MODELS OF GALACTIC NUCLEI

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Various models of the galactic nuclei are discussed. The dense star cluster model for the galactic center is treated in greater detail. It turns out that such model can explain many of its observed properties, i. e. emission of gravitational waves, number density and energy of cosmic rays in the Galaxy, γ and X-ray emission, infrared and radio spectrum.

1. General remarks

In this paper we will discuss some models of the structure of galactic nuclei. Many new experimental facts have been discovered in the last few years which indicate that the majority of galactic nuclei is very active: they exhibit in fact strong emission in the infrared, γ and radio band of the spectrum. According to Weber's interpretation of his experimental data [1], it appears that the nucleus of our galaxy is a very strong source of gravitational waves.

There is much discussion about this interpretation which is justified by the revolution it would cause in astrophysics if it turned out to be correct. In fact it would imply a loss of about 1 $M_{\odot}$ per pulse, i.e. a few hundred solar masses per year. With this loss the life of the Galaxy would be $\sim 10^9$ years. We are therefore faced with some possibilities:

a) the mass of the Galaxy is one or two orders of magnitude greater than the current estimate;

b) the emission of gravitational waves from the galactic center is not isotropic;

c) the interpretation of Weber's data is wrong.

This last possibility is real in the light of recent searches for gravitational pulses in Moscow University [2] which have not confirmed Weber's data. One of the main features of Braginski's experiment is the detailed examination of the pulse which indicates that the supposed coincidences are not real because the shapes of coincident pulses are different.

If, on the other hand, Weber's interpretation is correct, the present density of gravitons in the Universe would be greater than the photon density. In fact with the conservative

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estimate that about $10^{-1} \div 10^{-2}$ of the galaxies have an active nucleus similar to that of our galaxy, we get for the energy density of the gravitons ϱ_γ:

$$\varrho_\gamma = \frac{M_\odot c^2 n \tau N}{V} \sim 10^{-11} \div 10^{-10} \text{ erg/cm}^3,$$

where n is the number of pulses per year; τ is the age of the Universe, N is the number of galaxies similar to our, V is the volume of the Universe.

Such figure is very interesting because of its cosmological implications and is many orders of magnitude greater than a previous estimate we made before Weber's results were known.

Let us now take into account the infrared and radio emission from galactic nuclei. The infrared spectrum of the nucleus of our Galaxy is peaked at $\sim 4 \cdot 10^{12}$ Hz and the observed flux at this frequency is $\sim 10^{-18}$ erg/cm^2·sec Hz [4] on the Earth; while the radio spectrum is almost flat with a mean flux $\sim 10^{-21}$ erg/cm^2·sec·Hz [5]. It is very interesting to notice that the infrared spectrum of all the Galaxies so far examined is peaked at the same frequency of $4 \cdot 10^{12}$ Hz, with, however, a different intrinsic luminosity.

In order to explain these features, it has been assumed that in the nucleus of the Galaxy there is:

a) a supermassive collapsed body (black-hole) on which the accretion of the surrounding gas takes place [6]

b) a supermassive rotating magnetoplasmic body called Magnetoid [7, 8]

c) a compact star system [9].

As to model a), where the energy released (at least in the case of stationary radiation) is the result of spiralling of interstellar gas to the central hole and the infrared radiation is due to the infall of matter from outside [6], Ambartsumian [11] emphasized that the idea of a collapsed body as a source of energy encounters the foundamental difficulty that we observe only an ejection of matter rather than an infall as, for instance, in nuclei of Seyfert Galaxies and Quasars.

Besides, we observe that the bulk of radiation from a black-hole is of thermal origin while the spectrum and the polarization of the infrared radiation suggests rather a non-thermal origin.

In model b) the supermassive body is outside its Schwarzschild radius and endowed with rotation and magnetic field [7, 8]. According to Ozernoy, who calls this supermassive body a "magnetoid", in order to explain the energy released and the active phase of Seyfert Galaxies the mass of the object must range between 10^4 and 10^9 $M_\odot$; the emitted energy must be a not too small fraction of the rest mass. It is known in fact that Seyfert Galaxies are about 1% of normal spirals and this fact gives evidence to the fact that the active phase lasts about 10^8 years. This life-time can be considered as a mean-harmonic in the sense that the active phase is recurrent and one can have 100 active phases lasting each about 10^6 years every $\sim 10^8$ years.

With this model one can explain thermal, synchrotron and infrared emission, as well as other features. According to Ozernoy, the evidence for a single body in the nuclei of galaxies rests on the quasi-periodic character of the optical variability discovered in a number of quasars (for instance 3C 273) and active nuclei. This periodicity is not very strong but the regular component prevails on the chaotic one. Radio variability data (as, for instance, in 3C 279) also seem to support this model.

Ozernoy remarks also that among 82 extragalactic radio sources there are at least 5 sources consisting of two pairs of components which are lying along the same axis and this fact is in favour of a single rather than a multiple source producing double-double radio components.

Here we will consider in more details model c), which allows foı a theoretical explanation of a greater number of expeiimental data: infrared, radio, and X-ray emission, as well as cosmic rays and gravitational waves.

