pd AND π^+d COLLISIONS AT 100 GeV/c*

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From an exposure of the 30-inch deuterium bubble chamber at Fermilab we examine 7600 events with $\geqslant 3$ charged prongs. Multiplicity distributions for π^+n, pn, and pd collisions are presented and are in general agreement with those expected based on knowledge of π^-p and pp collisions at the same energy. The pd distribution is slightly wider than expected from a combination of free pp and pn collisions, and we estimate from this that the fraction of double inelastic collisions is about 5%. From the fraction of events with spectator protons we find that in about 15% of the inelastic break-up collisions both nucleons participate. We find no significant N-dependence in the double interaction effects so that the odd-prong multiplicities we present should correspond closely to free pn and π^+n collisions. An interpretation of these results is suggested.

I would like to give some preliminary results from an exposure of a 100 GeV/c mixed π^+, p beam in the 30-inch deuterium chamber at FNAL.

The identification of the individual beam particles as π^+ or p was accomplished by a Cerenkov counter along with a proportional wire tagging system built by the PWHBCS consortium. The beam composition was 56% proton, 41% π^+, 2% μ^+, and 1% K^+.

We will be primarily interested in the odd-prong events in which the spectator protons remains invisible in the bubble chamber. This constitutes 29% of the events and this sample will closely approximate free pn and π^+n interactions.

The odd-prong probability distributions for $N \geqslant 3$ (N is the number of charged prongs) are given in Tables I and II for pn and π^+n collisions, respectively. The even-prong sample (Table III) is defined as the sum of the observed even-prongs plus the odd prongs with one prong (the unobserved spectator proton) added to the prong count. We have made the usual small corrections to the data for Dalitz pairs, scan biases, un-

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TABLE I

Comparison of odd-prong pn data with predictions

N	Events	P_N^{pn} (observed)	P_{N+1}^{pp}	P_N^{pn} (predicted, $X=.5$)	P_N^{pn} (predicted, $X=.6$)	S_N $X=.5$
3	304	$.203\pm.012$	$.242\pm.01$	$.205\pm.02$	.220	$.99\pm.10$
5	396	$.264\pm.013$	$.236\pm.01$	$.257\pm.006$	.260	$1.02\pm.06$
7	326	$.217\pm.012$	$.177\pm.005$	$.222\pm.006$	.219	$.98\pm.06$
9	241	$.161\pm.010$	$.118\pm.005$	$.159\pm.005$	.155	$1.01\pm.08$
11	119	$.079\pm.007$	$.056\pm.003$	$.093\pm.003$	.088	$.87\pm.09$
13	73	$.049\pm.006$	$.0214\pm.002$	$.042\pm.003$	.038	$1.17\pm.15$
15	29	$.019\pm.004$	$.0078\pm.001$	$.0157\pm.0015$	.0144	$1.21\pm.20$
17	9	$.006\pm.002$	$.0016\pm.0006$	$.0050\pm.002$	.0044	$1.20\pm.50$
19	1	$.0007\pm.0007$	$.0006\pm.0003$	$.001\pm.0005$	.0011	—
Total	1498	1.00	1.00	1.00	1.00	

TABLE II

Observed and predicted odd-prong π^+n data

N	Events	P_N(observed)	P_N(predicted) $X = .5$	$S_N(X = .5)$
3	140	$.203\pm.017$	$.170\pm.008$	$1.18\pm.12$
5	164	$.238\pm.019$	$.250\pm.004$	$.95\pm.09$
7	155	$.255\pm.018$	$.237\pm.004$	$.94\pm.09$
9	99	$.144\pm.015$	$.171\pm.004$	$.85\pm.09$
11	73	$.106\pm.012$	$.098\pm.003$	$1.08\pm.12$
13	36	$.052\pm.009$	$.051\pm.002$	$1.03\pm.16$
15	17	$.025\pm.006$	$.019\pm.001$	$1.30\pm.25$
17	3	$.004\pm.002$	$.0046\pm.0008$	$.90\pm.45$
19	1	$.0015\pm.0015$	$.0015\pm.0005$	—
Total	688	1.00	1.00	

certain prong counts, etc. The total cross sections for producing $\geqslant 3$ prongs are (54.0 ± 1.5) mb and (32.0 ± 1.5) mb for pd and π^+d, respectively.

As we will show, we believe the data in Tables I and II very nearly approximate what would be obtained in free pn and π^+ n collisions at 100 GeV/c.

We have examined the data given in the Tables for evidence of events in which both nucleons in the deuteron participate. The effects of such double interactions might show up in the following two ways:

1) The odd-prong pn events might be relatively depleted in the higher prong numbers, because the larger the number of particles produced off the neutron the greater might be the probability of a secondary elastic or inelastic interaction which knocks out the spectator proton. This effect could only be detected if it is N-dependent.

