to L^- && + \text{hadrons} \\ & \qquad\qquad \hookrightarrow \nu_L && + \text{hadrons (or } e^- + \bar{\nu}_e) \\ & \qquad\qquad\quad \hookrightarrow \mu^- && + \text{hadrons (or } e^+ + \nu_e) \end{aligned}$$

$$\begin{aligned} \text{B)}\quad & \nu_\mu + N \to L^- && + \text{hadrons} \\ & \qquad\qquad \hookrightarrow \nu_L && + \text{hadrons (or } e^- + \bar{\nu}_e) \\ & \qquad\qquad\quad \hookrightarrow \mu^- + \mu^+ + \nu_\mu && \end{aligned}$$

$$\begin{array}{llll} \text{C)} & \nu_\mu + \text{N} \to \text{L}^- & & +\text{hadrons} \\ & \quad\; \hookrightarrow \nu_\text{L} & & +\mu^- + \bar{\nu}_\mu \\ & \qquad\quad \hookrightarrow \bar{\mu} & & +\text{hadrons (or } e^+ + \nu_e) \end{array}$$

$$\begin{array}{llll} \text{D)} & \nu_\mu + \text{N} \to \text{L}^- & & +\text{hadrons} \\ & \quad\; \hookrightarrow \nu_\text{L} & & +\mu^- + \bar{\nu}_\mu \\ & \qquad\quad \hookrightarrow \mu^- + \mu^+ + \nu_\mu & & \end{array}$$

The final channel A is not particularly interesting. It is practically indistinguishable from the usual process in which μ^- is produced directly.

Channel B is an additional source of opposite sign dimuons. The analysis of the invariant mass distribution of $\mu^- \mu^+$ pairs [16] shows that it is not possible to attribute all the dimuon events to the decay of one kind of neutral lepton, however it is perfectly admissible that some fraction of them is produced via this mechanism. In fact a closer inspection of the invariant mass distribution diagrams in Ref. [16, 17] shows some concentration of events below ~ 2 Gev. It fits qualitatively our picture if we assume $m_{\nu_\text{L}} \approx 2 \div$ $\div 2.5$ Gev. We do not attempt to make a precise fitting because there are too many unknown factors and the statistics are not very good. However, we should check the consistency of this estimate at least with the condition that both L and ν_L have to be sufficiently short living particles. With ν_L itself there is no problem. Neglecting the muon mass in comparison with ν_L the formula for partial leptonic with of ν_L can immediately be written down

$$\Gamma^{\nu_\text{L} \to \text{leptons}} = \sin^2 \alpha \left(\frac{m_{\nu_\text{L}}}{m_\mu} \right)^5 \frac{1}{\tau_\mu} \tag{12}$$

which for $m_{\nu_\text{L}} \gtrsim 20\, m_\mu$ and $\sin^2 \alpha = 0.04$ gives $\Gamma^{\nu_\text{L} \to \text{leptons}} > 1/(10^{-11}\text{s})$. For the decay of L, the ν_L mass must be taken into account and integration of the spectral distribution calculated in [18] for this kind of decay leads to the expression

$$\Gamma^{\text{L} \to \nu_\text{L} + \mu + \bar{\nu}_\mu} = \frac{1}{\tau_\mu} \left(\frac{m_\text{L}}{m_\mu} \right)^5 G\left(\frac{m_{\nu_\text{L}}}{m_\text{L}} \right), \tag{13}$$

where

$$G(\xi) = 1 - 8\xi^2 + 8\xi^6 - \xi^8 - 24\xi^4 \ln \xi. \tag{14}$$

With the help of this expression we find that for $m_{\nu_\text{L}} \approx 2 \div 2.5$ GeV and $m_\text{L} > 4$ GeV the lifetime of L is even shorter than that of ν_L. These lifetimes are safely outside the region excluded experimentally [14] for particles which are the source of additional muon in dimuon events.

We have argued that $m_\text{L} > 4$ GeV but if the proposed mechanism of dimuon production has to be effective, the mass of L cannot be much larger than this limit, becuase otherwise the probability of heavy lepton production would be much smaller than $\sin^2 \alpha$. The relevant kinematical factor was numerically calculate in [19]. For typical energy of the incident neutrino $E_\nu = 100$ GeV and $m_L = 4$ GeV the suppression factor is 0.38 and it decreases rapidly for $m_L > 5$ GeV. Therefore our guess is

$$4\text{ GeV} \lesssim m_L \lesssim 5\text{ GeV}.$$

With the suppression factor $\approx \frac{1}{3}$ the actual probability of L production (relatively to μ) is therefore $\sim 1\%$. The probability that ν_L will be produced as a result of L decay is practically the same. Now, to estimate the number of $\mu^-\mu^+$ pairs we have to know the branching ratio

$$\frac{\Gamma(\nu_L \to \mu^- + \mu^+ + \nu_\mu)}{\Gamma(\nu_L \to \mu^- + \text{anything})}. \tag{15}$$

