QUARK FRAGMENTATION FUNCTIONS IN HADRONIC AND DEEP INELASTIC SINGLE PARTICLE INCLUSIVE PROCESSES*

By B. Flume-Gorczyca

Deutsches Elektronen-Synchrotron DESY, Hamburg**

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In the framework of Feynman's quark-parton model for inclusive electroproduction and quark model of Ezawa, Maharana and Miyazawa for hadronic inclusive processes, relations between invariant cross sections for electroproduction of pions and purely hadronic production of pions in the beam fragmentation regions are derived. They are satisfied by data. The ratio of parton fragmentation functions $\eta = D_u^{\pi^+}(z)/D_d^{\pi^+}(z)$ is estimated from purely hadronic processes independently from the parton model.

The purpose of this paper is to relate the inclusive electroproduction of pions in the parton fragmentation region with the inclusive production of pions in purely hadronic processes in the beam fragmentation region.

For electroproduction we use the framework of Feynman's quark parton model [1], [2], and for hadronic inclusive processes the additive quark model the starting idea of Ezawa, Maharana and Miyazawa model [3].

In Feynman's quark-parton model [1] $N_{ep}^{\pi}(x, z) = \frac{1}{\sigma_{\text{tot}}}\frac{d\sigma}{dxdz}$ is written as the product of probability to hit a certain type of quark in the proton times the probability that this quark decays into a pion plus anything:[1]

$$N_{ep}^{\pi}(x, z) f_1^{ep}(x) = \tfrac{4}{9}\, u(x) D_u^{\pi}(z) + \tfrac{4}{9}\, \bar{u}(x) D_{\bar{u}}^{\pi}(z)$$
$$+ \tfrac{1}{9}\, d(x) D_d^{\pi}(z) + \tfrac{1}{9}\, \bar{d}(x) D_{\bar{d}}^{\pi}(z) + \tfrac{1}{9}\, s(x) D_s^{\pi}(z) + \tfrac{1}{9}\, \bar{s}(x) D_{\bar{s}}^{\pi}(z), \tag{1}$$

where

$$f_1^{ep}(x) = \tfrac{4}{9}\,(u(x) + \bar{u}(x)) + \tfrac{1}{9}\,(d(x) + \bar{d}(x)) + \tfrac{1}{9}\,(s(x) + \bar{s}(x)).$$

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** Address: Deutsches Elektronen-Synchrotron DESY, 2 Hamburg 52, Notkestieg 1, BRD.

[1] In the following we will omit $p_\perp^2$ dependence of D's. It is understood that all relations are integrated over $p_\perp^2$.

The variable x is equal to $1/\omega$ and the variable z is the energy of the pion measured in terms of the energy of the photon in the laboratory system. These D functions depend neither on q^2 nor on x [1, 2]. Six functions $u(x)$, $d(x)$, ... give us the average number of quarks u, d, s, ..., in the proton with a fraction x of the proton momentum. The six $D_q^\pi(z)$ are the parton (quark) fragmentation functions. Here the produced π carries a fraction z of the quark momentum.

Ezawa, Maharana and Miyazawa (EMM) [3] proposed a simple quark model describing inclusive cross sections for hadron-hadron interactions: $a+b \to c+$ anything. We take over the starting idea of this model. In this scheme one quark in hadron a and one in b collide and start a multiple production: quark+quark $\to c+$anything. This process contributes mainly to the central region. Other spectator quarks are shaken off and decay into hadrons: quark $\to c+$anything. Processes of the second type contribute to the fragmentation regions of hadrons a and b.

We shall be concerned with the description of the fragmentation regions in this model. In the fragmentation region particle c is produced as a fragment of a quark. It is expected that distribution of the particle c is isotropic in the quark's rest frame and is a function of $\vec{p}_c^{\,2}$ ($\vec{p}_c$ being the momentum of particle c in quark rest frame). It is easy to show that $\vec{p}_c^{\,2} = f(z, p_\perp^2)$, where z is a finite fraction of the quark momentum taken by particle c, $p_\perp$ is its transverse momentum. We suggest that this process should be described by the same quark fragmentation functions $D(z)$ which appear in the Feynman quark parton model[1].

Now we confine ourselves to the inclusive meson M' production in the meson M fragmentation region. In this case we can write for $N(x_F) = 1/\sigma_{\text{tot}}\, d\sigma/dx_F$ the following expression

$$N_{MT}^{M'}(x_{\text{F}}) = \tfrac{1}{2}\,(D_q^{M'}(z)+D_{\bar{q}}^{M'}(z)), \tag{2}$$

where x_F is the Feynman's variable, T denotes a target.

