DIFFRACTIVE DISSOCIATION IN A HADRONIC CLUSTER BREMSSTRAHLUNG MODEL

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Using the uncorrelated cluster emission as a model for non-diffractive processes, the diffractive production is estimated. Multiplicity and leading particle distributions in diffraction dissociation are discussed. The cross-section for diffractive production is calculated and appears consistent with the data.

1. Introduction

In a recent paper [1] the relation between the diffractive and non-diffractive processes was discussed. It was suggested that, using the unitarity condition, the diffractive production can be calculated from amplitudes of non-diffractive production. Assuming the uncorrelated jet model for the production of particles in non-diffractive processes, the explicit formula for amplitudes of diffractive production was found.

In the present paper we continue this study of diffractive dissociation in models of uncorrelated production. Taking into account the results of Ref. [2], we assume that the non-diffractive production is described by a hadronic cluster bremsstrahlung model [3, 4]. As shown in Ref. [3] and [4] this model describes very reasonably the data and, conse-

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quently, seems a good first approximation to reality. Using the formulae of Ref. [1] and [2], we calculated the high-energy limit of diffractive cross-sections and distributions.

The most important result is that the cross-section for diffractive dissociation calculated in the model is of the order of the elastic one and thus comparable to that observed experimentally. This is important because it suggests that the mechanism considered may be indeed of importance in reality.

Out second result concerns the scaling properties of the diffractively excited mass spectrum. We found that, within the approximations of the model, the spectrum scales in the high-energy limit. This is independent of the specific parameters of the model, in particular of the coupling constants and of the average value of transverse momentum.

We discussed also the multiplicity distribution of diffraction dissociation and we found that the cross-section for diffractive production of N clusters can be approximated by the formula

$$\sigma_N = \sigma_{\mathrm{el}} \frac{1}{1+\beta_N w N}, \tag{1.1}$$

where β_N is a constant depending on the shape of the transverse momentum distribution, $\frac{1}{4} \leqslant \beta_N \leqslant \frac{1}{2}$. Furthermore,

$$w = A_{\mathrm{el}} \langle q_\perp^2 \rangle \omega, \tag{1.2}$$

where A_{el} is the slope of the elastic cross-section, $\langle q_\perp^2 \rangle$ is the average value of the transverse momentum squared and ω is given by Eq. (4.10).

The result (1.1) is in marked difference with the predictions of the short-range correlation models which predict σ_N approximately independent of N [5, 6].

Finally, for the multiplicity distribution at fixed value of

$$\xi = \mathcal{M}^2/s \tag{1.3}$$

where $\mathcal{M}$ is the diffractively excited missing mass, we improved the result of Ref. [1] and obtained

$$\frac{d\sigma_N}{d\xi} = \frac{\sigma_{\mathrm{el}} N}{\xi(1+\beta_N w N)} \left[\frac{\ln(\mathcal{M}^2/\bar{\bar{\mu}}^2)}{\ln(s/\bar{\bar{\mu}}^2)} \right]^{N-1}, \tag{1.4}$$

where $\bar{\bar{\mu}}$ is defined by Eq. (A.8).

The paper is organized as follows. In Section 2 we describe the uncorrelated cluster emission model used for non-diffractive processes. In Section 3 the diffractive amplitudes (as derived in Ref. [1]) are given. The diffractive multiplicity distribution is discussed in Section 4 and the leading particle spectrum in Section 5. The results and conclusions are summarized in the last Section. Appendix A is devoted to a presentation of the calculation of the generalized overlap function using the method of de Groot [7]. In Appendix B the formula for diffractive amplitudes is analyzed.

2. Non-diffractive collisions in the Uncorrelated Cluster Emission model

The non-diffractive amplitude is written in the form [1]

$$T_{\mathrm{ND}} = t_{\mathrm{L}} S_{\pi} \delta(P_{\mathrm{in}} - P_{\mathrm{f}}), \tag{2.1}$$

where

$$S_{\pi} = \exp\left\{ i \int \frac{d^3k}{E} \left(\varrho(k) a^{+}(k) + \varrho^{*}(k) a(k) \right) \right\} \tag{2.2}$$

describes the emission and absorption of clusters, and

$$t_{\mathrm{L}} = \int \frac{d^3k_1}{E_1} \frac{d^3k_2}{E_2} \frac{d^3k_3}{E_3} \frac{d^3k_4}{E_4} b^{+}(k_3) b^{+}(k_4) b(k_1) b(k_2) \Psi(k_1, k_3; k_2, k_4) \tag{2.3}$$

describes the scattering of leading particles. Here $a^{+}(k)$ is the creation operator of the cluster and $\varrho(k)$ is the probability amplitude for its creation. $b^{+}(k)$ is the creation operator of a leading particle and $\Psi(k_1, k_3; k_2, k_4)$ is the corresponding scattering amplitude.