2. *Cluster of dense stars*

The assumption of the existence of a massive cluster of dense stars in galactic nuclei is acquiring more and more support by experimental results. It is not unlikely that the bulk of the mass of a galaxy is concentrated in a very small volume in the center. We believe that the mass estimate of the nuclei of galaxies derived from the rotation curves must be taken very cautiously. In fact these estimates are valid only in the assumption that a galaxy is in an almost equilibrium state. The recent work, however, does not seem to support the almost equilibrium state assumption. For instance, if one tries to calculate the mass and density of M 31 from the rotation curves, one finds quite contradictory results, a situation which could probably be explained if one does not take into account the assumption of an equilibrium state [12]. Other data supporting greater estimates of galactic masses can be derived from the study of the motion of galaxies in clusters of galaxies. Applying the virial theorem, one can deduce that the mass of the whole cluster is one order of magnitude greater than the sum of the masses of the single galaxies as deduced from luminosity measurements [13].

On the grounds of these considerations we are encouraged to make the assumption that in the center of our galaxy there is a very massive cluster of dense stars. These stars are assumed to be neutron stars with the mass $\sim 1\ M_\odot$ and radius $\sim 10^6$ cm. The cluster has a radius $R\ \sim 10^{17}$ cm and is formed by $N = 1.2\ 10^{11}$ neutron stars.

The choice of $R = 10^{17}$ cm is due to energetic requirements: all the energy emitted by the cluster is at expenses of the kinetic energy of the stars, which increases with decreasing radius (virial theorem). On the other hand, with a radius less than three gravitational radii, the cluster becomes gravitationally unstable.

The further assumption is made that 10% of the total number of neutron stars has the maximum allowed magnetic moment, i.e. $\mu \sim 10^{33}$ erg/G [14]. This 10% ratio is the same as the ratio between magnetic and normal stars in our galaxy. The mean velocity of a star can be deduced from non-relativistic virial theorem

$$2T+V = 0,$$

i.e.

$$mv^2 = \frac{GMm}{R} = \frac{GMmc^2}{Rc^2} = zmc^2\,,$$

where $z = \frac{GM}{Rc^2}$ is the red-shift of a photon emitted by the cluster. For our cluster $z \cong 0.16$ and the mean velocity is $v \cong 0.4\,c$.

3. *Acceleration process. Cosmic ray injection*

The model outlined in the previous section can, in the first place, explain the injection of cosmic rays in the galaxy [10]. As to the acceleration mechanism of cosmic rays we follow the Fermi mechanism [15]. The main difficulty of this model is the very high injection energy required in order that the protons can gain energy by scattering against the galactic random magnetic field inhomogeneities and this gain is greater than the losses due to nuclear interaction. This treshold energy is for protons ~200 MeV. Such injection energy is provided in our model by the motion of the magnetic neutron stars through the ionized interstellar gas inside the cluster. We assume for the interstellar gas inside the cluster a density $N_H = 10^8$ hydrogen atoms cm^{-3} with 1% ionized.

The acceleration process of the ionized interstellar gas inside the cluster takes place as follows. We call E-system the reference system of the Earth which pratically coincides with that of the whole galaxy and therefore with that of the cluster. In this system the ionized particles, before being scattered by the wandering magnetic fields of the neutrons stars, move with thermal velocity, while in the Star-system (S-system) they have a velocity $v \cong 0.4\,c$. In the S-system, after the interaction with the magnetic field, the particles do not change their energies, but only the directions of their momenta. Then in the E-system, in respect to which the star moves with a velocity ~0.4 c, the speed u of the particle will be:

$$u = \frac{v' + 0.4c}{1 + \frac{v' 0.4c}{c^2}} = 0.69c,$$

where $v' = 0.4c$ is the velocity of the particle in the S-system. Then in the E-system the kinetic energy will be:

$$T = m_p c^2 \left(\frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} - 1 \right) \left(\frac{1 - \cos\chi}{2} \right) = 0.19 m_p c^2 (1 - \cos\chi),$$

where χ is the scattering angle in the S-system. Then the maximum energy gained in the first scattering is

$$T_{\max}^{(1)} = 0.38 m_p c^2.$$

If now the protons, with this energy, knock a second time against a magnetic star, they will gain an energy:

$$T_{\max}^{(2)} = 1.81 m_p c^2.$$

In this calculation we have used $T_{\max}^{(1)}$ as initial energy because for a scattering angle in the range $0 \leqslant \chi \leqslant \frac{\pi}{2}$, which is the range in which presumably the major part of the deflections occurs, the energy gained is of the same order of magnitude as the maximum energy.

For subsequent scatterings we have

$$T_{\max}^{(3)} \cong 5.4 m_p c^2, \quad T_{\max}^{(4)} \simeq 13.8 m_p c^2$$

and so on.

A proton in leaving the cluster loses energy because of the gravitational potential barrier and of ionization of neutral hydrogen. The energy lost in overcoming the gravitational barrier is $\sim \frac{3}{2} \frac{GMm}{R} = 3.75 \cdot 10^{-4}$ erg and that lost in the ionization process is [16]:

$$\frac{dE}{dx} R = -R N_H \phi_0 m_e c^2 \frac{3 m_p c^2}{4T} \log \frac{4 m_e T}{I m_p} \simeq 1.14 \cdot 10^{-4} \text{ erg},$$

where $\phi_0 = 6.65 \cdot 10^{-25}$ cm^2 (Thomson cross-section); m_e = electron mass; m_p = proton mass; T = kinetic energy of the protons $\sim 0.38\, mc^2 = 5.47 \cdot 10^{-4}$ erg; I = ionization energy of the hydrogen atom. The energy which is left to proton after traversing the cluster is therefore $E_K \sim 30$ MeV. This energy is less than the threshold energy ~ 200 MeV required by the Fermi injection process.