In the pion fragmentation region ($x_F > 0$) the additive quark model predicts the following:[2] (it is assumed that D functions are related among themselves by the charge symmetry and charge conjugation)

$$N_{\pi^+p}^{\pi^+}(x_{\text{F}}) = N_{\pi^-p}^{\pi^-}(x_{\text{F}}) = D_u^{\pi^+}(z), \tag{4}$$

$$N_{\pi^+p}^{\pi^-}(x_{\text{F}}) = N_{\pi^-p}^{\pi^+}(x_{\text{F}}) = D_{\bar{d}}^{\pi^+}(z), \tag{5}$$

$$N_{\pi^0p}^{\pi^-}(x_{\text{F}}) = \tfrac{1}{2}\,(D_u^{\pi^+}(z)+D_{\bar{d}}^{\pi^+}(z)), \tag{6}$$

where $z = \dfrac{p_c}{p_{\text{quark}}}$.

If we look for fragments of the beam then single inclusive distributions in hadronic

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[2] We will not quote here predictions for inclusive production of kaons (or pions) by pion, kaon or proton beams, but one can use them (additionally with help of SU(3) relations, and with presence of a strange quark fragmentation function D_s^π) also to test the consistency of the EMM model with Feynman's quark parton model.

processes are described just by the same quark fragmentation functions which are used for description of the single inclusive spectra in ep scattering.

Cleymans and Sehgal [4] have considered arguments supporting the assumption that in the quark parton model "functions $\bar{u}(x)$, $\bar{d}(x)$, $s(x)$ and $\bar{s}(x)$ may be neglected in comparison to $u(x)$ and $d(x)$ in an interval $x_0 \leqslant x \leqslant 1$, where x_0 is an unspecified, but presumably small number". Using this assumption they were able to extract $u(x)$ and $d(x)$ from the inelastic electron scattering data. So they have for $\pi^{\pm}$ inclusive distributions in electron--proton scattering (excluding the small x region) the following relations:

$$N_{ep}^{\pi^+}(x, z) \cong [\tfrac{4}{9}\, u(x)D_u^{\pi^+}(z)+\tfrac{1}{9}\, d(x)D_d^{\pi^+}(z)]/f_1^{ep}(x), \tag{7}$$

$$N_{ep}^{\pi^-}(x, z) \cong [\tfrac{4}{9}\, u(x)D_d^{\pi^+}(z)+\tfrac{1}{9}\, d(x)D_u^{\pi^+}(z)]/f_1^{ep}(x), \tag{8}$$

where $f_1^{ep}(x) \cong \frac{4}{9}u(x)+\frac{1}{9}d(x)$.

Knowing the probabilities of hitting different kinds of partons (functions $u(x)$ and $d(x)$) and invariant cross sections $N_{ep}^{\pi^{\pm}}$ Cleymans and Rodenberg [5] extracted the quark fragmentation functions $D_u^{\pi^+}(z)$ and $D_d^{\pi^+}(z)$. Another result of this paper [5] is that now if $N_{ep}^{\pi}(x, z)$ is known (e.g. experimentally) for one particular value of x, it is easy to calculate it for values of x (excluding the small x region).

Combining relations (4)–(6) and (7)–(8) we can write down the following sum rules:

$$N_{ep}^{\pi^+}(x, z)\,(4u(x)+d(x)) = 4u(x)N_{\pi^+p}^{\pi^+}(x_F)+d(x)N_{\pi^+p}^{\pi^-}(x_F), \tag{9}$$

$$N_{ep}^{\pi^-}(x, z)\,(4u(x)+d(x)) = 4u(x)N_{\pi^+p}^{\pi^-}(x_F)+d(x)N_{\pi^+p}^{\pi^+}(x_F). \tag{10}$$

The comparison should be made at an experimental point where the momentum carried by the quarks is the same in ep and in πp.

Moreover, from purely hadronic inclusive cross sections in the fragmentation region in the framework of EMM model we can calculate the ratio $\eta(z) = D_u^{\pi^+}(z)/D_d^{\pi^+}(z)$. So it can be done independently from the parton model. This ratio η is an important variable as in the framework of quark parton model one needs it to calculate the charge ratio $N_{ep}^{\pi^+}/N_{ep}^{\pi^-}$. Dakin and Feldman [6] have pointed out that in the McElhaney and Tuan [7] fit to the Kuti–Weisskopf model η (average over z) gives the upper limit of $N_{ep}^{\pi^+}/N_{ep}^{\pi^-}$ for $x = 1$ (similar conclusion can be drawn from Refs [4–5] as from graphs given there for $u(x)$ and $d(x)$ for $0.1 < x < 0.8$ one can expect that $\lim_{x\to 1}(d(x)/u(x)) = 0$).

