ON THE APPROXIMATE CALCULATION OF REGGE POLES BY FORMAL SERIES

By A. A. Atanasov

Department of Theoretical Physics, University of Plovdiv*

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A method of solving ordinary nonlinear differential equations through a formal series is applied to the approximate calculation of Regge poles. In particular, the nonrelativistic potential theory, Dirac theory and a problem in quasipotential approach are considered.

1. *Introduction*

In potential scattering theory, the position of singularities of the S_l-matrix in the complex l-plane are described by the continuous movement of a pole of the S_l-matrix, called a Regge pole. The physical bound states occur at a discrete energy where these poles pass through an integer angular momentum l. Regge poles are generalized bound states with complex angular momenta and they are closely connected with resonances. In potential scattering theory any potential has an infinite number of Regge poles, corresponding to different values of the radial quantum number.

Many approximate methods have been developed by a number of authors [1–3] for the calculation of Regge poles. In the present paper we are going to use the method of formal series to calculate Regge poles for a wide class of potentials. We apply this method to solve nonlinear variable phase equation for partial amplitude. An exact expression for the partial amplitude may be represented by the ratio of two power series of the coupling constant. The zeros of the denominator of the partial amplitude determine the Regge poles. When the series in the numerator and denominator are approximated by two polynomials, instead of the corresponding series the Padé approximant is obtained. The approximation is valid for strong and weak coupling.

In Section 2. Regge poles for the nonrelativistic potential theory are considered. The trajectories of Regge poles are found for scattering by a Yukawa potential in the case of strong coupling. Section 3 is devoted to a discussion of Regge poles for particles described by the Dirac equation. In Section 4 the equation for scalar particles of equal

* Address: Department of Theoretical Physics, University of Plovdiv, 24 Tsar Assen str., Bulgaria.

masses and for a quasipotential, having in the coordinate representation the form $V(r) = gr^{-1}$ in discussed. The corresponding problem for Regge poles is considered in detail by applying the method of formal series to the variable phase equation for Green's function.

2. *Regge poles for nonrelativistic local potential theory*

In an earlier paper [4] it was shown that in the case of a central regular potential $V = gV_0(r)$ with the coupling constant g the variable phase equation

$$\frac{df_l}{dr} = -\frac{V(r)}{k}[j_l(kr)+if_l(r)h_l^{(1)}(kr)]^2, \tag{1}$$

$$f_l(0) = 0, \tag{2}$$

can be solved by the method of formal series. In equation (1) $f_l(r)$ is a nonrelativistic partial amplitude defined by the relation $f_l(r) = e^{i\delta_l(r)} \sin \delta_l(r)$, where $\delta_l(r)$ is the phase shift produced by the potential up to the distance r. The functions $j_l(kr)$ and $h_l^{(1)}(kr)$ are two linearly independent solutions of the corresponding Schrödinger equation when $V(r) = 0$:

$$j_l(kr) = \sqrt{\frac{\pi kr}{2}}\, J_{l+\frac{1}{2}}(kr), \qquad h_l^{(1)}(kr) = \sqrt{\frac{\pi kr}{2}}\, H_{l+\frac{1}{2}}^{(1)}(kr). \tag{3}$$

The functions (3) are defined for complex l and k so that (I) must be taken as the definition of the partial amplitude for arbitrary l and k, as long as a solution exists. The solution of equation (I) with condition (2) takes the form

$$f_l(r) = \frac{N_l(g, k; r) - \varphi_l(r)D_l(g, k)}{D_l(g, k)}, \tag{4}$$

where

$$N_l(g, k; r) = \sum_{n=0}^{\infty} N_l^{(n)}(g, k; r) = \Gamma \exp\left\{-g \int dy \frac{\delta G[\varphi_l, y]}{\delta\varphi_l(y)}\right\} \varphi_l(r), \tag{5}$$

$$D_l(g, k) = \sum_{n=0}^{\infty} D_l^n(g, k) = \Gamma \exp\left\{-g \int dy \frac{\delta G[\varphi_l, y]}{\delta\varphi_l(y)}\right\} \cdot 1, \tag{6}$$

$$\varphi_l(r) = -ij_l(kr)[h_l^{(1)}(kr)]^{-1}, \tag{7}$$

$$G[\varphi_l(r), r] = -\int_0^r \frac{V_0(x)}{k}[h_l^{(1)}(kx)]^2 \varphi_l^2(x)dx. \tag{8}$$

The symbol Γ indicates that all functional derivatives must be on the left, acting thus on all functionals which are put on their right.