It seems that there are reasons to believe that this number should be $\sim \frac{1}{10} \div \frac{1}{5}$. Exactly 1/5 follows from the simple counting of channels open to the decay of the virtual W^+ created in the process $\nu_L \to \mu^- + W^+$. The W may decay either into $\mu^+\nu_\mu$ or into $e^+\nu_e$, $p^i n^i_C$ where $i = 1, 2, 3$ numbers the various colours of quarks. This is the same kind of argument which implies $R = 2$ for $\sigma(\gamma^{\text{virtual in } e^+e^-} \to \text{hadrons})/(\gamma \to \mu^+\mu^-)$ (below charm threshold). However the actual value of R in the vicinity of $E \approx 2$ GeV is somewhat higher than the predicted value — similarly the considered branching ratio for ν_L decay into leptons is expected to be smaller than $\frac{1}{5}$. The other reason for the possible suppresion of this branching ratio may be connected with the charm threshold. Im e^+e^- this threshold is ~ 3 GeV because two charmed quarks have to be created. In the weak process, however, only one is involved and the threshold is lower, possibly as low as ~ 2 GeV. In any case, the reduction by a factor of more than 2 is unlikely — hence our estimate $\frac{1}{10} \div \frac{1}{5}$.

Finally, we come to the conclusion that the amount of $\mu^-\mu^+$ pairs from the decays of ν_L may contribute about $1\% \times (\frac{1}{10} \div \frac{1}{5}) \approx 0.1 \div 0.2\%$ of single muon events. This is not much (the observed value is $\sim 0.8\%$). But we should remember that it is only one of the four mechanisms

(a) production of charmed particles from the sea of λ quarks
(b) production of c quarks from the valence n quarks
(c) production of t quarks from the valence n quarks
(d) $\mu^-\mu^+$ from the decays of ν_L.

The (c) and (d) mechanisms operate only in the six quark, but not in the four quark, model.

Although process (d) is not very important for the $\mu^-\mu^+$ events it is crucial for the understanding of the origin of $\mu^-\mu^-$ pairs. Once we agreed that process (d) occurs at the level $\sim 0.1 \div 0.2\%$ of single muon production we immediately come to the conclusion that a similar amount of $\mu^-\mu^-$ pairs should be produced via channel C. The reported cross section for the $\mu^-\mu^-$ pairs is actually of the order $\sim 0.1\%$ of the single muon production — fully consistent with our picture. A similar discussion can be carried out for the antineutrino reactions, but since the various uncertainties are even larger in this case, we omit it here.

Finally, let us observe that if both L^- and ν_L decay muonically (the final channel D), the trimuon event should be observed. Since in this case two small branching ratios are involved, the number of trimuon events is expected on the level of $\sim 10\%$ of $\mu^-\mu^-$ pairs. As the number of $\mu^-\mu^-$ pairs actually observed is of the order of 10, the prediction is that if statistics increase, some trimuon events will be observed soon.

5. *CP violation*

Any realistic model of elementary particles has to be able to describe if not predict, the phenomenon of CP violation. In the gauge theory of weak and electromagnetic interactions, the most natural way to have the CP violation is through the mechanism of spontaneous breaking, and not by a CP noninvariant Lagrangian [20]. Although such a principle restricts the number of possibilities there still remains too much freedom in specifying the actual form of the effective CP violating interaction.

On purely phenomenological ground, the effects of CP violation may be satisfactorily described by a single parameter ε measuring the nonorthogonality of K_L and K_S. With our present understanding of the mechanism of spontaneous breaking there is little hope of calculating this parameter in terms of some already known quantities only. In such a situation the mere explanation why CP violation manifests only through $K_S K_L$ mixing and why ε is not of order of unity, but $\sim 10^{-3}$ would be a satisfactory result.

In the most general case, the effective CP nonconserving interaction is caused by both Higgs scalars and gauge bosons exchange. However, it would be very inconvenient to associate any physical effect with Higgs particles exchange. Their masses are completely arbitrary and their theoretical status uncertain. It is not excluded that they are only an artificial device to maintain the spontaneous symmetry breaking and will be eliminated completely from a future theory. Therefore we restrict ourselves of the effects of CP violation which are explicitly contained in the form of weak currents. The role of Higgs particles in our approach is to produce a quark mass matrix which in the case of spontaneous CP violation is not symmetric, but a general hermitian matrix. This implies that the matrix $\hat{C}$ considered previously cannot be orthogonal, but in general must be a unitary matrix. This generalization does not affect our previous discussion, because we made use only of the property $\sum_j |c_{ij}|^2 = 1$ which is equally valid for any orthogonal or unitary matrix.