If we want to check experimentally relations (9)–(10) or to calculate $\eta(z)$ from hadronic processes we meet one difficulty, namely the quark fragmentation function $D_u^{\pi^+}$ is always connected with the inclusive processes for which the leading particle effect [8, 9] plays important role (in above relations processes $\pi^+ \xrightarrow{p} \pi^+$ and $\pi^0 \xrightarrow{p} \pi^-$). Here the result of the paper given by Cleymans and Sehgal [4] is very useful. Namely for $x \geqslant 0.8$ we can assume that $d(x) \cong 0$. It gives us the interesting test of the relation (10):

$$N_{ep}^{\pi^-}(x \geqslant 0.8;\, z) = N_{\pi^+p}^{\pi^-}(x_F)\,(= N_{\pi^-p}^{\pi^+}(x_F)) \tag{11}$$

because in the process $\pi^- \xrightarrow{p} \pi^+$ (or $\pi^+ \xrightarrow{p} \pi^-$)$\pi^+$ (or π^-) is non-leading particle.

If we want to compare with data relation (9) or to calculate the ratio η we must avoid the contamination of the pure fragmentation mechanism by the leading particle effect in other way. To reach it we use the data for inclusive reaction $\gamma p \to \pi^- +$anything with elastic ϱ^0 production excluded [10]. To relate the invariant cross section for the process $\gamma p \to \pi^- +$anything (with ϱ^0 elastic excluded) with hadronic inclusive production of pion we use the relation obtained by combining vector meson dominance with the quark model [11]. In this way we have:

$$N_{\gamma p}^{\pi^-} = N_{\gamma p}^{\pi^+} = \frac{\gamma \xrightarrow{p} \pi^-}{\sigma_{\text{tot}}(\gamma p)} = \frac{\pi^0 \xrightarrow{p} \pi^-}{\sigma_{\text{tot}}(\pi^0 p)} \tag{12}$$

and next from the equation (6) we get

$$N_{\gamma p}^{\pi^-}(x_F) = \tfrac{1}{2}(D_u^{\pi^+}(z)+D_d^{\pi^+}(z)). \tag{13}$$

Now we can calculate the ratio $\eta(z) = D_u^{\pi^+}(z)/D_d^{\pi^+}(z)$ e.g. from the relation

$$N_{\gamma p}^{\pi^-}(x_F)/N_{\pi^+ p}^{\pi^-}(x_F) = \tfrac{1}{2}(1+\eta(z)). \tag{14}$$

Moreover from relations (7), (8) and (13) we obtain an interesting sum rule:

$$N_{ep}^{\pi^+}(z)+N_{ep}^{\pi^-}(z) = 2N_{\gamma p}^{\pi^-}(x_F) \quad \text{(with } \varrho^0 \text{ excluded)}. \tag{15}$$

Comparison with the data

a) For the relation (11):

$$N_{ep}^{\pi^-}(x = 0.8; z) = N_{\pi^+ p}^{\pi^-}(x_F).$$

The comparison of the relation (11) with the data is presented in Fig. 1.

$N_{ep}^{\pi^-}(x = 0.8; 0.3 < z < 0.7)$[3] is taken from the graph in Ref. [5] and $N_{\pi^+ p}^{\pi^-}(x_F)$ from Ref. [12] (at $p_{\pi^+} = 3.7$ GeV).

We have chosen these data as in this case the momentum carried by quarks is the same in ep and πp ($\simeq 1.25$ GeV). We see that relation (11) is well attest in the region $0.3 < x_F < 0.7$.

b) For the relation (15):

$$N_{ep}^{\pi^+}(z)+N_{ep}^{\pi^-}(z) = 2N_{\gamma p}^{\pi^-}(x_F) \quad \text{(with } \varrho^0 \text{ excluded)}.$$

To compare the relation (15) with data we use the fit from Refs [5, 13] namely (at $Q^2 = 1.20$ and 2.02 GeV^2, where W average over these two points is equal 2.4 GeV):

$$2N_{ep}^{\pi^-}(x = 0.25; z) = N_{ep}^{\pi^+}(x = 0.25; z) = \frac{\pi}{Bz} e^{-3.17z+0.45} \quad 3$$

In Fig. 2 the points put on the experimental data for $N_{\gamma p}^{\pi^-}$ (at $E_\gamma = 2.8$ GeV and $\sigma_{\text{tot}} = 133$ μb) [10] are given by above fit to the data for $\frac{1}{2}(N_{ep}^{\pi^-}+N_{ep}^{\pi^+})$ with $B = 5.23 \pm 0.68$

[3] The variable z is essentially identical with the variable x_F for considered kinematical region of electroproduction.