The behaviour of the pole positions of S_l in l-plane, as functions of energy is determined by the equation

$$D_l(g, k) = 0. \tag{9}$$

It is easy to verify that

$$D_l^{(0)}(g, k) = 1, \; D_l^{(1)}(g, k) = -\frac{ig}{k}\int_0^\infty dx V_0(x) j_l(kx) h_l^{(1)}(kx), \tag{10}$$

$$D_l^{(2)}(g, k) = -\frac{g^2}{2k^2}\int_0^\infty \Big\{ V_0(x) h_l^{(1)}(kx) j_l(kx) \times \int_0^\infty V_0(y) j_l(ky) h_l^{(1)}(ky) dy$$

$$+2V_0(x)\,[h_l^{(1)}(kx)]^2 \int_0^x V_0(y) j_l^2(ky) dy \Big\}\, dx. \tag{11}$$

We shall apply the above method to the case of the Yukawa potential, defined as

$$V(r) = 2m\alpha r^{-1} e^{-\mu r}, \tag{12}$$

having in mind that the obtained expression for Regge trajectories can be generalized to the superposition of Yukawa potentials. In first order approximation for Regge trajectories we obtain

$$\frac{ie^{i\pi l}}{k}\int_0^\infty j_{-l-1}(kr) j_l(kr) V(r) dr - \frac{1}{k}\int_0^\infty j_l^2(kr) V(r) dr = ie^{i\pi l}\cos \pi l \tag{13}$$

or

$$\frac{ie^{i\pi l}\alpha m}{k} R_l\left(1+\frac{\mu^2}{2k^2}\right) - \frac{\alpha m}{k} Q_l\left(1+\frac{\mu^2}{2k^2}\right) = ie^{i\pi l}\cos \pi l, \tag{14}$$

where

$$Q_l\left(1+\frac{\mu^2}{2k^2}\right) = 2\int_0^\infty j_l^2(kr)\frac{e^{-\mu r}}{r}\, dr, \tag{15}$$

$$R_l\left(1+\frac{\mu^2}{2k^2}\right) = 2\int^\infty j_l(kr) j_{-l-1}(kr)\frac{e^{-\mu r}}{r}\, dr. \tag{16}$$

$Q_l(z)$ is the Legendre function of the second kind. To investigate the pole positions of S_l in l-plane as functions of the k^2 for large negative k^2, we shall use the pole approximation in equation (14) from the relation

$$Q_l(z) = Q_{-l-1}(z) - \pi \operatorname{tg} \pi(l+\tfrac{1}{2}) P_{l-1}(z). \tag{17}$$

Thus the zero of $D_l^{(0)}+D_l^{(1)}$ approaches $l = -n-1$ as

$$\frac{\frac{\alpha m}{\kappa} P_n\left(1+\frac{\mu^2}{2k^2}\right)}{l+n+1} = -i \tag{18}$$

and the equation for a Regge trajectory is an expression of the form

$$l_n = -n-1-\frac{\alpha m}{ik} P_n\left(1+\frac{\mu^2}{2k^2}\right), \quad k = i\kappa, \quad \kappa > 0. \tag{19}$$

For the high energy limit, relation (19) reduces to the approximation obtained by a modified perturbation procedure, valid for strong coupling [5]:

$$l_n = -n-1-\tfrac{1}{2}\cdot\frac{M_0}{\kappa} \tag{20}$$

where M_0 is determined from the power series in r

$$V(r) = \sum_{i=-1}^{\infty} M_{i+1}(-r)^i. \tag{21}$$

The problem of the Regge poles is considered in detail on the basis of the perturbation approaches for weak coupling in paper [3]. Expression (19) is distinguished from the corresponding equation for a Regge trajectory, obtained for weak coupling [3], only by the sign of the potential.