It is important that the unitarity (as opposed to orthogonality) of the $\hat{C}$ matrix does lead to an observed CP violation in the six quark model, but not in the standard four quark model. To see this let us consider the Feynman diagram contributing to $\langle K^0|H_{\rm eff}|\overline{K^0}\rangle$

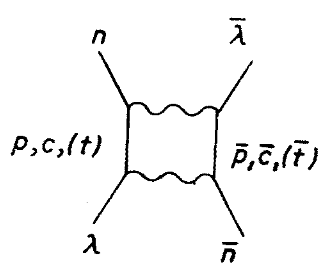

Fig. 1. The Feynman diagram contributing to $\langle K^0|H_{\rm eff}|\overline{K^0}\rangle$

The addition of such diagrams with various intermediate quarks effects in replacing the quark propagator by the sum

$$c_{\mathrm{pn}}c^*_{\mathrm{p}\lambda}\frac{1}{\not k-m_{\mathrm{p}}}+c_{\mathrm{cn}}c^*_{\mathrm{c}\lambda}\frac{1}{\not k-m_{\mathrm{c}}}+\left(c_{\mathrm{tn}}c^*_{\mathrm{t}\lambda}\frac{1}{\not k-m_{\mathrm{t}}}\right). \tag{16}$$

In the four quark model, the unitarity condition

$$c_{\mathrm{pn}}c^*_{\mathrm{p}\lambda}+c_{\mathrm{cn}}c^*_{\mathrm{c}\lambda}=0 \tag{17}$$

allows us to rewrite the sum of propagators in the form

$$c_{\mathrm{pn}}c^*_{\mathrm{p}\lambda}\left(\frac{1}{\not k-m_{\mathrm{p}}}-\frac{1}{\not k-m_{\mathrm{c}}}\right). \tag{18}$$

But the on-shell matrix elements $\langle K^0|H^{\mathrm{W}}|2\pi\rangle$, $\langle K^0|H^{\mathrm{W}}|3\pi\rangle$, ... which determine the antihermitian part of the $K^0-\overline{K^0}$ mass matrix are also proportional to $c_{\mathrm{pn}}c^*_{\mathrm{p}\lambda}$ therefore by appropriate choice of the K meson wave functions phase, we can make both M_{12} and Γ_{12} in the $K^0-\overline{K^0}$ mass matrix real. Alternatively, we can say, that by appropriate change of λ quark phase the $c_{\mathrm{pn}}c^*_{\mathrm{p}\lambda}$ can be made real which proves that no observable effect of CP violation in the $K^0-\overline{K^0}$ system appears.

In the six quark model, the unitary condition

$$c_{\mathrm{pn}}c^*_{\mathrm{p}\lambda}+c_{\mathrm{cn}}c^*_{\mathrm{c}\lambda}+c_{\mathrm{tn}}c^*_{\mathrm{t}\lambda}=0 \tag{19}$$

leads to the "effective" propagator

$$c_{\mathrm{pn}}c^*_{\mathrm{p}\lambda}\left[\left(\frac{1}{\not k-m_{\mathrm{p}}}-\frac{1}{\not k-m_{\mathrm{t}}}\right)+\frac{c_{\mathrm{cn}}c^*_{\mathrm{c}\lambda}}{c_{\mathrm{pn}}c^*_{\mathrm{p}\lambda}}\left(\frac{1}{\not k-m_{\mathrm{c}}}-\frac{1}{\not k-m_{\mathrm{t}}}\right)\right] \tag{20}$$

with a complex (in general) parameter

$$z=\frac{c_{\mathrm{cn}}c^*_{\mathrm{c}\lambda}}{c_{\mathrm{pn}}c^*_{\mathrm{p}\lambda}} \tag{21}$$

which is independent of any change of phase of any quark field, and which does lead to an observable CP violation. The relevant imaginary part of z cannot be calculated "from first principles" in the kind of theory we are considering but from the known phenomenological facts and conditions of unitarity, we can calculate its maximal value. With some elementary algebra, it can be shown that from the constrains

$$\frac{|c_{\mathrm{p}\lambda}|}{|c_{\mathrm{pn}}|}=\mathrm{tg}\,\theta_{\mathrm{C}}\ll 1,\qquad |c_{\mathrm{pb}}|\leqslant|c_{\mathrm{p}\lambda}| \tag{22}$$

the following result is obtained

$$\mathrm{Max\,Im}\,z=\tfrac{1}{2}. \tag{23}$$

One can easily check that $c_{\mathrm{pb}}=0\Rightarrow \mathrm{Im}\,z=0$ therefore CP violation vanish when both $e-\mu$ and Cabibbo universalities are exactly satisfied.