On the other hand, the energy gained in the second scattering is much greater than the minimum needed, and therefore, even by taking into account the factor $(1 - \cos \chi)$, they have an energy which is always sufficient for the injection.

Now we can evaluate the number of particles which are accelerated per second per star. In order to do this we must evaluate the volume swept by a star together with its magnetic field. An estimate of the depth reached by the particles in the magnetic field of the star is given by

$$\frac{1}{8\pi} \left(\frac{\mu}{r^3} \right)^2 = T_S N_p,$$

where the left hand side is the energy density of the magnetic field at a distance r from the dipole center and the right hand side is the proton energy density in the S-system. For thermal protons we have $T_S = 0.09\, mc^2$, then, with $N_p = 10^6$ cm^{-3} we have $T_S N_p = 130$ erg/cm^3. Substituting this in the preceding equation we get with $\mu = 10^{33}$ erg/G

$$r = 2.6 \cdot 10^{10} \text{ cm}$$

from which the effective accelerating surface $\pi r^2 = 2 \cdot 10^{21}$ cm^2. The volume swept per second is therefore $V = v \pi r^2 = 2.4 \cdot 10^{31}$ cm^{3}sec^{-1} and the number of protons scattered the first time per star per second is $\sim 2.4 \cdot 10^{37}$ sec^{-1}. The total number of protons scattered in the whole cluster is therefore

$$n_p^{(1)} \sim 3 \cdot 10^{47} \text{ particles sec}^{-1}. \quad (1)$$

To get particles with enough energy it is necessary that they undergo a second scattering. In order to calculate the density of such particles it is necessary to know the density of particles which had the first scattering. This last quantity is easily computed equating the rate of production to the rate at which the first scattering particles diffuse out of the cluster; and we get [17]

$$\nu = \tfrac{4}{3}\pi v^{(1)} lRn,$$

where $v^{(1)}$ is the mean velocity of the first scattering protons, l is the mean free path of these protons inside the cluster and n is their density.

As to l, we notice that the ionization process does not perturb substancially the trajectory of a proton which, therefore, goes on a straight line trough the cluster; on the other hand, the mean free path for a second scattering by a magnetic star is

$$l \sim \frac{l}{\pi r^2 N} \sim 10^{20}\ \mathrm{cm},$$

where $N \sim 0.3 \cdot 10^{-41}\ \mathrm{cm}^{-3}$ is the number density of magnetic stars and $r = 2.6 \cdot 10^{10}\,\mathrm{cm}$ is the radius of the magnetosphere. In this way there is no diffusion and we can take $l \sim R$. Therefore, we have

$$\nu \sim \tfrac{4}{3}\pi R^2 v^{(1)} n = 3 \cdot 10^{47},$$

from which we get

$$n \sim \frac{3 \cdot 10^{47}}{4R^2 v} \sim 4 \cdot 10^2\ \mathrm{cm}^{-3}.$$

Such equilibrium density is reached in a time

$$t = \frac{nV}{n_{\mathrm{p}}^{(1)}} = \frac{4 \cdot 10^2 4 \cdot 10^{51}}{3 \cdot 10^{47}} = 5 \cdot 10^6\ \mathrm{sec},$$

which is very small compared with the mean life of the cluster and we can always assume an equilibrium configuration with $4 \cdot 10^2$ protons which can have a second scattering. Remembering that the radius of the magnetosphere is $2.6 \cdot 10^{10}$ cm, we get for the number of protons which undergo the second scattering

$$n_{\mathrm{p}}^{(2)} \sim 10^{44}\ \text{protons sec}^{-1}, \tag{2}$$

These protons have a kinetic energy $\sim 10^{-3}$ erg; then the total energy carried by cosmic ray protons is $\sim 10^{41}$ erg. sec^{-1}. Both numbers are in a very good agreement with standard estimates of the present density of cosmic rays in the Galaxy [17]. Clearly the number of particles which undergo subsequent scatterings is smaller and smaller because the density in the cluster decreases. The energy of these protons is greater than 200 MeV and, therefore, the main difficulty of the Fermi theory is overcome.

To be notice also that the total energy of the particles ($\sim 10^{41}$ erg sec^{-1}) is just the energy necessary to keep the present density of cosmic rays in our Galaxy [17].

The particles heavier than hydrogen are also accelerated by the same process. The minimum injection energy for α particles, oxygen and iron is respectively [15] about

1 BeV, 20 BeV and 300 BeV. Our model predicts 7 BeV for α particles, 29 BeV for oxygen and 100 BeV for iron. Subtracting from these values the loss of energy due to overcoming the gravitational potential barrier, we get 6 BeV, 23 BeV and 83 BeV, respectively. The second scattering is therefore not enough to give the heaviest particles the minimum energy: this energy can be reached in subsequent scatterings in which, however, the number of accelerated particles is reduced.

As to the abundance of heavy elements, it is difficult to give an estimate: in fact their number depends upon the relative abundance in the interstellar gas of the cluster on which nothing precise can be said.

The supply of material must be $3 \cdot 10^{47}$ particles sec^{-1}, i.e. $\sim 10^{-2}\ M_\odot$/year and can be given without difficulty either by the infalling of gas from the surrounding parts of the Galaxy or by tidal effects during collisions of stars [18].

4. *Gravitational waves*

Let us now take into account the mean life of the cluster. The energy tansferred to cosmic rays is negligible with respect to the total kinetic energy of the stars and therefore the mean life of the cluster is not affected by this loss of energy. The life is dominated by the following processes [18], [19]:

a) emission of gravitational waves in the pulsating mode due to binary collisions of stars,

b) evaporation of stars.