TABLE III

Comparison of even-prong pd data with predictions

N	Events	P_N^d(observed)	$P_N^d(R=0)$ $X=.5$	$P_N^d(R=1)$	M_N(obs)
4	1243	.235 ±.007	.242 ±.005	.022	.97±.04
6	1377	.260 ±.007	.265 ±.005	.077	.98±.04
8	1150	.217 ±.006	.214 ±.004	.140	1.01±.04
10	754	.142 ±.005	.148 ±.005	.180	.96±.05
12	415	.078 ±.004	.080 ±.003	.182	.98±.08
14	211	.040 ±.003	.0336±.0026	.153	1.19±.13
16	118	.022 ±.002	.0125±.0012	.110	1.76±.30
18	24	.005 ±.001	.0035±.0007	.068	1.43±.35
20	4	.0008±.0005	.0010±.0003	.037	1.10±.60
22	1	.0002±.0002	—	.018	—
24	1	.0002±.0002	—	.0074	—
26	0	—	—	.0027	—
28	0	—	—	.0008	—
Total	5298	1.00	1.00	1.00	

2) The even-prong pd data might have a surplus of high prong number events if one or more particles coming from an inelastic interaction off the first nucleon made another inelastic interaction off the second nucleon. For simplicity we will assume such an effect is N-independent.

In order to look for these effects, we need to have precise knowledge of the multiplicity distributions in free pp and pn collisions. Results from pp collisions at 100 GeV/c exist in the literature [1]. We can use these results to predict the multiplicity distribution for free pn collisions, by making some assumptions based on known properties of high energy nucleon-nucleon collisions.

We let a_N, b_N, c_N, d_N be the probabilities that in an N-prong event from a pp collisions the two final state (non-produced) nucleons are pp, pn, np, nn, respectively. Then $b_N = c_N$ (from symmetry) and $a_N+b_N+c_N+d_N = 1$, by definition. The average number of (non-produced) protons and neutrons in the final state will be

$$\langle \mathrm{p}\rangle_N^{\mathrm{pp}} = 2(a_N+b_N),$$
$$\langle \mathrm{n}\rangle_N^{\mathrm{pp}} = 2(c_N+d_N). \tag{1}$$

We now assume that the final state nucleons are far enough separated in rapidity so that their charge states are uncorrelated with each other, but that they can be identified as belonging to leading particle states from the beam or target, respectively. If we let the probability that a target proton changes to a neutron be equal to the probability that a target neutron changes to a proton [2], and call this probability $1-X_N$, we can then identify

$$X_N = a_N+b_N = \tfrac{1}{2}\langle \mathrm{p}\rangle_N^{\mathrm{pp}},$$
$$1-X_N = c_N+d_N = \tfrac{1}{2}\langle \mathrm{n}\rangle_N^{\mathrm{pp}}. \tag{2}$$

The odd-prong pn topological cross sections are then related to the even-prong pp cross sections as follows:

$$\sigma_N^{pn} = X_{N+1}\sigma_{N+1}^{pp} + (1 - X_{N-1})\sigma_{N-1}^{pp}, \quad (N = 3, 5, 7 \ldots). \tag{3}$$

In reality we expect X_N to be at most a slowly varying function of N (except perhaps for small N), so from here on we will just call it X. (Our results are insensitive to this simplification.) Thus we have

$$\sigma_N^{pn}(\text{predicted}) = X\sigma_{N+1}^{pp} + (1 - X)\sigma_{N-1}^{pp}, \quad (N = 3, 5, 7 \ldots) \tag{4}$$

with $X = \langle\text{protons}\rangle/2$ in pp collisions, which is known to be in the range $X = 0.5$ to 0.6 [3, 4].

We can now compare our observed odd-prong distribution by means of the ratio

$$S_N = \sigma_N^{pn}(\text{observed})/\sigma_N^{pn}(\text{predicted}). \tag{5}$$

If there were no secondary interactions off the spectator proton we would expect $S_N \approx 0.7$ for all N, since about 30% of the spectator protons are visible and these events are classified as even-prongs. In practice we can only hope to detect an N-dependence of S_N, and not its absolute value, since the spectator visibility fraction is hard to define in an absolute way experimentally. Hence, we will arbitrarily normalize the observed and predicted σ_N to the same values and rewrite

$$S_N = P_N^{pn}(\text{observed})/P_N^{pn}(\text{predicted}) = (1 - NR_N)/(1 - \langle N\rangle R_N) \tag{6}$$