Now we are able to find (approximately) the explicit form of the effective Hamiltonian leading to the CP violation in $K^0-\overline{K^0}$ system and to estimate the parameter ε. (Since the CP violation may appear in our model only as a result of interference between contributions from various quarks in the intermediate states, it is obvious that in the free quark approximation it does not manifest itself in any usual weak process except in the $K^0-\overline{K^0}$ system. The theory is therefore effectively of the "superweak" type.) The general form of the mass matrix consistent with CPT is

$$\mathscr{M} = \begin{bmatrix} M-i\dfrac{\Gamma}{2}, & M_{12}-i\dfrac{\Gamma_{12}}{2} \\ M_{12}^*-i\dfrac{\Gamma_{12}^*}{2}, & M-i\dfrac{\Gamma}{2} \end{bmatrix}, \tag{24}$$

where

$$\Gamma_{12} = 2\pi\sum_n \langle\overline{K^0}|H^{\mathrm{W}}|n\rangle\,\langle n|H^{\mathrm{W}}|K^0\rangle\delta(E_n-m_K). \tag{25}$$

As we have mentioned earlier, by an appropriate phase convention, one can make both Γ_{12} and the coefficient in front of (20) real. This means that the CP violation is connected with Im M_{12} which in turn is induced by Im z only.

In the four quark model where Im $M_{12}=0$ the real part Re M_{12} was calculated by Gaillard and Lee [21]. Following their arguments we can calculate now in an analogous way both Re M_{12} and Im M_{12}. In our case, the situation is even simpler, because ε depends only on the ratio of these quantities, therefore we can avoid the dubious procedure of estimating the matrix element $\langle K^0|H_{\mathrm{eff}}\,|\overline{K^0}\rangle$ for free quarks. The result of Lee and Gaillard is

$$H_{\mathrm{eff}}^{\Delta S=2} = \frac{G_{\mathrm{F}}}{\sqrt{2}}\,\frac{\alpha}{4\pi}\,\frac{\delta}{(38\ \mathrm{GeV})^2}\cos^2\theta_{\mathrm{C}}\sin^2\theta_{\mathrm{C}}\{\bar{\lambda}\gamma_\mu\tfrac{1}{2}(1-\gamma_5)n\}^2+\mathrm{h.c.} \tag{26}$$

where

$$\delta\cos^2\theta_{\mathrm{C}}\sin^2\theta_{\mathrm{C}} = \int_0^\infty x^2dx\left(\frac{\sin\theta_{\mathrm{C}}\cos\theta_{\mathrm{C}}}{x+m_{\mathrm{p}}^2}+\frac{\cos\theta_{\mathrm{C}}(-\sin\theta_{\mathrm{C}})}{x+m_{\mathrm{c}}^2}\right)^2. \tag{27}$$

According to the previous discussion, in our case, we have to replace the integrand in (27) by

$$|c_{\mathrm{pn}}c_{\mathrm{p}\lambda}^*|^2\left[\left(\frac{1}{x+m_{\mathrm{p}}^2}-\frac{1}{x+m_{\mathrm{t}}^2}\right)+z\left(\frac{1}{x+m_{\mathrm{t}}^2}=\frac{1}{x+m_{\mathrm{c}}^2}\right)\right]^2. \tag{28}$$

The square of the first term (including the nonessential contribution from Re z) gives the old result, the second term squared is negligible ($m_{\mathrm{t}}\approx m_{\mathrm{c}}$) and the interference term gives the effective CP violating Hamiltonian which we are looking for

$$H_{\rm eff}^{\rm CP=1} = 2i({\rm Im}\, z)\frac{G_{\rm F}}{\sqrt{2}}\,\frac{\alpha}{4\pi}\,\frac{\sin^2\theta_{\rm C}\cos^2\theta_{\rm C}}{(38\ {\rm GeV})^2}\{\bar{\lambda}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)_n\}^2$$

$$\times \int_0^\infty x^2 dx\left(\frac{1}{x+m_{\rm p}^2}-\frac{1}{x+m_t^2}\right)\left(\frac{1}{x-m_t^2}-\frac{1}{x+m_{\rm c}^2}\right)+{\rm h.c.} \tag{29}$$