Let us take into account in turn these processes.

a) The number of interactions per second between two neutron stars with impact parameter b which are responsible for the emission of gravitational waves is

$$N_{\text{int}} = \pi b^2 \frac{N^2 v}{V}\,.$$

Assuming for b the value of a few stellar radii, we get

$$N_{\text{int}} \sim 10\ \text{yr}^{-1}$$

which is less than Weber's estimate. Assuming that the whole kinetic energy of a star ($\sim 10^{53}$ erg) is radiated as gravitational waves, and taking into account that the total kinetic energy of the cluster is $\sim 10^{64}$ erg, we get a mean-life $\sim 10^{10}$ years.

b) The rate of escape of stars from the cluster is given by [20]

$$\frac{dN}{dt} = -0.007\,\frac{N}{T_{\text{E}}} \sim 2 \cdot 10^{-7}\ \text{stars sec}^{-1},$$

where

$$T_{\text{E}} = \tfrac{1}{16}\left(\frac{3\pi}{2}\right)^{1/2}\left(\frac{NR^3}{GM_\odot}\right)^{1/2}\frac{1}{\ln N} \sim 5 \cdot 10^{15}\ \text{sec}$$

is the relaxation time.

The total number of neutron stars ejected in 10^{10} years is therefore $\sim 6 \cdot 10^{10}$. If this model is correct, the number of neutron stars in our Galaxy is some order of magnitude greater than estimated on the ground of pulsar counting. Besides this, such number is comparable with the number of normal stars. This fact is suggestive of a possible role that neutron stars could play in the formation of normal stars acting as accretion centers of diluted hydrogen in galactic clouds. Such idea is only a proposal, which is in line with Ambartsumyan view. It would be therefore extremely interesting to investigate the various steps (if they exist) along which such an object could become a normal star. It is our opinion that the deep perturbations caused by the accretion should modify the equilibrium conditions of the neutron star, which therefore after a certain transition period could not exist any more as such and would evolve towards a normal star.

5. *γ-ray production*

The second scattering protons have an energy above the treshold of π^0 production and therefore in their inelastic collision with the interstellar matter can produce γ-rays by the π^0 decay into photons. Because of the different matter density in the cluster ($\sim 10^8$ protons cm^{-3}) and outside (~ 1 proton cm^{-3}) the major part of the production will take place inside the cluster. The density of these second scattering protons can be computed, as before, by equating the rate at which they are produced (i. e. 10^{44} protons sec^{-1}) to the rate at which they leave the cluster (i. e. $4\, R^2\, n_p^{(2)}\, c$).

We get in this way

$$n_p^{(2)} \sim 10^{-1} \text{ protons cm}^{-3}.$$

Assuming a cross-section for π^0 production $\sigma_{\pi^0} \sim 3 \cdot 10^{-27}\, cm^2$, the total number of π^0 is $\sim N_H\, \sigma_{\pi^0}\, n_p^{(2)}\, c \frac{4}{3} \pi R^3 \sim 4 \cdot 10^{42} \pi^0\, sec^{-1}$.

If we take into account that every π^0 can decay in a few photons, we have for the total production of γ-rays $\sim 10^{42} - 10^{43}\, \gamma\, sec^{-1}$. The flux on the earth is $\sim 10^{-4} - 10^{-3}$ $\gamma\, cm^{-2}\, sec^{-1}$, in agreement with the observed values [21].

The number of photons lost to cosmic ray flux in nuclear collisions is $\sim 10^{-2}$ of the total number and this does not influence the production of cosmic rays.

6. *Infrared emission: the dust-shell model*

The recent observations in the far infrared by Low [4] and Becklin and Neugebauer [22] show that the central core from which much of the far infrared radiation is emitted is an object with a size $\sim 10^{17} \div 10^{18}$ cm. The flux increases steeply with frequency to a maximum at about $3 \cdot 10^{12}$ Hz and then falls again. The total energy emitted in the infrared was measured to be $\sim 10^{40} - 10^{41}$ erg sec^{-1}.

For higher frequencies ($\sim 10^{14}$ Hz) a dust-shell model was proposed by Wickramasinghe [23] in which the dust grains are heated by cosmic rays of low energy. We shall now investigate whether this model can also account for the emission at the main peak at $3 \cdot 10^{12}$ Hz.

In our model the protons which suffered the first scattering can play this role through electronic excitation of the lattice of a dust grain and causing thereby the dust to be heated.

We shall assume that the dust consists of graphite grains with a radius a $\sim 10^{-5}$ cm, density $\varrho \sim 2.5$ gr $\cdot$ cm^{-3}. The energy lost by a proton in a dust grain is

$$\Delta E \sim \frac{2a}{E_K} \frac{2\pi e^4 m_H}{m_e} \frac{N_0 \varrho}{A} \ln\left(\frac{2E}{I}\right) \sim 5 \cdot 10^{-10} \text{ erg},$$

where $A = 12$ is the atomic weight for carbon, I is a mean ionization potential and $N_0 \sim 6 \cdot 10^{23}$ is the Avogadro number.

If $n_p^{(1)} \sim 3 \cdot 10^{47}$ (see formula (1)) is the number of protons which leave the cluster per second, and R_D the distance from the galactic center of a dust grain, then such a grain absorbs the energy

$$\pi a^2 \frac{m_p^{(1)}}{4\pi R_D^2} \Delta E \text{ erg sec}^{-1} \tag{3}$$

which must be reemitted as thermal radiation according to

$$4\pi\, a^2 \varepsilon \sigma T^4 \text{ erg sec}^{-1} \tag{4}$$

(ε is the absorption efficiency of the grain and its value is ~ 1 for very low temperatures).