The cluster creation probability amplitude $\varrho(k)$ is approximately independent of the longitudinal momenta (to guarantee scaling of the inclusive spectrum). To fix the normalization we introduce the parameter λ defined by the formula

$$\lambda = \int d^2q_{\perp} |\varrho(q_{\perp})|^2. \tag{2.4}$$

In the following it is convenient to introduce the function $f(q_{\perp})$ defined by

$$|\varrho(q_{\perp})|^2 = \lambda f(q_{\perp}); \quad \int d^2q_{\perp} f(q_{\perp}) = 1. \tag{2.5}$$

The matrix elements of the operator (2.1) are given by

$$\begin{aligned} &\langle q_1, \ldots, q_N; k_C, k_D | T_{\mathrm{ND}} | k_A, k_B \rangle \\ &= i^N \varrho(q_1) \ldots \varrho(q_N) \Psi(k_A, k_C; k_B, k_D) \, [\Lambda/\bar{\mu}]^{-\lambda}, \end{aligned} \tag{2.6}$$

where

$$\ln \bar{\mu} = \int d^2q_{\perp} \ln(\mu_{\perp} e^{\gamma}) f(q_{\perp}) \tag{2.7}$$

and Λ is the cut-off parameter[1]. Following the suggestions of Ref. [2] we take Λ = total energy of the system.

The cross-section for non-diffractive production of N particles is

$$\begin{aligned} d\sigma_N = {} & \frac{4\pi^2}{M k_{\mathrm{lab}}} \frac{1}{N!} \delta^4(k_A + k_B - k_C - k_D - q_1 - \ldots - q_N) \\ & |\langle q_1, \ldots, q_N, k_C, k_D | T_{\mathrm{ND}} | k_A, k_B \rangle|^2 \frac{d^3k_C}{E_C} \frac{d^3k_D}{E_D} \frac{d^3q_1}{E_1} \ldots \frac{d^3q_N}{E_N}. \end{aligned} \tag{2.8}$$

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[1] γ is the Euler constant equal 0.5772.

To obtain an approximately independent emission of clusters, the leading particle scattering amplitude $\Psi(k_A, k_C; k_B, k_D)$ should be of the form

$$\Psi(k_A, k_C; k_B, k_D) = \sqrt{E_A^{(+)} E_C^{(+)} E_B^{(-)} E_D^{(-)}}\, \zeta(k_{A\perp}, k_{C\perp}; k_{B\perp}, k_{D\perp}), \tag{2.9}$$

where $E^{(\pm)} = E \pm p_{\parallel}$. Indeed, after integration over the longitudinal momenta of the leading particles C and D we obtain

$$d\sigma_N = \frac{4\pi^2}{Mk_{\text{lab}}} 2 \frac{E_A^{(+)} E_B^{(-)}}{N!} \delta^2(k_{C\perp} + k_{D\perp} + q_{1\perp} + \ldots + q_{N\perp}) d^2k_{C\perp} d^2k_{D\perp}$$

$$|\varrho(q_1)|^2 \ldots |\varrho(q_N)|^2 \frac{d^3q_1}{E_1} \ldots \frac{d^3q_N}{E_N} |\zeta(k_{A\perp}, k_{C\perp}; k_{B\perp}, k_{D\perp})|^2 \tag{2.10}$$

and we see that there are no rapidity correlations between clusters. There remain, however, the correlations between transverse momenta. These are of long-range character and imply that the Pomeranchuk singularity calculated in our model is a cut rather than a pole. This feature of the model has several phenomenological consequences [1].

The leading particle spectrum obtained from the formula (2.8) is [4]

$$\frac{d\sigma^{(\text{ND})}}{dx} = \tfrac{1}{2} \lambda \sigma^{(\text{ND})} (1-x)^{\lambda-1}, \tag{2.11}$$

where

$$\sigma^{(\text{ND})} = \int_{-1}^{1} \frac{d\sigma^{(\text{ND})}}{dx} dx \tag{2.12}$$

is the non-diffractive cross-section.

Thus, to represent approximately the experimentally observed flat spectrum, it is necessary to take [3, 4]

$$\lambda \simeq 1. \tag{2.13}$$

3. *Diffractive amplitudes*

It was shown in Ref. [1] that the amplitude for diffractive production of N clusters is given by the formula

$$\langle q_1, \ldots, q_N; k_C, k_D |A| k_A, k_B \rangle = i^N \varrho(q_1) \ldots \varrho(q_N) F_N\{\Phi\}, \tag{3.1}$$

where

$$F_N\{\Phi\} = \sum_{n=0}^{N} (-1)^n \sum_{\binom{N}{n}\text{combinations}} \Phi(k_A + k_B - q_{l_{n+1}} - \ldots - q_{l_N}). \tag{3.2}$$

and $\Phi(R)$ is the generalized overlap function:

$$\Phi(R;\, k_A, k_C;\, k_B, k_D) = \tfrac{1}{2} \int \frac{d^3k_1}{E_1} \frac{d^3k_2}{E_2}\, \Psi^*(k_1, k_C;\, k_2, k_D)$$

$$\times\, \Psi(k_A, k_1;\, k_B, k_2) \sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{j=1}^{m} \frac{d^3Q_j}{dE_j}\, |\varrho(Q_j)|^2 \delta^4(R-Q_1-\,\dots\,-Q_m-k_1-k_2). \tag{3.3}$$