(0.5 < $x' \approx z$ < 0.7). These points fit the photoproduction data satisfactorily in the region $0.2 < x_F < 0.7$.

c) Calculation of the $\eta(z)$ from the relation (14):

$$N_{\gamma p}^{\pi^-}(x_F)/N_{\pi^+ p}^{\pi^-}(x_F) = \tfrac{1}{2}(1+\eta(z)).$$

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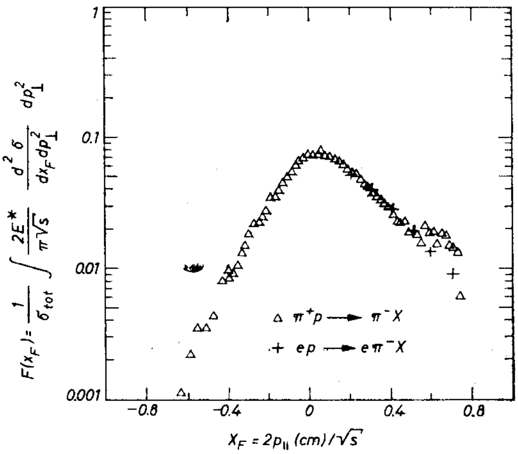

Fig. 1. Experimental check of the relation (11), see the main text for details

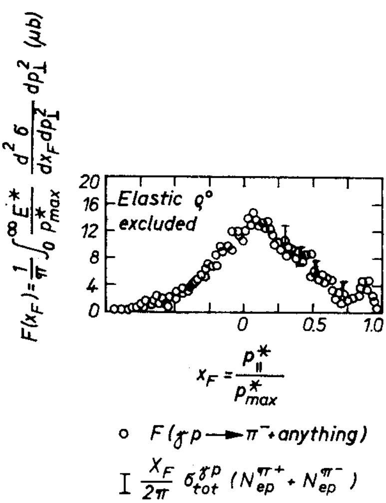

Fig. 2. Experimental check of the relation (15), see the main text for details

We use data from Ref. [10] for $\gamma p \to \pi^-$ + anything (at E_γ = 2.8 and 4.7 GeV) and data from Ref. [12] for $\pi^+ p \to \pi^-$+ anything (at E_{π^+} = 3.7 GeV). In this way we get η = 3.0 with the error of 20% for $0.4 \leqslant z \leqslant 0.8$. This number is identical with one obtained by Dakin and Feldman [6] and similar with one obtained by Cleymans and Rodenberg (η = 2.59 for $0.4 < z < 0.8$) [5].

Conclusions

In the framework of Feynman's parton-quark model and additive quark model for inclusive hadronic processes, relations between inclusive electroproduction of pions and purely hadronic inclusive production of pions in the beam fragmentation region are found. These relations are obtained through quark fragmentation functions.

We have considered in detail the case of pion production. We have obtained relations between $N_{ep}^{\pi^\pm}$ and $N_{\pi\pm p}^{\pi^\pm}$, $N_{\pi\mp p}^{\pi^\pm}$ or $N_{\gamma p}^{\pi^\pm}$ for beam fragmentation region. We found that these relations are satisfied by data in the broad region of x_F, namely $0.2 \lesssim x_F \lesssim 0.7$. This fact is remarkable in spite of simplicity of the additive quark model assumed here for the hadronic processes.

Moreover we calculated from purely hadronic processes the ratio $\eta(z) = D_u^{\pi^+}(z)/D_d^{\pi^+}(z)$. The value obtained in this way agrees with the one obtained by other authors from data on electroproduction analyzed in the framework of parton model.

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REFERENCES

[1] R. P. Feynman, *Photon-Hadron Interactions*, W. A. Benjamin, Reading, Massachusetts, 1972.
[2] M. Gronau, F. Ravndal, Y. Zarmi, *Nucl. Phys.* **B51**, 611 (1973).
[3] Z. F. Ezawa, J. Maharana, H. Miyazawa, *Prog. Theor. Phys.* **51**, 877 (1974).
[4] J. Cleymans, L. M. Sehgal, *Nucl. Phys.* **B61**, 479 (1973).
[5] J. Cleymans, R. Rodenberg, *Phys. Rev.*, to be published.
[6] J. T. Dakin, G. J. Feldman, *SLAC preprint*, SLAC-PUB-1273, July 1973.
[7] J. Kuti, V. Weisskopf, *Phys. Rev.* **D4**, 3418 (1971); R. McElhaney, S. F. Tuan, *Preprint Hawaii*, UH-511-156-73, May 1973.
[8] H. Satz, *Phys. Lett.* **25B**, 220 (1967).
[9] D. R. O. Morrison, *Proc. of the 4th Intern. Conf. Oxford*, 1972, p. 253.
[10] Stanford-Berkeley-Tufts Collaboration, *Phys. Rev.* **D5**, 1603 (1972).
[11] H. Satz, *Phys. Lett.* **25B**, 27 (1967).
[12] M. Alston-Garnjost et al., *Phys. Lett.* **39B**, 402 (1972).
[13] Harvard-Cornell Collaboration, C. J. Bebek et al., *Phys. Rev. Lett.* **30**, 624 (1973).