For small k^2, when $|k^2/\mu^2| \ll 1$, the asymptotic behaviour of $Q_l(z)$ and $R_l(z)$ is

$$R_l\left(1+\frac{\mu^2}{2k^2}\right) \sim \frac{k}{\mu}, \quad Q_l\left(1+\frac{\mu^2}{2k^2}\right) \sim \left(\frac{k^2}{\mu^2}\right)^{l+1}, \tag{22}$$

and the equation of a Regge trajectory is now

$$i\frac{\alpha m}{k}\frac{e^{i\pi l}}{\sin \pi l}\sqrt{\pi}\frac{\Gamma(-l-\frac{1}{2})}{\Gamma(-l)}\left(\frac{k^2}{\mu^2}\right)^{l+\frac{1}{2}} = 1. \tag{23}$$

This expression is distinguished too from the one obtained previously for weak coupling by the sign of the potential. The investigation of the motion of the Regge poles in details is analogical in the case of weak coupling [3].

3. *Regge poles for Dirac particles*

We now want to extend the results of the previous section to the case of a particle of spin 1/2 in the central field $V(r)$, described by the Dirac equation

$$[-i\alpha\nabla+\beta m+V(r)-E]\psi = 0. \tag{24}$$

The phase shifts, determined by the behaviour

$$u_\pm \xrightarrow[r\to\infty]{} \text{const}\ \sin\left(kr-\frac{\pi l}{2}+\delta_{l\pm\frac{1}{2},l}\right). \tag{25}$$

in the case of a central regular potential satisfy the phase equations

$$\frac{d}{dr} t_{l\pm\frac{1}{2},l} = -\lambda^{-1} V(r) [j_l(kr) - n_l(kr) t_{l\pm\frac{1}{2},l}]^2 - \lambda V(r) [j_{l\pm 1}(kr) - n_{l\pm 1}(kr) t_{l\pm\frac{1}{2},l}]^2,$$

$$t_{l\pm\frac{1}{2}}(0) = 0, \tag{26}$$

where

$$t_{l\pm\frac{1}{2},l}(r) = \operatorname{tg} \delta_{l\pm\frac{1}{2},l}(r), \qquad n_l = \sqrt{\frac{\pi kr}{2}}\, N_{l+\frac{1}{2}}(kr), \tag{27}$$

$$k = \sqrt{E^2 - m^2}, \qquad \lambda = \sqrt{\frac{E-m}{E+m}}. \tag{28}$$

In equation (26) the term $\lambda V(r)[j_{l\pm 1}(kr) - n_{l\pm 1}(kr) t_{l\pm\frac{1}{2},l}]$ describes the relativistic effect of the spin-orbit force of Dirac particles [6], [7]. Introducing new unknown functions by the relation

$$t_{l+\frac{1}{2},l} = F_l + X_l, \qquad X_l = \frac{j_l(kr) n_l(kr) + \lambda^2 j_{l+1}(kr) n_{l+1}(kr)}{n_l^2(kr) + \lambda^2 n_{l+1}^2(kr)}, \tag{29}$$

we obtain

$$F_l(r) + \lambda^{-1} g \int_0^r dx V_0(x) (\alpha_l^2(kx) F_l^2(x) + \lambda^2 \alpha_l^{-2}(kx)) = -X_l(r), \tag{30}$$

where

$$\alpha_l^2(kr) = n_l^2(kr) + \lambda^2 n_{l+1}^2(kr). \tag{31}$$

The equation (30) is of the type

$$F_l(r) + G[F_l(r), r] = \varphi(r), \tag{32}$$

where

$$\varphi(r) = -X_l(r) - g\lambda \int_0^r dx V_0(x) \alpha_l^{-2}(kx), \qquad G[F_l(r), r] = g\lambda^{-1} \int_0^r dx V_0(x) \alpha_l^2(kx) F_l^2(x) \tag{33}$$

and its solution takes the form (4).