This generalized overlap function can be estimated in the high-energy limit using methods developed by de Groot [7]. Such an estimate is presented in Appendix A. Assuming a Gaussian cut-off on the momenta of the nucleons, the result is

$$\Phi(R;\, k_A, k_C;\, k_B, k_D) = \mathcal{M}_{\mathrm{el}}(0) \sqrt{\frac{E_C^{(+)}E_D^{(-)}}{E_A^{(+)}E_B^{(-)}}} \exp\left\{-\frac{(k_{C\perp}^2+k_{D\perp}^2)A_{\mathrm{el}}}{4}\right\}$$

$$\times \exp\left\{-\left[R_\perp - \frac{k_{C\perp}+k_{D\perp}}{2}\right]^2 \Big/ \Omega_1^2\right\}, \tag{3.4}$$

where $\mathcal{M}_{\mathrm{el}}(0)$ is the forward elastic amplitude, and

$$\Omega_1^2 = 2A_{\mathrm{el}}^{-1} + \lambda\langle q_\perp^2\rangle \left[\ln\left(\frac{s}{\bar{\mu}^2}\right) - 2\psi(\lambda+1)\right], \tag{3.5}$$

where $\psi(z)$ is the digamma function.

Using formula (3.4) it can be shown [1] that the leading contribution to $F_N\{\Phi\}$ is given by

$$F_N\{\Phi\} = \Phi\{R = k_A+k_B;\, k_A, k_C;\, k_B, k_D\}$$

$$\times \left(\frac{2}{\Omega_1^2}\right)^{N/2} \sum (\vec{q}_{\mu_1\perp} \cdot \vec{q}_{\mu_2\perp}) \dots (\vec{q}_{\mu_{N-1}\perp} \cdot \vec{q}_{\mu_N\perp}), \tag{3.6}$$

where the summation extends over all possible pairs of clusters, each of the $(N-1)!!$ combinations taken only once. Note that formula (3.6) makes sense only for even N. The contributions for odd N are zero in the leading order. Thus this model predicts that the diffractively produced clusters must occur in pairs.

Eq. (3.6) implies that the differential cross-section for diffractive production of N clusters is given by the formula

$$d\sigma_N = \frac{4\pi^2}{Mk_{\mathrm{lab}}} |\Phi(R = k_A+k_B;\, k_A, k_C;\, k_B, k_D)|^2 \frac{1}{N!} \left(\frac{2}{\Omega_1^2}\right)^N$$

$$\times Z(q_1, \dots, q_N)\delta^4(k_A+k_B-k_C-k_D-q_1-\,\dots\,-q_N)$$

$$\times \frac{d^3k_C}{E_C} \frac{d^3k_D}{E_D} \sum_{i=1}^{N} \frac{d^3q_i}{E_i}, \tag{3.7}$$

where

$$Z(q_1, \ldots, q_N) = |\sum (\vec{q}_{\mu_1\perp} \cdot \vec{q}_{\mu_2\perp}) \ldots \vec{q}(_{\mu_{N-1}\perp} \cdot \vec{q}_{\mu_N\perp})|^2$$

$$= \sum (\vec{q}_{\mu_1\perp} \cdot \vec{q}_{\mu_2\perp}) \ldots (\vec{q}_{\mu_{2N-1}\perp} \cdot \vec{q}_{\mu_{2N}\perp}). \tag{3.8}$$

Here again the second sum extends over all combinations of the pairs of particles (each particle occurring twice).

4. Multiplicity distribution of diffraction dissociation

Using Eq. (3.7) we arrive at the following formula for the asymptotic value of the cross-section for diffractive production of N clusters:

$$\sigma_N = \frac{A_{el}\sigma_{el}}{2\pi}\frac{1}{N!}\int \frac{d^3k_C}{E_C}\frac{d^3k_D}{E_D}\frac{d^3q_1}{E_1}\ldots\frac{d^3q_N}{E_N}\left(\frac{2\lambda}{\Omega_1^2}\right)^N$$

$$\times \delta^4(P-k_C-k_D-q_1- \ldots -q_N)\exp\left[-(k_{C\perp}^2+k_{D\perp}^2)A_{el}/2\right]$$

$$\times E_C^{(+)}E_D^{(-)}f(q_1)\ldots f(q_N)Z(q_1, \ldots, q_N). \tag{4.1}$$

As already mentioned, this formula is valid for even N. For odd N the diffractive cross-section vanishes in the high-energy limit.

