A GLAUBER MODEL CALCULATION OF THE σ (πd $\to \pi\Delta\Delta$) CROSS SECTION FROM THE POINT OF VIEW OF A POSSIBLE $\Delta\Delta$ ADMIXTURE TO THE GROUND STATE DEUTERON WAVE FUNCTION

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The reaction πd $\to \pi\Delta\Delta$ is analysed from the point of view of a $\Delta\Delta$ admixture to the deuteron ground state wave function. A simplified Glauber model approach is used to obtain $\sigma(\pi d \to \pi\Delta\Delta) = 0.3$ μb. This value has been calculated for 21 GeV/c incident pions but it is not sensitive to the primary pion energy.

A priori one should expect that excited nucleons exist inside the atomic nuclei but their contribution to the wave function is at present unknown.

According to our present understanding, the static properties of nuclei are not influenced by internal excitations of nucleons [1]. But it is possible that some high energy reactions can be sensitive to this extra part of the ground-state wave-function of nuclei. We show here that this is the case for the reaction $\pi d \to \pi\Delta\Delta_s$, which may be a good probe of the $\Delta\Delta$ configuration of a deuteron. The recent analysis of this reaction [2], in which the ratio $R = \sigma(md \to m\Delta\Delta_s)/\sigma(md \to m pn)$ was used as an estimate of the $\Delta\Delta$ fraction of the deuteron, has given an upper limit of 0.7% of the d($\Delta\Delta$) configuration. This result seems to be too much pessimistic due to the fact that the $\sigma(md \to m\Delta\Delta_s)$ cross section is strongly reduced by the form factor effects as compared to the $\sigma(md \to m pn)$ cross section. We demonstrate here an important role of the deuteron form factor in calculating a probability of the d($\Delta\Delta$) state.

A conventional reaction $\pi d \to \pi\Delta\Delta$ needs a double inelastic scattering of an incident pion by both nucleons according to the following diagram.

In this paper we assume that the production of two Δ's in a final state interaction (Fig. 2) is negligible due to the fast decrease of the deuteron wave function with momen-

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tum. However, we are aware of the fact that this assumption should be verified by a detailed calculation.

If we accept the possibility of the $\Delta\Delta$ configuration [3] of a deuteron, we can draw still other diagrams, such as those of Fig. 3.

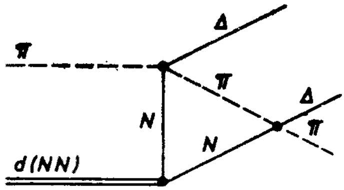

Fig. 1. Diagram corresponding to a double Δ production in πd scattering

These diagrams are identical with those corresponding to the well known break-up reaction of a deuteron, i. e. $\pi d \to \pi np$ [4]. If we can separate the two different mechanisms of $\Delta\Delta$ production from deuteron, we can the nmeasure the probability of the $\Delta\Delta$ configuration.

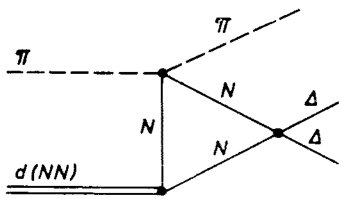

Fig. 2. Diagram for $\Delta\Delta$ production by the final state NN interaction

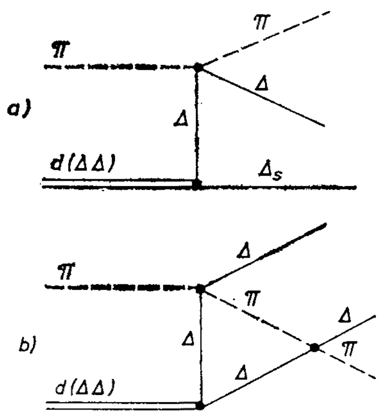

Fig. 3. Break-up reaction diagrams of a deuteron in the $\Delta\Delta$ configuration: a) single scattering term, b) double scattering term

Here we propose to evaluate the diagrams in Figs 1 and 3 with the Glauber model [5], assuming its applicability to strongly bound systems[1]

[1] The binding energy of a deuteron in the $\Delta\Delta$ configuration is about 600 MeV. We believe, however, that when the incident momentum is sufficiently high, the off-mass-shell effects are negligible.