The spectrum must have a peak at $\nu_m = 3 \cdot 10^{12}$ Hz, which corresponds, according to Wien's law $\frac{kT}{4\nu_m} = 0.201$ to a temperature $T = 30$°K. Equating (3) and (4) we get, with the above value for T, the distance R_D of the dust shell

$$R_D = \left(\frac{n_p^{(1)} \Delta E}{16\pi\varepsilon\sigma T^4}\right)^{1/2} \sim 2.5 \cdot 10^{17} \text{ cm}.$$

This equation is valid only if self-absorption of the infrared radiation is not an important source of heating for the grains. This implies that the maximum luminosity which is radiated at a frequency $\nu_m = 3 \cdot 10^{12}$ Hz corresponds to that emitted by a black-body of radius R_D and temperature $T \sim 30$°K. The total energy radiated is in this way:

$$4\pi R_D^2 \sigma T^4 \simeq 4 \cdot 10^{37} \text{ erg sec}^{-1}$$

The efficiency of this process, i. e. the fraction f of energy carried by the protons which is radiated as infrared is

$$f = \frac{\Delta E}{4\varepsilon E_K} \sim 2 \cdot 5 \cdot 10^{-6}.$$

The total mass of the dust which is necessary in order to have our infrared luminosity is

$$M_D = \frac{n_p^{(1)} \Delta E}{12\varepsilon^2 \sigma T^4} a\varrho \simeq 7 \cdot 10^{30} \text{ gr} \simeq 3 \cdot 5 \cdot 10^{-3} M_\odot.$$

As one can see, the energy emitted ($\sim 4 \cdot 10^{37}$ erg/sec) is far below experimental data ($\sim 10^{40} \div 10^{41}$ erg/sec).

On a purely energetic ground this model cannot be considered as a satisfactory explanation of the emission at a frequency $3 \cdot 10^{12}$ Hz.

7. *Infrared emission: the proton-synchrotron model*

As one can see from the preceding section a dust shell model is not suitable to explain the strong infrared emission at 3×10^{12} Hz from the galactic center. We shall therefore propose another model based on the synchrotron emission by protons in the magnetic field of a neutron star. The basic idea is that neutral hydrogen falling on a magnetic neutron star experiences an electric field induced by the relative motion between the dipole magnetic field and the atom. As the latter moves towards regions of increasing magnetic field, at a certain point the induced electric field becomes so strong to ionize the atom itself. The quantum treatment of such a phenomenon is lacking, because of the breakdown of the time-dependent perturbation theory in very strong magnetic field (as we shall see presently the ionization of hydrogen takes place at $H \sim 5 \cdot 10^9$ Gauss). O'Connel [24] and Kadomsev [25] studied the problem of hydrogen atom in very strong magnetic fields[1], but their results are not applicable in our case, because they consider static magnetic fields, while for our process is essential that the field is variable in time. So we shall limit ourselfs to an estimate of orders of magnitude based on a purely classical theory. On the other hand a rigorous quantum treatment is not likely to change these orders of magnitude: for our purposes a classical calculation is therefore sufficient.

From the induction law

$$\text{rot}\, E = -\frac{\dot{H}}{c}$$

one can compute the work W done by the electric field, in the assumption that the magnetic field is a slowing varying quantity during a time of the order of the rotation period of the electron. This condition is satisfied in our situation. In fact

$$\frac{|\Delta H|}{H} = \frac{3\Delta s}{r} \simeq 10^{-13},$$

where Δs is the distance travelled by the atom during a period of revolution 10^{-16} sec, that is $\Delta s \sim 10^{-5}$ cm and $r \sim 7 \cdot 10^7$ cm is the minimum distance from the neutron star reached by the neutral hydrogen.

So, integrating on one orbit of radius a, which we will assume to be the Bohr radius,

$$W_{\text{orbit}} = \oint e\, E\, dr = e \int (\text{rot}\, E)_n, df \cong e\, (\text{rot}\, E)_n\, \pi r^2$$

[1] We can, however, notice that, according to O'Connel et al. [24], in a strong magnetic field the lowest states become more bounded. This effect may increase a little the ionization energy and for a field strength of 10^9 Gauss the ionization energy may be greater by a factor of 2.8, which does not modify our results.

and the work done in one second is:

$$\frac{e(\text{rot } E)_n \pi r^2}{\tau}$$

where $\tau = 2\pi/\omega$ is the revolution period.