In the approximation obtained by using first degree terms of g in the numerator and denominator we get

$$t_{l+\frac{1}{2},l}(\infty) = \frac{-\int_0^\infty V(r) [\lambda^{-1} j_l^2(kr) + \lambda j_{l+1}^2(kr)] dr}{1 + \int_0^\infty [\lambda^{-1} j_l(kr) n_l(kr) + \lambda n_{l+1}(kr) j_{l+1}(kr)]}, \tag{34}$$

$$t_{l-\frac{1}{2},l}(\infty) = \frac{-\int_0^\infty V(r) [\lambda^{-1} j_{l-1}^2(kr) + \lambda j_l^2(kr)] dr}{1 + \int_0^\infty [\lambda^{-1} j_{l-1}(kr) j_{l-1}(kr) + \lambda j_l(kr) n_l(kr)] dr}. \tag{35}$$

The behaviour of the pole positions of S_l in l-plane, as functions of energy is determined by the equation

$$i\,\mathrm{tg}\,\delta_{l\pm\frac{1}{2},l} = 1. \tag{36}$$

In the first order approximation for Regge trajectories we obtain

$$i\int_0^\infty V(r)[\lambda^{-1}j_l^2(kr)+\lambda j_{l+1}^2(kr)]dr+1+\int_0^\infty V(r)\,[\lambda^{-1}j_l(kr)n_l(kr)+\lambda n_{l+1}(kr)j_{l+1}(kr)]dr = 0 \tag{37}$$

for the states with the total angular momentum $j = l+\frac{1}{2}$, and

$$i\int_0^\infty V(r)\,[\lambda^{-1}j_{l-1}^2(kr)+\lambda j_l^2(kr)]dr+1$$

$$+\int_0^\infty V(r)\,[\lambda^{-1}j_{l-1}(kr)n_{l-1}(kr)+\lambda j_l(kr)n_l(kr)]dr = 0 \tag{38}$$

for the states with the total angular momentum $j = l-\frac{1}{2}$.

In the nonrelativistic limit, when $E\to m(\lambda = k/2m)$ the term $\lambda V(r)[j_{l\pm1}(kr)-n_{l\pm1}(kr)t_{l\pm\frac{1}{2},l}]$ can be neglected and equations (37), (38) are reduced to the nonrelativistic phase equation for $\mathrm{tg}\,\delta_l$. The corresponding equations for Regge trajectories are reduced to the equations obtained in the previous section.

In the case of the ultrarelativistic limit, when $E \gg m(\lambda = 1)$, both terms in equations (26) are of the same order. For potentials of the Yukawa type (12), using the relation

$$n_l(kx) = \mathrm{ctg}\,\pi(l+\tfrac{1}{2})j_l(kx)-\frac{1}{\sin\pi(l+\frac{1}{2})}j_{l-1}(kx), \tag{39}$$

and pole approximation for $Q_l(z)$, we get

$$\frac{\alpha imP_n\left(1+\dfrac{\mu^2}{2k^2}\right)}{l+n+1}+\frac{\alpha imP_n\left(1+\dfrac{\mu^2}{2k^2}\right)}{l+n+2}+\mathrm{ctg}\,\pi(l+\tfrac{1}{2})\frac{\alpha mP_n\left(1+\dfrac{\mu^2}{2k^2}\right)}{l+n+1}$$

$$+\mathrm{ctg}\,\pi(l+\tfrac{3}{2})\frac{\alpha mP_n\left(1+\dfrac{\mu^2}{2k^2}\right)}{l+n+2} = 0. \tag{40}$$

Thus, the Regge trajectories are reduced to a family of straight lines parallel to axis $\mathrm{Re}\,l$:

$$l = -n-\tfrac{3}{2}. \tag{41}$$

The behaviour of the Regge poles for intermediate energies can be investigated by numerical calculations.