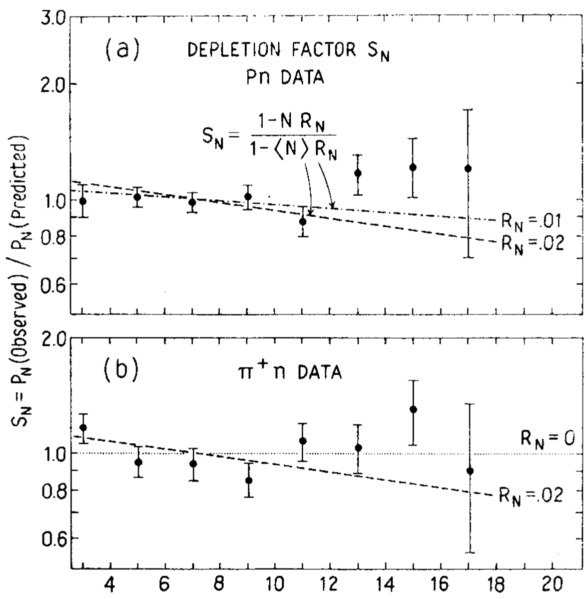


Fig. 1a. Odd prong pn data on the relative spectator invisibility fraction S_N. The dotted curves represent an interaction probability of 0.01 or 0.02 per charged track to make the spectator proton visible; b. Depletion factor for odd-prong π^+n data

where $\Sigma P_N = 1.0$ and R_N is a parameter to be extracted. The values we find for S_N are given in Table I which is based on equations (4)–(6). The input pp cross sections are taken from weighted averages of the values given in the two references [1].

The values of S_N from Table I are plotted in Figure 1a. We also show curves based on two values (0.01 and 0.02) for the R_N parameter. This can be interpreted as the probability, per charged track, for a particle produced on the neutron to interact with the spectator proton; thereby making the proton visible. Since the value of R_N is consistent with zero, we conclude there is no evidence in the present data for a secondary interaction with the spectator proton which depends on the number of particles produced off the neutron. Indeed the data appear to rule out values of R_N larger than about 0.01. Hence, to the extent that the points on Figure 1 fall on the line $S_N = 1.0$, the P_N^{pn} (observed) values in Table I can be taken as the free pn probability distribution.

Next we give, in Table II, our observed odd-prong distribution for π^+n collisions. Also given are the predicted π^+n values based on the measured π^-p cross section [7] and the analogy [8] to equation (4)

$$P_N^{\pi^+\mathrm{n}}(\text{predicted}) = XP_{N+1}^{\pi^-\mathrm{p}} + (1-X)P_{N-1}^{\pi^-\mathrm{p}}, \quad (N = 3, 5, 7\ldots). \tag{7}$$

The predicted values from (7) are given in Table II for $X = 0.5$, where $X = \langle\text{protons}\rangle^{\pi^-\mathrm{p}} = \langle\text{neutrons}\rangle^{\pi^+\mathrm{n}}$. The S_N values for π^+n are shown in Figure 1b. Again, we see no evidence for an N-dependent spectator depletion factor.

We turn now to a discussion of the even-prong pd data, which is formed from all the observed even-prongs, plus the odd-prongs with one added to the prong number. We look for a cascading effect; double interactions in which (one or more) high energy particles from the first interaction produce more particles off the second nucleon. Such events could be expected to have a multiplicity distribution similar to the independent convolution of the distributions from 100 GeV/c and 69 GeV/c pp collisions. (We assume that the leading nucleon from the first interaction is degraded in momentum and take its average momentum to be $\sim$69 GeV/c, a momentum at which the multiplicity distribution has been measured. We have also used 19 GeV/c as the momentum of the second interaction in order to see the sensitivity of our results to this assumption.) The probability distribution from such a convolution is given in Table III under $P_N(R = 1)$, where R is the fraction of collisions in which such double production takes place. We assume for simplicity that R is N-independent.

In order to look for such a double interaction effect, we need to compare our results with what would be expected from an average of free pp and pn collisions. Assuming the total inelastic pp and pn cross sections are equal, we can write for the idealized even-prong deuterium ($R = 0$) probabilities

$$P_N^{\mathrm{d}}(R = 0) = (\tfrac{1}{2})P_{N-1}^{\mathrm{pn}} + (\tfrac{1}{2})P_N^{\mathrm{pp}}, \quad (N = 4, 6, 8\ldots). \tag{8}$$

The predicted probability distributions $P_N^{\mathrm{d}}(R = 0)$ are given in Table III. For P_{N-1}^{pn} we can use our own data from Table I or, what amounts to the same thing, the $X = 0.5$ prediction, since that is a smooth curve through the data (see Fig. 1a).