To estimate σ_N we use again the method of de Groot, writing

$$\sigma_N = \sigma_{N\parallel} \cdot \sigma_{N\perp}, \tag{4.2}$$

where

$$\sigma_{N\parallel} = \frac{A_{el}\sigma_{el}}{2\pi}\int dy_C dy_D E_C^{(+)}E_D^{(-)}dy_1 \ldots dy_N \delta^{(2)}(P-k_C-k_D-q_1- \ldots -q_N) \tag{4.3}$$

and

$$\sigma_{N\perp} = \frac{1}{N!}\left(\frac{2\lambda}{\Omega_1^2}\right)^N \int d^2k_{C\perp}d^2k_{D\perp}d^2q_{1\perp} \ldots d^2q_{N\perp}\delta^{(2)}(k_{C\perp}+k_{D\perp}+q_{1\perp}+ \ldots +q_{N\perp})$$

$$\times \exp\left[-(k_{C\perp}^2+k_{D\perp}^2)A_{el}/2\right]f(q_1)\ldots f(q_N)Z(q_1, \ldots, q_N). \tag{4.4}$$

$\sigma_{N\parallel}$ can be calculated either directly, or by de Groot's expansion. The result is

$$\sigma_{N\parallel} = \frac{A_{el}\sigma_{el}}{\pi}\left\{\frac{d^N}{dz^N}\left[\frac{\exp\left[z\log(s/\bar{\mu}^2)\right]}{[\Gamma(z+1)]^2}\right]\right\}_{z=0}. \tag{4.5}$$

For N fixed and $s \to \infty$ this gives

$$\sigma_{N\parallel} = \frac{A_{el}\sigma_{el}}{\pi}\left[\ln(s/\bar{\mu}^2)\right]^N. \tag{4.6}$$

To calculate $\sigma_{N\perp}$ we replace the transverse $\delta^{(2)}$ function by its Fourier representation and perform the integration over the transverse momenta of the nucleons. The result is

$$\sigma_{N\perp} = \frac{1}{N!}\left(\frac{2\lambda}{\Omega_1^2}\right)^N A_{\rm el}^{-2} \int d^2 b Z_N(b) e^{-b^2/A_{\rm el}}, \tag{4.7}$$

where

$$Z_N(b) = \int Z(q_1, \ldots, q_N) \prod_{i=1}^{N} [f(q_i) e^{-\vec{b}\cdot\vec{q}_{i\perp}} d^2 q_{i\perp}]. \tag{4.8}$$

$Z_N(b)$ is the sum of $[(N-1)!!]^2$ terms corresponding to different products $\vec{q}_{l\perp}\cdot\vec{q}_{m\perp}$. In Appendix B we show that $Z_N(b)$ can be approximated by

$$Z_N(b) = N!\left(\frac{\langle q_\perp^2\rangle}{2}\right)^N \exp\{-b^2 N\omega\langle q_\perp^2\rangle\beta_N\}, \tag{4.9}$$

where

$$\omega = \frac{\langle q_\perp^4\rangle}{2\langle q_\perp^2\rangle^2} \tag{4.10}$$

and β_N is a constant satisfying the condition

$$\tfrac{1}{4} \leqslant \beta_N \leqslant \tfrac{1}{2}. \tag{4.11}$$

Substituting Eq. (4.9) into (4.7) we obtain

$$\sigma_{N\perp} = \frac{\pi}{A_{\rm el}}\left(\frac{\lambda\langle q_\perp^2\rangle}{\Omega_1^2}\right)^N \frac{1}{1+\beta_N w N}. \tag{4.12}$$

Thus we have finally in the high-energy limit

$$\sigma_N = \sigma_{\rm el}\left[\frac{\ln(s/\bar{\mu}^2)\langle q_\perp^2\rangle\lambda}{\Omega_1^2}\right]^N \frac{1}{1+\beta_N w N} \to \frac{\sigma_{\rm el}}{1+\beta_N w N} \tag{4.13}$$

for N even, and zero for N odd.