So we have:

$$\frac{W_{\text{orbit}}}{\tau} = \text{rot } E \frac{er^2\omega}{2} = \frac{\dot{H}e\omega r^2}{2c}.$$

Integrating on the time we get for the work done by the electric field

$$W = \int_0^t \frac{W_{\text{orbit}}}{\tau} dt = \int_0^t \frac{e\omega a^2}{2c} \frac{dH}{dt} dt = \frac{e\omega a^2}{2c} \int_0^t dH = \frac{e\omega a^2}{2c} H.$$

Equating this work to the ionization energy $\frac{e^2}{a}$, we obtain the value of the magnetic field at which ionization takes place:

$$H_{\text{ion}} = 2\sqrt{\frac{mc^2}{a^3}} \simeq 5 \cdot 10^9 \text{ Gauss.}$$

This value of the field is reached at a distance

$$R_{\text{ion}} = \left(\frac{\mu}{H}\right)^{1/3} \simeq 7 \cdot 10^7 \text{ cm}$$

The volume swept per second by a disk of such a radius is

$$V_{\text{ion}} = \pi R_{\text{ion}}^2 v \simeq 2 \cdot 10^{26} \text{ cm}^3/\text{sec}$$

Therefore the total number of hydrogen atoms which are ionized per star is given by

$$N_{\text{ion}} = V_{\text{ion}} N_{\text{H}} = 2 \cdot 10^{34} \text{ atoms/sec.}$$

Protons and electrons lose all their kinetic energy ($E_K = 10^{-1}\,mc^2$) by synchrotron emission in 10^{-1} sec and 10^{-10} sec, respectively. The energy emitted in electromagnetic waves per star per second by protons and electrons is

$$E_{\text{protons}} = N_{\text{ion}}\, 10^{-1}\, m_p c^2 \simeq 2 \cdot 10^{30} \text{ erg/sec,}$$

$$E_{\text{electrons}} = N_{\text{ion}}\, 10^{-1}\, m_e c^2 \simeq 2 \cdot 10^{27} \text{ erg/sec}$$

and from the whole cluster we get

$$E_p = 3 \cdot 10^{40} \text{ erg/sec,}$$

$$E_e = 3 \cdot 10^{37} \text{ erg/sec.}$$

These figures are in agreement with Low [4] estimates.

The maximum in the proton synchrotron spectrum is at a frequency

$$\nu_{0p} = \frac{0.3eH}{2\pi m_p c}\left(1-\frac{v^2}{c^2}\right)^{-1} \sim 0.3\,\frac{eH}{2\pi m_p c(1-z)},$$

where z is the gravitational red-shift of the cluster.

The observed frequency is

$$\nu_p = \nu_{0p} - \Delta\nu = \nu_{0p}(1-z) = \frac{0.3eH}{2\pi m_p c} \sim 3 \cdot 10^{12}\ \text{Hz}$$

which is very close to the measured one.

The maximum in the electron synchrotron spectrum is at a frequency

$$\nu_{0e} \sim 5 \cdot 10^{15}\ \text{Hz}$$

These ultraviolet photons can ionize the neutral hydrogen. The photoelectric cross-section at the corresponding energy is

$$\sigma_{ph} = 10^{-18}\ \text{cm}^2$$

The density of such photons inside the sphere of radius equal to the distance travelled by light in one second, that is $3 \cdot 10^{10}$ cm, is $\sim 4 \cdot 10^5$ photons/cm^3. The number of processes is $\sim N_H\,\sigma_{ph}\,4 \cdot 10^5 c$ processes cm^{-3} sec^{-1}, which multiplied by the interaction time $\sim 3 \cdot 10^{10}/v \sim 1$ sec, gives the number density of ionized atoms.

From this calculation one sees that photons are absorbed and therefore the number of ionized atoms near the star is $\mathcal{N} \sim 10^5$ cm^{-3}. Therefore only about 0.1% of neutral hydrogen is ionized in the neighbourhood of a star: such a quantity, leaving substancially unchanged the density of neutral hydrogen, does not disturb the production of infrared radiation.

8. *Radio waves*

The observed emission of radio waves covers a frequency range between 10^2 MHz and 10^4 MHz: the flux density goes from $200 \div 300$ f. u. at $2 \cdot 10^2$ MHz to 10^2 f. u. at $2 \cdot 10^4$ MHz (1 f. u. $= 10^{-26}$ Watt/m$^2 \cdot$ Hz $= 10^{-23}$ erg/cm^{2}sec Hz).

The spectrum is extremely flat with spectral index $\alpha \sim -0.25$ [26]. Other observations fail to detect the source at 80 MHz [27] and at 85 MHz [5].

Let us show now that these characteristics of the radio spectrum can be explained by our model noticing that in the neutron star magnetic field, Van Allen belts are formed. As shown in Section 6, protons are produced as a result of ionization by ultraviolet photons coming from the synchrotron radiation of electrons. Protons produced in this way are trapped in the magnetic field and they therefore give rise to radiation belts around magnetic neutron stars.

An upper bound of the radiation belts is obtained equating the kinetic energy density of the protons to the energy density of the magnetic field.

The kinetic energy density of the protons must be calculated taking into account the number of protons produced by ionization (i. e. 10^5 cm^{-3} sec^{-1}) times their mean life in the field $\sim E_K/(dE_K/dt)$.

As we have

$$\frac{dE_K}{dt} = \dot{E}_K = -\frac{2e^4H^2v^2}{3m_p^2c^5(1-v^2/e^2)} = -\frac{2.3\cdot 10^{-4}}{m_p^2R^6},$$

we get for the total energy density of the protons

$$\mathcal{N}\frac{E_K^2}{\dot{E}_K}\ \mathrm{erg/cm^3},$$

where $E_K = 10^{-1}\ mc^2$. Equating this quantity to the magnetic energy density of the field, we get

$$R_{\mathrm{belt}} = \sqrt[12]{\frac{\mu^2\cdot 2.3\cdot 10^{-7}}{8\pi(m_pc)^4}} \sim 2\cdot 10^9\ \mathrm{cm}.$$

At a distance greater than R_{belt} the protons cannot be trapped and therefore the radio-waves spectrum has a cut-off at low-frequencies given by

$$\nu_{\mathrm{min}} = \frac{0.3eH}{2\pi m_pc} \sim \frac{0.3e\mu}{2\pi m_pcR_{\mathrm{belt}}^3} \sim 10^2\ \mathrm{MHz}.$$

This result is in agreement with experimental data.