Both integrals in (27) and (29) can easily be calculated, and their ratio determines the phase of M_{12}. This phase, together with experimental values of the K mesons mass difference and lifetimes leads in a straight-forward way to the following result for Re ε

$$\mathrm{Re}\,\varepsilon \approx \tfrac{1}{2}\frac{m_t-m_c}{m_t}\,\mathrm{Im}\,z \tag{30}$$

and to the phase of ε which is automatically good

$$\arg\varepsilon = \mathrm{arctg}\, 2(\Delta M)\tau_S. \tag{31}$$

We see that the CP violation is intimately connected in this model with X-spin symmetry. In the limit of exact X-spin conservation ε vanish and it remains small in the real world because this is a very good symmetry indeed. To estimate whether this connection is not only of a qualitive value, we have to know something about the X-spin nonconservation. I can see two possible approaches to this problem: one phenomenological, the other purely speculative.

If $m_t \neq m_c$ then obviously there is a mixing between the X-spin scalar and X-spin vector. The particle with small admixture of X-spin scalar is not in this case, completely decoupled from the photon. The mixing angle is proportional to the ratio of the quark mass difference and the relevant vector meson mass difference. If the quark mass difference were of the order 10–20 MeV the mixing would be complete and the whole picture spoiled. In fact, the search for sharp resonances in the vicinity of $\psi(3100)$ has been performed [22] with the result that if such resonance exists its coupling to e^+e^- is less than $\sim 10\%$ of the ψ coupling. It means in our picture that $m_t - m_c < \sqrt{0.1}\,(m_{X=0}-m_{X=1})$. Estimating the last mass difference by the analogous difference $m_\omega - m_\varrho = 13$ MeV we obtain

$$m_t - m_c \lesssim 4\ \mathrm{MeV}. \tag{32}$$

With $m_t \approx m_c \approx \frac{1}{2}(m_\psi - m_\varrho) \sim 1$ GeV the maximal value of ε we can obtain (when Im $z = \frac{1}{2}$) is

$$\mathrm{Re}\,\varepsilon \lesssim 1\times 10^{-3} \tag{33}$$

extremely close to the actual value $(1.03\pm 0.18)10^{-3}$. It is very interesting that the $m_t - m_c$ mass difference obtained in this way is of the same order of magnitude as the mass difference of hadrons, belonging to the same isomultiplet. This agrees with Wilczek's conjecture that both I-spin and X-spin symmetries have the same (unknown at present) origin. It is true that there is also an important difference between them. The I-spin is violated by the electromagnetic interactions while the X-spin is not. However, we should remember than in spite of many efforts, nobody has been able to explain the I-spin violating phenomena in terms of photon exchange only. The additional nonelectromagnetic (but of the same order of magnitude as electromagnetic ones) I-spin violation had to be assumed (under the names "tadpole", u_3-scalar density, or just the p-n quark mass difference). We are referring to this I-spin violation when comparing with X-spin.

6. *Conclusions*

I would like to conclude with the following comment. It is not surprising that increasing the number of quarks and therefore the number of the parameters we were able to achieve better agreement between experiment and the unified gauge theory. It is however interesting that the freedom we have gained by extending the charm scheme is very limited indeed. The values of various parameters ($\cos\alpha$, c_{pb}, $\mathrm{Im}\, z$, $m_t - m_c$, m_L, m_{ν_L}) estimated in the paper are very close to their limiting values allowed either by experiment or by the requirement of self-consistency. This means that very soon this model will be either confirmed or unambigously ruled out by experiment. There are many points where the above model can be crucially tested. The most obvious are: the violation of e–μ universality at the level $\sim 4\%$; the trimuon events at the level $\sim 10\%$ of $\mu^-\mu^-$; the existence of X-vector partner of $\psi(3100)$ coupled to photons with strength not much less than 10% of ψ coupling and many others.

After this work had been finished I received a paper by Ellis, Gaillard and Nanopoulos [23] "Left handed currents and CP violation", CERN preprint TH2116, where the detailed theoretical analysis of CP violation in various processes, implied by $\mathrm{Im}\, z \neq 0$ has been presented. However, they do not take into account the possibility that the present bound on $e-\mu$ universality allows a much higger value of $|c_{pb}|^2$ than that one can infer form the accuracy of the "Cabibbo universality" itself (0.045 instead of 0.003). Therefore they explore another region of various parametrs and the connection of their consideration with physical reality is different than that presented in this paper. I have learned from their paper that the possibility of description of CP violation through non-orthogonality of $\hat{C}$ matrix in the theory with at least 6 quarks had been discovered some time ago and that there exist already some works on this subject [24–26].

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