5. *Leading particle spectrum in diffraction dissociation*

To obtain the leading particle spectrum we have to integrate the differential cross--section for diffractive production (given by Eq. (4.1)) over the cluster momenta and the momentum of one of the leading particles. Using Eq. (2.8) we obtain

$$\frac{d\sigma_N}{dx_C d^2 k_C^\perp} = \frac{A_{\rm el}\sigma_{\rm el}}{2\pi} e^{-k_{C\perp}^2 A_{\rm el}/2} \frac{1}{N!}\left(\frac{2\lambda}{\Omega_1^2}\right)^N \int e^{-k_{D\perp}^2 A_{\rm el}/2}$$

$$\times E_D^{(-)} Z(q_1, \ldots, q_N)\delta^4(P-k_C-k_D-q_1-\ldots-q_N)\frac{d^3 k_D}{E_D}\prod_{i=1}^{N}\left[f(q_i)\frac{d^3 q_i}{E_i}\right] \tag{5.1}$$

and it is clear that the estimates of $d\sigma_N/dx$ and inclusive cross-section

$$\frac{d\sigma}{dx} = \sum_N \frac{d\sigma_N}{dx} \tag{5.2}$$

can be performed quite similarly to those presented in Section 4. We will thus not repeat these calculations but only quote the results.

The inclusive leading particle spectrum is

$$\frac{d\sigma}{dx_C d^2 k_{C\perp}} = \frac{\sigma_{el} e^{-k_{C\perp}^2 A_{el}/2}}{\pi\omega} \cdot \frac{1}{(1-x_C)\ln(1-x_C)}. \tag{5.3}$$

The multiplicity distribution integrated over the transverse momenta is

$$\frac{d\sigma_N}{dx_C} = \frac{\sigma_{el}}{\ln(s/\bar{\mu}^2)} \left[\frac{\ln(\mathscr{M}^2/\bar{\mu}^2)}{\ln(s/\bar{\mu}^2)}\right]^{N-1} \frac{N}{1+w\beta_N N} \frac{1}{1-x_C}. \tag{5.4}$$

The average multiplicity for given $\xi = \mathscr{M}^2/s = 1-x_c$

$$\bar{n}(\xi, s) = -\frac{\ln(s/\bar{\mu}^2)}{\ln \xi}. \tag{5.5}$$

6. *Discussion and comments*

The results described in the previous sections show that the Uncorrelated Cluster Emission model remains an attractive possibility for the description of not only non--diffractive, but also diffractive processes. Two points seem to us of particular importance.

A. The magnitude of the diffractive cross-section comes out rather close to the experimental value. We find this result non-trivial, particularly in view of the results of Ref. [2], where it was shown that in the uncorrelated pion production model (without clusters and with the cut-off parameter defined as in Ref. [1]) the diffractive cross-section is too large by more than one order of magnitude.

B. One obtains naturally the scaling of the diffractively excited mass spectrum. It should be emphasized that this scaling property is not sensitive to the parameters of the model. In particular, it does not depend on the value of the coupling constant λ.

Other results of interest are

a) The multiplicity distribution follows the rule given by Eq. (4.13) which can be approximated by

$$\sigma_N = \sigma_{el}/(1+N) \tag{6.1}$$

at present energies. This result differs from the one obtained in Regge-pole models [5, 6]. At asymptotic energies, the behaviour of σ_N depends on the behaviour of w, that is, on the shrinkage of the diffractive peak and on the transverse momentum cut-off in non--diffractive collisions.

b) The shape of the leading particle spectrum in diffractive collisions is given by Eq. (5.3), which shows two characteristic properties: (*i*) The missing mass distribution follows the formula derived in Ref. [1]:

$$\frac{d\sigma}{d\xi} \propto (\xi \ln \xi)^{-1}. \tag{6.2}$$

(*ii*) The slope of the inclusive transverse momentum distribution of the leading particles is predicted to be half that observed in elastic scattering. Both these properties are similar to those expected in Regge theory [5, 6].

c) The energy dependence of diffraction dissociation is governed by the behaviour of the parameter

$$w = A_{\text{el}} \langle q_\perp^2 \rangle \omega = A_{\text{el}} \frac{\langle q_\perp^4 \rangle}{2\langle q_\perp^2 \rangle}. \tag{6.3}$$

The asymptotic behaviour of the ratio of diffractive to elastic cross-section is

$$\frac{\sigma_{\text{diffr}}}{\sigma_{\text{el}}} \sim \frac{\ln \ln s}{\omega}. \tag{6.4}$$

This formula follows from Eq. (5.3).

d) The multiplicity distribution at fixed $\mathcal{M}$ is given by (cf. Ref. [1]):

$$\frac{d\sigma_N}{d\xi} = \frac{\sigma_{\text{el}}}{\xi \ln (s/\bar{\bar{\mu}}^2)} \left[\frac{\ln(\mathcal{M}^2/\bar{\bar{\mu}}^2)}{\ln (s/\bar{\bar{\mu}}^2)} \right]^{N-1} \frac{N}{1+\beta w N}. \tag{6.5}$$

As already noted in Ref. [1], this result differs in an important way from that of Regge models: The different energy dependence of cross-sections for different N is a reflection of the cut nature of the Pomeranchuk singularity in our model.

e) A peculiar feature of the model is that, at asymptotic energies, only an even number of clusters can be produced diffractively. The cross-section for production of an odd number of clusters vanishes like $(\ln s)^{-1}$. It would be amusing if this result were confirmed experimentally.