The Van Allen belt, which is formed at a distance less than R_{belt} (extended down to $R\sim 7\cdot 10^7$ cm, where we have infrared emission) emits electromagnetic waves of increasing frequency. The energy emitted per second per star by a shell of thickness dr at a distance r from the center of the cluster is

$$dE_{0\nu} = 10^5\frac{E_K}{\dot{E}_K}4\pi r^2dr\dot{E}_K, \qquad (\nu \geqslant \nu_{\mathrm{min}}).$$

The energy produced per second per unit frequency is

$$J_{0\nu}^1 = \frac{dE_{0\nu}}{d\nu} = -10^5E_K4\pi r^2\frac{dr}{d\nu}.$$

From the synchrotron foımula we have

$$r = \left(\frac{0.3e\mu}{2\pi m_pc}\right)^{1/3}\nu^{-1/3}.$$

whence it follows that

$$\frac{dr}{d\nu} = -\tfrac{1}{3}\left(\frac{0.3e\mu}{2\pi m_pc}\right)^{1/3}\nu^{-4/3}$$

and the final result is

$$J^{1}_{0\nu} = 4\pi \cdot 10^{5} E_{K} \frac{0.3e\mu}{2\pi m_{p}c} \nu^{-2} \sim 10^{38}\nu^{-2}, \quad (\nu \geqslant \nu_{\min})$$

which multiplied by the total number of stars gives

$$J^{1}_{0\nu} = 10^{48}\, \nu^{-2}, \quad (\nu \geqslant \nu_{\min}).$$

The total power emitted at the source, without plasma effects which will not be taken into account presently, is

$$E_{0\nu} = 10^{48} \int_{\nu_{\min}}^{\infty} \nu^{-2} d\nu = 10^{48}\nu_{\min}^{-1} \sim 10^{40} \text{ erg/sec}.$$

Let us now evaluate the absorption effect of the plasma. The optical thickness of the source is [17]

$$\tau_{\nu} = 10^{-2} \frac{n_e^2 l}{T^{3/2}\nu^2} \left[17.7 + \ln \frac{T^{3/2}}{\nu} \right], \tag{3}$$

where n_e is the electron density, l is the distance travelled by the wave inside the cluster, T is the kinetic temperature of the gas which is determined by the usual theory of ionization equilibrium with $\delta = 10^{-2}$ as degree of ionization. In fact from the theory of ionization equilibrium we have [28]

$$\frac{1-\delta^2}{\delta^2} = N_{\mathrm{H}} \left(\frac{2\pi}{m_e} \right)^{3/2} \frac{\hbar^3}{(kT)^{3/2}} e^{\frac{I}{KT}},$$

where I is the ionization potential of Hydrogen atom.

The state of the gas in our cluster is such that for a path $l = 10^{17}$ cm (which is the radius of the cluster) the absorption is complete at all frequencies in the range considered here; therefore the observed radiation comes from the stars contained in a very thin shell on the border. The order of magnitude of the thickness of such a shell is given at an optical depth $\tau_{\nu} \simeq 1$, that is, using formula (3).

$$l = 10^{-6}\nu^2$$

in the range $10^2 \div 10^4$ MHz and therefore

$$10^{10} < l < 10^{14} \text{ cm}.$$

The number of stars inside a shell of thickness l is

$$n_s = \frac{4\pi R^2 l}{4\pi R^3/3} N_{\mathrm{M}} = \frac{3l}{R} N_{\mathrm{M}},$$

where N_{M} is the number of magnetic neutron stars.

For the flux density observed on the earth we get, $d = 3 \cdot 10^{22}$ cm being the distance of the galactic center,

$$J_\nu = \frac{3l}{R} N_M \frac{10^{38}\nu^{-2}}{4\pi d^2} = \frac{3 \cdot 10^{32}}{R 4\pi d^2} \sim 300 \cdot 10^{-26}\ \mathrm{Wm^{-2}\,Hz^{-1}} \sim 300\ \mathrm{f.u.}$$

From this calculation we get a flat spectrum and a flux density of the correct order of magnitude. The correct slope can be obtained by a more refined calculation as shown in Dulk [5].

9. X-rays

It should be noticed that accretion of matter on a neutron star can give rise to X radiation. The accretion radius due to the motion of the star through the ambient gas is given approximately by

$$r_{acc} = \frac{2GM}{v^2} = 2 \cdot 10^6 \text{ cm}$$

So the matter which falls on the star is $\sim \pi r_{acc}^2 N_H v \sim 10^{31}$ particles/sec. Therefore the energy given to a star is $\sim 10^{31}\, 10^{-1}\, m_p c^2 \sim 10^{27}$ erg/sec.

We notice however that also the matter trapped in the Van Allen belts is accreted on a region with area $\sim R^3/r_{belt} \sim 5 \cdot 10^8$ cm^2 around the magnetic poles of a star. This matter amounts to $\sim \pi r_{belt}^2 \mathcal{N} v \sim 10^{34}$ protons/sec with an energy $\sim 10^{31}\, 10^{-1}\, mc^2 \sim 10^{30}$ erg/sec ($\mathcal{N}$ is the number density of ions produced by ultraviolet, i. e. $\sim 10^5$ cm^{-3}). The accretion area around the poles acquires a temperature $\sim 10^7$ °K and therefore radiates, as a black-body, X-ray photons with an energy $\sim 10^{-9}$ erg. So the total number of X-photons is 10^{39} sec^{-1}.