The quasi-elastic character of the $\pi N \to \pi \Delta$ amplitude enables us to treat the diagram of Fig. 1 in the same way as in the case of the break-up reaction (Fig. 3b). Using a Gaussian parametrization of the scattering amplitudes and form factors, we are able to present our final results in the form of simple analytic expressions.

In order to calculate the break-up cross section, it is sufficient (according to the impulse approximation) to keep only a single scattering term (Fig. 3a). The Glauber model gives the following expression for the differential cross section:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{break-up}} = \langle i|\,|F_d(\bar{q}, \bar{s})|^2|i\rangle - |\langle i|F_d(\bar{q}, \bar{s})|i\rangle|^2, \tag{1}$$

where

$$F_d(\bar{q}, \bar{s}) = \frac{iK}{2\pi} d^2\bar{b}e^{i\bar{q}\bar{b}}\Gamma_d(\bar{b}, \bar{s})$$

is an amplitude of the πd scattering for a fixed neutron-proton separation $\bar{s}$ in the plane perpendicular to the incident momentum vector $\vec{K}$. Here $\Gamma_d(\bar{b}, \bar{s})$ is a profile function constructed from elementary πN elastic scattering amplitudes [5].

Using a Gaussian parametrization of the deuteron ground-state wave function

$$\Phi_d(\bar{r}) = (\pi R_d^2)^{-3/4} e^{-r^2/2R^2{}_d}$$

and the same parametrization of the elastic scattering amplitude

$$f(\bar{q}) = \frac{(i+\alpha)\sigma}{4\pi} K e^{-a\bar{q}^2/2},$$

we obtain (1) in an analytic form:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{break-up}} = \frac{|1-i\alpha|^2\sigma^2}{8\pi^2} K^2[1+e^{-\bar{q}^2R_d{}^2/4}-2e^{-\bar{q}^2R_d{}^2/8}]e^{-a\bar{q}^2}. \tag{2}$$

The parameters in (2) have the following meaning: σ — total πN cross section, a — slope parameter, α — ratio of real to imaginary part of elastic πN amplitude, R_d — radius of the deuteron.

As we neglect a possible difference between πN and $\pi \Delta$ elastic scattering, we take the following values [7]; σ = 26 mb, a = 7.5 (GeV/c)$^{-2}$, $\alpha = 0$, corresponding to P_{inc} = 21 GeV/c.

To estimate the R_d parameter in the case of a $\Delta\Delta$ configuration, we use the relation between the binding energy and the mean separation in space, assuming the same potential depth for the πN and $\pi \Delta$ interactions. The resulting value is R_d ($\Delta\Delta$) = 0.33 fm [8].

Fig. 4 shows the differential cross section (2) for two different values of R_d which correspond to a "normal" and an "internaly excited" deuteron. One sees that a decrease of R_d produces a strong drop in the cross section and a broadning of the dip at small momentum transfer. Integrating (2) gives the total break-up cross section [10] for Fig. 3a.

$$\sigma_{\text{break-up}} = \frac{|1-i\alpha|^2}{8\pi a}\sigma^2\left[1+\frac{1}{1+{R_d}^2/4a}-\frac{2}{1+{R_d}^2/8a}\right]. \tag{3}$$

Substituting the quoted numerical values of the parameters, one obtains $\sigma(\pi d\,(\Delta\Delta) \to \pi\Delta\Delta) = 0.03$ mb. If we accept a 1% probability of the $\Delta\Delta$ configuration [1], we arrive at the final value of 0.3 μb.