We would like to close this paper with several comments.

(*i*) It should be realized that our calculations are only semiquantitative. It may be worthwhile to undertake a more detailed analysis and comparison with experimental data. This would, however, require better formulation of the cluster emission model, and determination of cluster properties.

(*ii*) A particularly important feature of the model is that it provides a possibility to calculate the absolute value of the ratio of the diffractive to the elastic cross-sections. We would like to emphasize that this is not possible in most approaches to diffraction dissociation [8].

(*iii*) Our results differ in several points from the standard predictions of Regge models, [5, 6]. This is not surprizing because, as discussed already in Ref. [1], the Pomeranchuk singularity in our model is a cut rather than a pole. It is important to keep in mind these differences as they may provide experimental possibilities of discrimination between the underlying ideas. The diffractive multiplicity distribution is perhaps the best candidate in this respect[2].

(iv) It should be stressed that our calculation gives information on the scaling part of the diffractively excited missing mass spectrum only. In the particular version we consider the nonscaling part vanishes in the high-energy limit. However, it may be easily added by introduction into the model the possibility of nucleon excitations (leading clusters [3, 4]).

To summarize, we feel that our asymptotic estimates indicate that the uncorrelated cluster emission model may be a serious candidate for the correct description of not only non-diffractive [3, 4] but also diffractive processes.

We would like to thank J. Benecke, P. Breitenlohner and E. de Groot for helpful comments.

APPENDIX A

Generalized Overlap Function in the high-energy limit[3]

We use the method developed by de Groot [7], i.e. we introduce the Laplace transform of the δ function in Eq. (3.3), and estimate the resulting integral.

The first step gives:

$$\Phi(R) = \frac{\sqrt{E_A^{(+)}E_C^{(+)}E_B^{(-)}E_D^{(-)}}}{2} \frac{(\bar{R}^2/\bar{\mu}^2)^{-\lambda}}{(2\pi)^4} \int d^4x \exp\{Rx\}$$

$$\times \sum_{N/0}^{\infty} \frac{\lambda^N}{N!} \int \frac{d^3k_1}{k_{01}} \frac{d^3k_2}{k_{02}} k_{01}^{(+)}k_{02}^{(-)} \frac{d^3q_1}{E_1} \cdots \frac{d^3q_N}{E_N} e^{-(q_1+\ldots+q_N+k_1+k_2)x}$$

$$\times \zeta(k_{A\perp}, k_{1\perp}; k_{B\perp}, k_{2\perp})\zeta^*(k_{C\perp}, k_{1\perp}; k_{D\perp}, k_{2\perp}) f(q_1)\ldots f(q_N), \tag{A.1}$$

where, according to suggestions of Ref. [2], we have taken $\Lambda = \bar{R} = \sqrt{R_0^2 - R_\parallel^2}$. The integration over x is performed along the straight line parallel to the imaginary axis. Using the identities

$$\int_{-\infty}^{\infty} dy e^{-(Ex_0 - p_\parallel x_\parallel)} = 2K_0(m_\perp \bar{x}),$$

$$\int_{-\infty}^{\infty} dy e^{-(Ex^0 - p_\parallel x_\parallel)} E^{(\pm)} = 2m_\perp \frac{x^{(\mp)}}{\bar{x}} K_1(m_\perp \bar{x}), \tag{A.2}$$

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[2] For example, formula (4.13) implies that the KNO scaling should be violated at high energies in the region of small multiplicities [9].

[3] The results presented here were obtained in collaboration with J. Benecke and E. H. de Groot.

where

$$\bar{x}^2 = x_0^2 - x_{\parallel}^2 = x^{(+)}x^{(-)}$$

we can rewrite Eq. (A.1) in the form

$$\Phi(R) = 2\sqrt{E_A^{(+)}E_C^{(+)}E_B^{(-)}E_D^{(-)}}\,\frac{(\bar{R}^2/\bar{\mu}^2)^{-\lambda}}{(2\pi)^2}\int d^2b \exp\{i(\vec{R}_\perp - \vec{k}_{1\perp} - \vec{k}_{2\perp})\vec{b}\}$$

$$\times \int d^2k_{1\perp} d^2k_{2\perp} \zeta(k_{A\perp}, k_{1\perp}; k_{B\perp}, k_{2\perp})\zeta^*(k_{C\perp}, k_{1\perp}; k_{D\perp}, k_{2\perp})\left(\frac{1}{2\pi i}\right)^2 \int dx_0 dx_{\parallel}$$

$$\times K_1(m_{1\perp}\bar{x})K_1(m_{2\perp}\bar{x}) \exp[2\lambda \textstyle\int d^2q_\perp K_0(\mu_\perp \bar{x})e^{-i\vec{q}_\perp \vec{b}}f(q_\perp)]. \qquad (A.3)$$