The last column in Table III gives M_N (observed), where

$$M_N(\text{observed}) = P_N^{\text{d}}(\text{observed})/P_N^{\text{d}}(R = 0). \tag{9}$$

With this definition, M_N can be thought of as the multiplication factor due to the effect of double inelastic collisions in the deuteron.

The experimental values of M_N are plotted in Figure 2. Keeping in mind possible uncertainties in the predictions from equation (7), to the extent the M_N values depart from the line $R = 0$, we see definite evidence for double inelastic collisions.

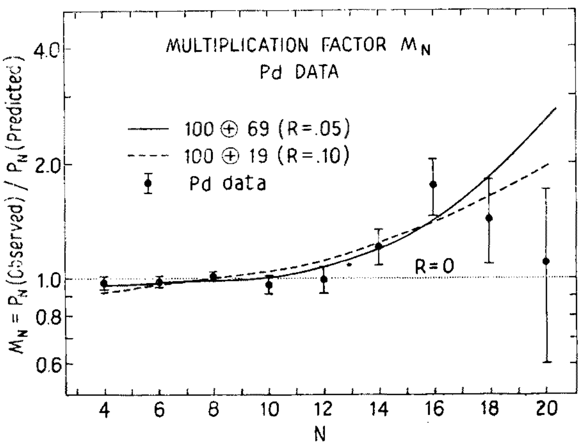


Fig. 2. Even-prong pd data on the multiplication factor M_N. The curves are the predictions of a double inelastic interaction model

In order to estimate the size of the double inelastic collision fraction R, we can use a linear combination of the $R = 0$ and the convoluted $R = 1$ distributions from Table II

$$M_N(R) = [(1-R)P_N^{\text{d}}(R = 0)+RP_N^{\text{d}}(R = 1)]/P_N^{\text{d}}(R = 0). \tag{10}$$

The predictions of equation (10) for M_N are shown on Figure 2 for $R = 0.05$ assuming a 69 GeV/c second inelastic collision and for $R = 0.10$ assuming a 19 GeV/c second inelastic collision. We conclude from this that there are indeed doubly inelastic collisions taking place at the level of about 5 to 10% of all pd inelastic interactions. This is in good agreement with the estimate one gets from a simple geometrical picture of the deuteron in which the fraction of double collisions by the beam particle would be given by $R \approx (\sigma_{\text{pp}}+\sigma_{\text{pn}}-\sigma_{\text{pd}})/\sigma_{\text{pd}}$. The experimental value for this is $R \approx 0.05$ at 100 GeV/c [5].

To summarize, by comparing our observed pd multiplicity distribution with the observed pp and estimated pn distributions, we are able to measure the fraction of double inelastic pd collisions to be $R \approx 0.05$ to 0.10. With our present odd-prong data we are not able to conclude much about a possible interaction of the spectator proton being proportional to the number of particles produced off the neutron, except to say that the probability R_N of such an interaction is less than about 0.01 per charged track. A small value for R_N is not inconsistent with our non-zero value for R since the probability of a double collision may very well be nearly N-independent [6]. We will say more about this later.

So far we have discussed what information can be gleaned about double interactions from the multiplicity distributions alone. There is perhaps more and better information contained in the angle and momentum distributions of the "spectator" protons, as well as in the produced particles themselves.

The fraction of all events which behave as if they were on free neutrons can be estimated from the distribution of backward spectators. Let us measure the fraction of true spectator events as follows:

$$f_{(\text{spec})} = \frac{(\text{observed odd}) + 2.5(\cos\theta < -0.2)1.11}{(\text{all } N \geq 3)} .$$

Here θ is the angle of the spectator proton with respect to the beam and 1.11 is a correction for the Moeller flux factor [10] associated with visible spectators. We use $\cos\theta < -0.2$ to get a pure sample of spectators uncontaminated by protons from peripheral pp collisions. Our preliminary value for $f_{(\text{spec})}$ is 0.43 ± 0.02. Since in the absence of double interactions one would expect about 51% of the $N \geq 3$ sample to come from neutrons, this indicates that in about 8% of the "neutron" interactions the proton is somehow also participating. Presumably, the same is true for the "proton" interactions; so in all about 15% of all the deuteron break-up reactions with $N \geq 3$ are in some respect double interactions in which both nucleons are active.