4. *Regge poles for a quasi-potential problem*

The quasi-potential equation for scalar particles [8] of equal masses

$$T(\vec{p}, \vec{p}') = V[(\vec{p}-\vec{p}')] + \int \frac{d^3q}{\sqrt{q^2+m^2}} \frac{V[(\vec{p}-\vec{q})]T(\vec{q}, \vec{p}')}{k^2-q^2}, \tag{42}$$

and quasi-potential

$$V[(\vec{p}-\vec{p}')] = g \int \frac{d^3r}{(2\pi)^3} \frac{e^{-i(\vec{p}-\vec{p}')\cdot\vec{r}}}{r}, \tag{43}$$

can be reduced to a differential boundary value problem of the second order in the momentum representation [9]. Expanding the total amplitude and the potentials in partial waves

$$T(p, p') = \frac{1}{4\pi pp'} \sum_{l=0}^{\infty} (2l+1) f_l(p, p') P_l\left(\frac{\vec{p}\cdot\vec{p}'}{pp'}\right), \tag{44}$$

$$V(p, p') = \frac{1}{4\pi pp'} \sum_{l=0}^{\infty} (2l+1) V_l(p, p') P_l\left(\frac{\vec{p}\cdot\vec{p}'}{pp'}\right), \tag{45}$$

we find an equation for the partial amplitude

$$f_l(p, p') = V_l(p, p') + \int_0^{\infty} \frac{dq}{\sqrt{q^2+m^2}} \frac{V_l(p, q) f_l(q, p')}{k^2-q^2}. \tag{46}$$

For potential (43) in the momentum representation equation (46) is reduced to the differential equation

$$\frac{d^2 f_l(p, p')}{dp^2} - \left\{\frac{l(l+1)}{p^2} + \frac{g}{\sqrt{p^2+m^2}\,(k^2-p^2)}\right\} f_l(p, p') = g\delta(p-p') \tag{47}$$

with the boundary conditions:

$$p^{l+1} \frac{df_l(p, p')}{dp} - (l+1) p^l f_l(p, p') \xrightarrow[p\to 0]{} 0, \tag{48}$$

$$p^{-l} \frac{df_l(p, p')}{dp} + l p^{-(l+1)} f_l(p, p') \xrightarrow[p\to\infty]{} 0. \tag{49}$$

Green's function $G_l(p, p') = g^{-1} f_l(p, p')$ can be represented as a superposition

$$G_l(p, p') = A_l(p, p') \mathscr{U}_l(p) \mathscr{V}_l(p') + B_l(p, p') \mathscr{V}_l(p) \mathscr{U}_l(p'), \tag{50}$$

where

$$\mathcal{U}_l(p) = p^{-l} \quad \text{and} \quad \mathcal{V}_l(p) = p^{l+1} \tag{51}$$

are two solutions of the potential free equation (47). For the Wronskian of the solutions $\mathcal{U}_l(p)$ and $\mathcal{V}_l(p)$ we get

$$\mathcal{U}_l \frac{d\mathcal{V}_l}{dp} - \mathcal{V}_l \frac{d\mathcal{U}_l}{dp} = 2l+1. \tag{52}$$

The boundary conditions (48) and (49) follow from the boundary conditions for the coefficients A_l and B_l:

$$B_l(+\infty, p') = 0, \qquad A_l(+\infty, p') = \text{const.} \tag{53}$$

The equations for $A_l(p, p')$ and $B_l(p, p')$ will be considered for the regions $p > p'$ and $p < p'$. In the region $p > p'$ Green's function is chosen of the type

$$G_l(p, p') = A_l(p, p') = [\mathcal{U}_l(p)\mathcal{V}_l(p') + C_l(p, p')\mathcal{V}_l(p')\mathcal{V}_l(p)], \tag{54}$$

where

$$C_l(p, p') = \frac{B_l(p, p')}{A_l(p, p')} \cdot \frac{\mathcal{U}_l(p')}{\mathcal{V}_l(p')}. \tag{55}$$

Similarly in the region $p < p'$

$$G_l(p, p') = B_l(p, p') [\mathcal{V}_l(p)\mathcal{U}_l(p') + F(p, p')\mathcal{U}_l(p)\mathcal{U}_l(p')], \tag{56}$$

where

$$F_l(p, p') = \frac{A_l(p, p')}{B_l(p, p')} \cdot \frac{\mathcal{V}_l(p')}{\mathcal{U}_l(p')}. \tag{57}$$