Fig. 5 shows the σ — R_d relation (3), which reflects the break-up probability-binding energy dependence. In order to estimate the cross section for the double Δ — production

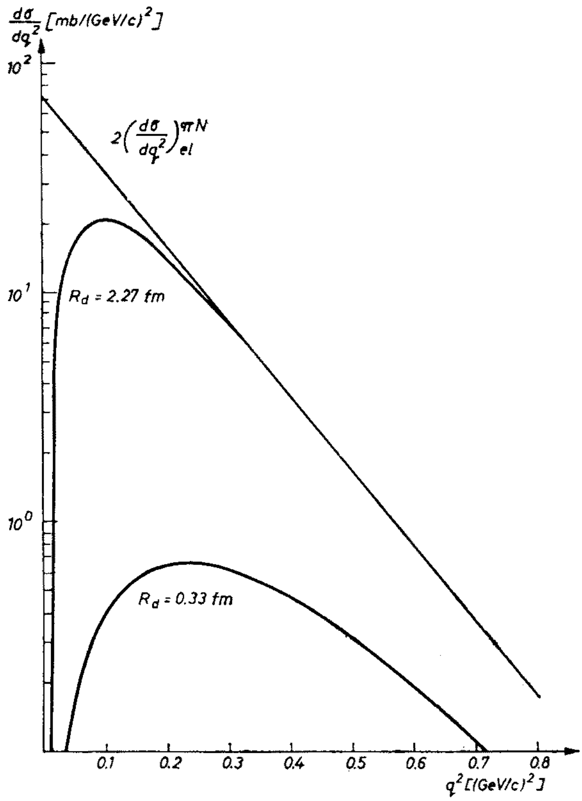

Fig. 4. The differential cross section for the break-up reaction for two different values of R_d: 2.27 fm and 0.33 fm. For comparison, a double elastic πN cross section is also drawn

off a deuteron (Fig. 1), we apply the same formalism, but now we calculate only a double--scattering term. The result is:

$$\sigma(\pi d(\text{NN}) \to \pi\Delta\Delta) = \frac{\sigma^2(\pi\text{N}\to\pi\Delta)}{2\pi a}\left[\frac{1}{1+{R_d}^2/2a}-\frac{1}{(1+{R_d}^2/4a)^2}\right]. \tag{4}$$

Using the values [11]: $\sigma(\pi\text{N} \to \pi\Delta) = 25$μb, $a = 10(\text{GeV}/c)^2$ and a "normal" R_d value (2.27 fm), we obtain $\sigma(\pi d \to \pi\Delta\Delta) = 2\cdot10^{-3}$ μb. We see that the double Δ production of Fig. 1 is negligible in comparison with the previously estimated cross section (Fig. 2a).

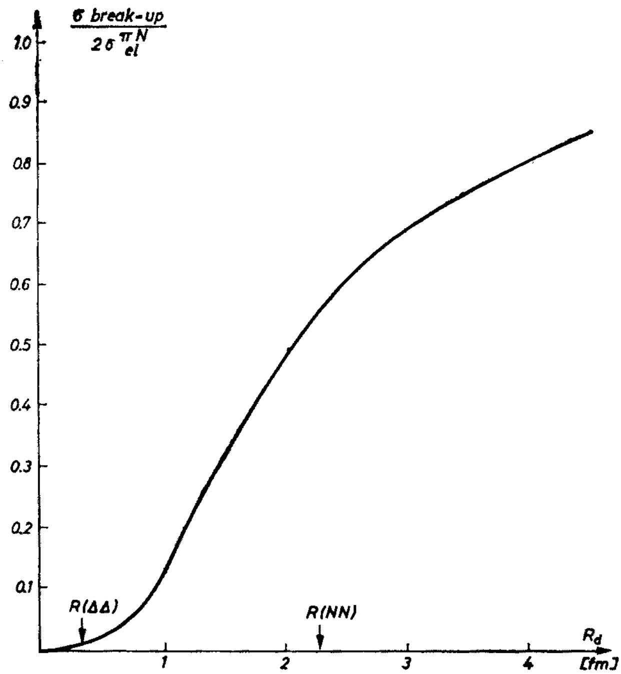

Fig. 5. The total break-up deuteron cross section as a function of the R_d parameter

It means that the precise measurement of the $\pi d \to \pi\Delta\Delta$ cross section should give a $P(\Delta\Delta)$ probability if the $d(\Delta\Delta)$ form factor is well known. A question of whether it is possible to separate the $\pi\Delta\Delta_s$ state from the directly observed $NN\pi\pi\pi$ final state is at the moment an open one. The predicted momentum distribution of a spectator Δ_s [9] and realistic Monte Carlo calculations may help perhaps in estimating the background [12].

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