Summary and possible interpretations of results

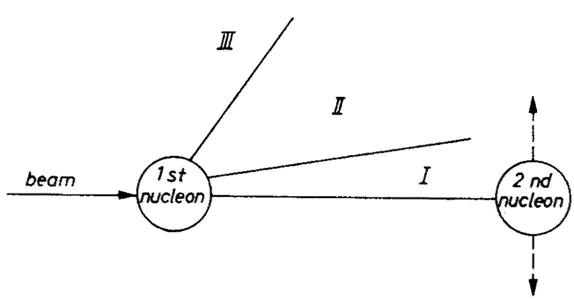
We have seen evidence for the following:

(1) Even-prong pd multiplicity data indicate about 5%–10% double inelastic collisions: ($R \sim 0.05$ to 0.10).

(2) $f_{(\text{spec})} \sim 0.43$ indicates about 15% total double (inelastic + elastic) interactions.

(3) Odd-prong multiplicity distributions show little or no evidence of N-dependent double interactions: ($R_N \gtrsim 0.01$ per charged track).

How can we interpret these very preliminary results? Taking a hint from recent experiments [9] we can suggest the following picture.



Let us call Z_N the probability that when N particles are produced off the first nucleon the second nucleon will also get struck. We define three regions of lab angle (or perhaps rapidity y) for the produced particles:

Region I. This is the small-angle region ($\theta \lesssim 3^\circ$, $y \gtrsim 3.5$). If $N_{\rm I}$ particles are produced in this cone they interact like just one (beam-like) particle; so $N_{\rm I}$ (effective) $= 1$. The probability that the second nucleon will be struck by these particles is just geometric and can be estimated from the total cross-section defect for the beam on deuterium. Such collisions are largely high energy and inelastic and give rise to about 5% double inelastic collisions ($R \approx 0.05$).

Region III. This is the wide-angle or target fragmentation region ($\theta > 30^\circ$, $y < 1.5$). If $N_{\rm III}$ particles are produced in this region they can interact individually, so that $N_{\rm III}$ (effective) $= N_{\rm III}$. However, the effects of these particles on Z_N will not depend on beam energy since the inclusive density function $(1/\sigma)\, d\sigma/dy$ is independent of beam energy in this region [11]. Moreover, these particles in Region III will cause very little dependence of Z_N on N. This is because the semi-inclusive functions $(1/\sigma_N)\, d\sigma_N/dy$ happen to be practically N independent in this region (see Fig. 68 of Ref. [4]). However, the particles in region III are very effective in making elastic and almost-elastic collisions with the second nucleon and contribute a lot to the 15% double collisions we find in deuterium.

Region II. If it were only for region I and region III, there would be practically no N-dependence to Z_N and we would find $R_N \approx 0$. However, we know there are a lot of particles produced in the region intermediate between I and III. These must obey $1 < N_{\rm II}$ (effective) $< N_{\rm II}$. They will produce some dependence of Z_N on N since the semi-inclusive density functions do depend on N in this region.

For the future, we hope to be able to use deuterium to study these effects in more detail in order to explore and understand the space-time development of produced hadronic matter. Unfortunately, the double interaction effects are quite small in the loosely-bound deuteron. On the other hand, the advantages of having at most only one extra collision — and then being able to study the recoiling second target — are considerable in comparison to the complicated effects which can go on in heavy nuclei.

I would like to thank my many colleagues in the Carnegie-Mellon, FNAL, Stony Brook, Michigan collaboration for data, discussions, and support. Particular thanks are due to Dr C. T. Murphy at FNAL who, as spokesman for the experiment, keeps such a wide-ranging collaboration viable. I must express my appreciation to the organizers of the XV Cracow School for a most enjoyable and informative conference.

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[6] Such N-independence would be in agreement with the smallness of cascading effects seen in high energy collisions in heavy nuclei. See Ref. [9].

[7] E. L. Berger et al., *Nucl. Phys.* **B77**, 365 (1974).

[8] In equation (10), unlike (4), no assumption of independent vertices is necessary, since $\pi^- p$ and $\pi^+ n$ are charge symmetric.

[9] Barcelona-Batavia–Belgrade–Bucharest–Lund–Lyons–Montreal–Nancy–Ottawa–Paris–Rome–Strasbourg–Valencia Collaboration, *Phys. Lett.* **48B**, 467 (1974); J. R. Elliott et al., *Phys. Rev. Lett.* **34**, 607 (1975); W. Busza et al., *Phys. Rev. Lett.* **34**, 836 (1975); K. Gottfried, *Phys. Rev. Lett.* **32**, 957 (1974) (and earlier references therein).

[10] At these energies we can take the flux factor to be proportional to the effective c. m. energy squared of the free pn collision.

[11] Note that at wide angles $d\sigma/dy$ is proportional to $d\sigma/d\Omega$; the latter probably being of more fundamental importance in region III.