Since the main contributions to the integral are from the region $\bar{x} \simeq 0$, we approximate the Bessel functions as $K_0(z) = -\ln\left(\frac{ze^\gamma}{2}\right)$ and $K_1(z) \simeq \frac{1}{z}$. Substituting these formulae into Eq. (A.3), and applying the identity

$$\frac{1}{2\pi i}\int \frac{dz}{z^\alpha} e^{qz} = \frac{q^{\alpha-1}}{\Gamma(\alpha)}\,\theta(q) \qquad (A.4)$$

we obtain

$$\Phi(R) = \sqrt{E_A^{(+)}E_C^{(+)}E_B^{(-)}E_D^{(-)}}\,\frac{(\bar{R}^2/\bar{\mu}^2)^{-\lambda}}{(2\pi)^2}\int d^2b \exp\{i(\vec{R}_\perp - \vec{k}_{1\perp} - \vec{k}_{2\perp})^2\vec{b}\}$$

$$\times \textstyle\int d^2k_{1\perp} d^2k_{2\perp} \zeta(k_{A\perp}, k_{1\perp}; k_{L\perp}, k_{2\perp})\zeta^*(k_{C\perp}, k_{1\perp}; k_{D\perp}, k_{2\perp})$$

$$\times \left[\frac{R^{\lambda(b)}}{\Gamma(\lambda(b)+1)}\right]^2 \theta(R^{(+)})\theta(R^{(-)})$$

$$\times \exp\left\{-2\lambda \int d^2q_\perp \ln\left(\frac{\mu_\perp e^\gamma}{2}\right) e^{-i\vec{q}_\perp \vec{b}}f(q_\perp)\right\}, \qquad (A.5)$$

where

$$\lambda(b) = \textstyle\int e^{i\vec{q}\vec{b}}|\varrho(q_\perp)|^2.$$

The final step is to perform the integration over $\vec{b}$ using the saddle-point method. Using

$$\lambda(b) \simeq \lambda(1 - b^2\langle q_\perp^2\rangle/4) \qquad (A.6)$$

we obtain

$$\int d^2q_\perp \ln\left(\frac{\mu_\perp e^\gamma}{2}\right) e^{-i\vec{q}_\perp \vec{b}}|\varrho(q_\perp)|^2 = \lambda\left[\ln(\bar{\mu}/2) - \frac{\langle q_\perp^2\rangle}{4} b^2 \ln(\bar{\mu}/2)\right],$$

where

$$\langle q_\perp^2 \rangle = \int f(q_\perp) q_\perp^2 \, d^2 q_\perp, \tag{A.7}$$

$$\langle q_\perp^2 \rangle \ln \bar{\mu} = \int d^2 q_\perp \ln(\mu_\perp e^\gamma) q_\perp^2 f(q_\perp) \tag{A.8}$$

and from this the formula

$$\Phi(R) = \sqrt{E_A^{(+)} E_C^{(+)} E_B^{(-)} E_D^{(-)}} \frac{1}{\pi[\Gamma(\lambda+1)]^2} \int d^2 k_{1\perp} d^2 k_{2\perp} \zeta(k_{A\perp}, k_{1\perp}; k_{B\perp}, k_{2\perp})$$

$$\times \zeta^*(k_{C\perp}, k_{1\perp}; k_{D\perp}, k_{2\perp}) \exp\{-(R_\perp - k_{1\perp} - k_{2\perp})^2/\Omega_0^2\}/\Omega_0^2, \tag{A.9}$$

where

$$\Omega_0^2 = \lambda \langle q_\perp^2 \rangle \left[\ln\left(\frac{R}{\bar{\mu}}\right)^2 - 2\psi(\lambda+1) \right] \tag{A.10}$$

follows.

Finally, let us note that this formula can be further simplified for gaussian nucleon wave functions ζ. Assuming

$$\zeta(k_{A\perp}, k_{1\perp}; k_{B\perp}, k_{2\perp}) = \zeta_0 \exp[-(k_{A\perp} - k_{1\perp})^2 A_{\text{el}}/2] \exp[-(k_{B\perp} - k_{2\perp})^2 A_{\text{el}}/2] \tag{A.11}$$

we obtain

$$\Phi(R) = \mathcal{M}_{\text{el}}(0) \exp[-(k_{C\perp}^2 + k_{D\perp}^2) A_{\text{el}}/4] \sqrt{\frac{E_C^{(+)} E_D^{(-)}}{E_A^{(+)} E_B^{(-)}}} \exp\left[-\left(R_\perp - \frac{k_{C\perp} + k_{D\perp}}{2}\right)^2 \Big/ \Omega_1^2 \right], \tag{A.12}$$

where

$$\Omega_1^2 = \Omega_0^2 + 2A_{\text{el}}^{-1} \tag{A.13}$$

and $\mathcal{M}_{\text{el}}(0)$ is the forward elastic amplitude.