With a calculation similar to that followed in the case of ultraviolet photons, and taking into account that the ionization cross section for 10^{-9} erg X-rays is $\sim 10^{-22}$ cm^2, we get a degree of ionization which is less than that produced by ultraviolet.

We therefore reach the conclusion that X-ray emission by accreting matter does not change the average picture.

This X-radiation, having a mean free path $\lambda_X \sim 10^{14}$ cm in a gas with number density 10^8 cm^{-3} is completely absorbed inside the cluster. The total energy given by X-rays to the gas of the cluster is $L_X \sim 10^{40}$ erg/sec. It should be noted however that the fraction of X-rays produced in an outer shell, with a thickness $l_X \sim 10^{14}$ cm can escape from the cluster. Such fraction is

$$\frac{3\lambda_X}{R} L_X \sim 3 \cdot 10^{37} \text{ erg/sec.}$$

This figure is in agreement with recent measurements which have detected an X-ray source in the galactic center with a power $\sim 10^{37}$ erg/sec and practically coinciding with the infrared source of the center of the Galaxy.

The gas in the cluster is heated mainly by the energy transferred to it by the first scattering protons which is $\sim 10^{43}$ erg/sec (see § 3). If thermal equilibrium of the gas in the cluster is reached, then its temperature turns out to be of a few thousand degrees which is in agreement with the choice of a ionization degree $\sim 10^{-2}$ assumed at the beginning.

10. Concluding remarks

Our model can therefore account for the following observed properties of the galactic center:

1) Gravitational waves (if Weber's estimates are too high).
2) Correct number and energy of protons in cosmic rays.
3) Correct flux of γ-rays.
4) Mean features of the radio spectrum.
5) Mean features of the infrared spectrum.
6) X-ray emission.

We can summarize in Fig. 1 the various processes.

Finally we make the following remarks:

a) All the processes involving the acceleration of protons need only an energy negligible compared with that necessary to the production of gravitational waves. In fact the

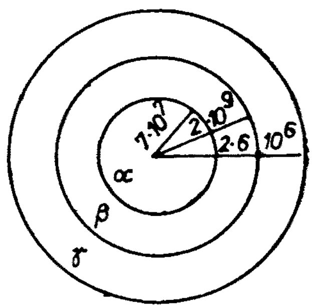

Fig. 1. Pictures of the various processes to which a neutron magnetic star, moving in the interstellar gas gives rise. Radio waves and X-rays come from a very thin shell on the border of the whole cluster, while γ-rays are produced in the whole cluster. α-zone in which ionization takes place giving infrared synchrotron radiation, β-radiation belts (radio waves), γ-outer boundary of the magnetosphere

maximum energy carried out by the protons is of the order of 10^{43} erg/sec, while the flux of gravitational waves is $3 \cdot 10^{46}$ erg/sec.

As to the mass loss, we observe that evaporation of stars is the dominating process ($6\, M_\odot\, \text{yr}^{-1}$ emitted) while cosmic rays carry out a mass of the order of $10^{-2}\, M_\odot\, \text{yr}^{-1}$. One can therefore draw the conclusion that the energy and the mass carried out by the protons does not disturb the stability of the cluster.

b) All the infrared spectra of compact objects so far observed by experimental astrophysicists have a maximum at $4 \cdot 10^{12}$ Hz, independent of the total power emitted. It seems therefore that in the production of such waves a common process is involved which is not linked to the particular structure of the object. Such process could be the ionization of hydrogen in a very strong magnetic field: in fact the peak frequency obtained in this way is independent of the parameters of the model and, on account of the red--shift independence, such frequency is not modified by the particular structure of the cluster.

A dust shell model, on the contrary, besides encountering the above mentioned difficulty with the energy, does not give a general explanation of the occurrence of the same peak frequency occuring in the spectrum of different objects as our galaxy, Seyfert galaxies and Quasars.

c) As to the radio emission, we notice that the proton synchrotron, in radiation belts, can account also for the slope and intensity of the spectrum and predicts also a cut-off at $\sim 10^2$ MHz which is actually observed. The electrons also contribute to the radio emission, but their energy is negligible compared with that emitted by protons, because their kinetic energy is negligible compared with that emitted by protons, because their kinetic energy is smaller by a factor 10^{-3} ($\sim 10^{-1}\, m_e c^2$).

In the end we would like to stress the peculiarity of the process of proton-synchrotron emission which might happen in some other astronomical objects and therefore could be very diffuse in the Universe.

For instance this process may take place in some peculiar binary systems in which one of the stars is a neutron star. In fact, during the accretion phenomenon, it is possible to have hydrogen ionization in the presence of the strong magnetic field of a neutron star and therefore infrared emission.

Obviously it is essential that there is a substancial fraction of neutral hydrogen, otherwise the process is not effective. For this reason, we must not have X-ray production, which would ionize the hydrogen preventing our effect.

We think it possible that under suitable conditions X-ray production does not occur and calculations in this sense are now in progress. Possible candidates for infrared emission by proton-synchrotron radiation seem to be some peculiar red giant and symbiotic stars (composite spectrum stars) as Z and/or AG Pegasus: very likely such systems consist of a binary systems in which one of the component may be a neutron star.

Consequently it would be very interesting to search for infrared radiation at $4 \cdot 10^{12}$ Hz from the above systems.

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