Introducing in the spirit of the phase-function approach the condition

$$\frac{dA_l(p, p')}{dp} \mathcal{U}_l(p)\mathcal{V}_l(p') + \frac{dB_l(p, p')}{dp} \mathcal{V}_l(p)\mathcal{U}_l(p') = 0, \tag{58}$$

we get the variable phase equation for F_l in region $p < p'$

$$\frac{dF_l(p, p')}{dp} = -\frac{1}{2l+1} \cdot \frac{g}{\sqrt{p^2+m^2}} \cdot \frac{1}{k^2-p^2}\, p(^{l+1} + F_l(p, p')p^{-l})^2. \tag{59}$$

An analogical equation can be obtained in region $p > p'$. At the point $p = p'$ for the discontinuities of the coefficients A_l and B_l we find

$$A_l(p'+0, p') - A_l(p'-0, p') = -\frac{1}{2l+1}, \tag{60}$$

$$B_l(p'+0, p') - B_l(p'-0, p') = \frac{1}{2l+1}. \tag{61}$$

The boundary conditions for variable phase equations are

$$C_l(+\infty, p') = 0, \quad F_l(0, p') = 0. \tag{62}$$

For the coefficients $A_l(p, p')$ and $B_l(p, p')$ the boundary conditions can be determined by the expressions (60) and (61):

$$C_l(p, p') = \frac{B_l(p'+0, p')}{A_l(p'+0, p)} \cdot \frac{\mathscr{U}_l(p')}{\mathscr{V}_l(p')}, \quad F_l(p, p') = \frac{A_l(p'-0, p')}{B_l(p'-0, p')} \cdot \frac{\mathscr{V}_l(p')}{\mathscr{U}_l(p')}. \tag{63}$$

Analogical to the case of the variable phase equation (1) the solution of equation (59) with condition (62) takes the form (4), where

$$\varphi_l(r) = -p^{2l+1}, \tag{64}$$

$$G[\varphi_l(p), p] = \frac{1}{2l+1}\int_0^p \frac{g}{\sqrt{x^2+m^2}} \cdot \frac{x^{-2l}}{k^2-x^2}\, \varphi_l^2(x) dx. \tag{65}$$

The functions $G_l(p, p')$ and S_l may have poles only in the zeros of function $D_l(g, k)$. For the first approximation of F_l we get

$$F_l(p, p') = \frac{1}{2l+1}\int_0^{p'} \frac{g}{\sqrt{x^2+m^2}} \cdot \frac{x^{2(l+1)}}{k^2-x^2}\, dx \left(1 + \frac{1}{2l+1}\int_0^{\infty} \frac{gx}{\sqrt{x^2+m^2}} \cdot \frac{dx}{k^2-x^2}\right)^{-1}. \tag{66}$$

The equation of a Regge trajectory is determined by the condition

$$D_l^{(1)}(g, k) + D_l^{(0)}(g, k) = 1 + \frac{1}{2l+1}\int_0^{\infty} \frac{gx}{\sqrt{x^2+m^2}} \cdot \frac{1}{k^2-x^2}\, dx = 0. \tag{67}$$

For negative k^2 the integral in expression (67) can be evaluated, and, therefore, in that case

$$l = -\tfrac{1}{2} + \frac{1}{4\sqrt{m^2+k^2}} \ln \frac{m+\sqrt{m^2+k^2}}{m-\sqrt{m^2+k^2}}. \tag{68}$$

Equation (68) is valid in the region $k^2 < -m^2$. The branch points of the functions (68) are $k^2 = -m^2$ and $k^2 = 0$. For $l = 0$ the expression for g as a function of k^2 is similar to that obtained by the method of standard equation [9]. A more accurate evaluation of Regge trajectories can be performed if we take the next order approximation for D_l.

5. Conclusions

We have presented the method of formal series for calculation of Regge poles for a wide class of problems. The partial amplitude is represened by the ratio of the power series of the coupling constant, valid for strong and weak coupling. So far, we have calculated the Regge trajectories for an ordinary Yukawa potential in the first order approximation, but the method can be applied in principle to obtain higher order approximations and to generalized Yukawa potentials. The method of formal series is a generalization of the method of Fredholm to the nonlinear equations. The possible applications are numerous but we have considered Regge poles for nonrelativistic local potential theory, Regge poles for particles described by the Dirac equation and a problem in quasipotential approach. The applications to quantum mechanical potential problems are mostly connected with bound states and resonances.

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