APPENDIX B

Approximate evaluation of the function $Z_N(b)$ for N even

From formulae (3.8) and (4.8)

$$Z_N(b) = \int |\sum (\vec{q}_{\mu_1} \cdot \vec{q}_{\mu_2}) \ldots (\vec{q}_{\mu_{N-1}} \cdot \vec{q}_{\mu_N})|^2 \prod_{i=1}^{N} f(q_i) e^{-i\vec{b}\vec{q}_i} d^2 q_i \tag{B.1}$$

where the summation extends over all the $(N-1)!!$ ways of arranging the N clusters into $N/2$ pairs, and $\vec{q}_{\mu_i}$ is the transverse momentum of the μ_i-th cluster.

According to (B.1) $Z_N(b)$ is a sum of $[(N-1!!)]^2$ terms. There is a one to one correspondence between these terms and diagrams defined as follows:

a) draw N points labelled $1, 2, \ldots, N$,

b) draw N red lines and N blue lines so that every line connects two points and every point is attached to exactly one red line and exactly one blue line. The correspondence between the diagrams and the contributions is made by relating each red (blue) line connecting points i and j with a factor $\vec{q}_i \cdot \vec{q}_j$ in F (F^*). Consequently the contribution of a graph to the integrand in (B.1) is the product of contributions corresponding to all the vertices and all the lines. For vertex i the contribution is $q_i^2 f(q_i) e^{-i\vec{b}\vec{q}_i}$ and for a line joining vertices i and j:

$$\cos \theta_{ij} = \cos \theta_i \cos \theta_j + \sin \theta_i \sin \theta_j. \tag{B.2}$$

Formula (B.2) implies that a single diagram with N lines contributes 2^N terms. Fortunately choosing the $\theta = 0$ axis along $\vec{b}$ one sees easily that only two terms survive the integrations over θ_i, and

$$Z_N(b) = \sum_{\text{diagrams}} \prod_{\text{loops}} [\langle \cos^2 \theta q^2 e^{i\vec{b}\vec{q}} \rangle^{n_l} + \langle \sin^2 \theta q^2 e^{i\vec{b}\vec{q}} \rangle^{n_l}]. \tag{B.3}$$

Here each product extends over all the disconnected loops forming a diagram, n_l denotes the number of vertices in the loop (thus $\sum n_l = N$) and the notation

$$\langle g(\vec{q}) \rangle = \int f(\vec{q}) g(\vec{q}) d^2 q \tag{B.4}$$

is used.

In order to simplify (B.3), we use the identity

$$\sum_{\text{diagrams}} \prod_{\text{loops}} 2 = N!. \tag{B.5}$$

This is easily proved by induction. For $N = 2$ it holds. Assuming that it holds for all even $M < N$, we prove it for N. Consider the contribution from graphs where vertex 1 is contained in an k-particle loop (k must be even according to the rules of constructing the diagrams). It is a product of

a) the number of ways of choosing the other $k-1$ particles in the loop, i.e. $\binom{N-1}{k-1}$,

b) the number of possible orderings of the particles in the loop, i.e. $(k-1)!$,

c) the contribution of the remaining loops equal according to the inductive assumption $(N-k)!$,

d) a factor 2 for the loop containing particle 1. The full contribution is $2(N-1)!$ independent of k. Since k can take any even value from 2 to N, (B.5) follows.

Since (B.3) is not of the form (B. 5), we only calculate the lower and upper bounds

$$2c_-^{n_l} \leqslant \langle \cos^2 \theta q^2 e^{-i\vec{b}\vec{q}} \rangle^{n_l} + \langle \sin^2 \theta q^2 e^{-i\vec{b}\vec{q}} \rangle^{n_l} \leqslant 2c_+^{n_l} \tag{B.6}$$

where

$$c_- = \tfrac{1}{2} \langle q^2 e^{i\vec{b}\vec{q}} \rangle \simeq \frac{\langle q_\perp^2 \rangle}{2} \exp\left[- \frac{\langle q_\perp^4 \rangle}{4 \langle q_\perp^2 \rangle} b^2 \right], \tag{B.7}$$

$$c_+ = \langle q^2 \sin^2 \theta e^{i\vec{b}\vec{q}} \rangle \simeq \frac{\langle q_\perp^2 \rangle}{2} \exp\left[- \frac{\langle q_\perp^4 \rangle}{8 \langle q_\perp^2 \rangle} b^2 \right]. \tag{B.8}$$

Substituting into (B.3), factoring $(c_\pm)^N$ out and using (B.5), we obtain two inequalities implying (4.9)–(4.11).

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